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GEOMETRIC PARAMETERS FOR FINITE ELEMENT MODEL UPDATING OF JOINTS AND CONSTRAINTS

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The sensitivity method is applied to update finite element models of welded joints and the boundary condition of a cantilever plate. Careful parameterisation is found to be critical in updating joints and boundary conditions, and the merits of geometric parameters are given special consideration. The use of nodal offset dimensions results in an updated model of the welded joint with physical interpretation. Similarly the "rigid" boundary in a cantilever plate is successfully updated using the effective length of the elements closest to the joint. In all cases an improvement on the analytical natural frequencies is demonstrated.

1. INTRODUCTION

The updating of finite element models by using vibration test data is an area which has received considerable attention from researchers in recent years. A recent survey article [1] reported many advances, although the problem of improving finite element mechanical joint models remains very difficult in many cases. In their book, Friswell and Mottershead [2] gave more detail on most of the common updating methods.

Wang and Liou [3] and Hong and Lee [4] described a method which can be used for the characterisation of joints, using only measured frequency response function (FRF) data, and avoiding the use of a finite element model altogether. The resulting joint stiffness and damping parameters were expressed by the difference between two dynamic stiffness matrices [3]. Although the FRF measurements did not have to be inverted, the method was rendered impractical in many cases, where the lack of accessibility meant that the joint interface FRF data could not be measured. Wang and Sas [5] used Rayleigh quotient iteration with an eigenvector which was sensitive to a joint parameter. An analytical model was constructed which differed from the physical system only in the characterisation of the joint. This appears to be the major drawback of the approach, since in most problems parametric uncertainty exists in both the joints and the main structure simultaneously. Nobari *et al*. [6] updated joint models from natural frequency and mode shape measurements, using stiffness and mass parameters. They recommended ignoring "very" stiff joints in the udpating process and updating the joint model based on accurate

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substructure models. If necessary these substructure models may be obtained from a preliminary updating phase. Baruh and Boka [7] represented boundary conditions by springs, and identified the spring stiffnesses. The problems of ill-conditioning in the identification process was not dealt with explicitly, although the lack of sensitivity of the measurements to changes in the spring stiffnesses were demonstrated. Kim *et al*. [8, 9] used the incomplete experimental modal data and a condensed finite element model to identify discrete, linear spring stiffnesses and damping coefficients. Arruda and Santos [10] and Yang and Park [11] used non-linear optimisation of a penalty function based on the error in the predicted FRF data to identify similar stiffness and damping parameters. Mottershead and Shao [12] used a two step updating procedure to reduce the effects of discretisation error from the identification of a joint, modeled as a discrete spring. In references [3–12] the joint was represented by using discrete springs and dashpots. Ahmadian *et al*. [13, 14] presented a more general approach. Essentially the mass and stiffness of an element, or group of elements, are decomposed into the local modes. These local modal properties may then be used for updating. For example, the first mode of a welded 'T' shaped joint will be a bending mode. The flexibility of this mode, given by its eigenvalue, may be updated. Because the method generally considers the lower frequency modes of the joint substructures, the measured data are sensitive to the chosen parameters.

In the present paper, model updating of mechanical joints by using eigenvalue sensitivities is considered. Geometric parameters will be sought which produce sensitive eigenvalues in the frequency range of a physical test. The extent to which physical meaning can be assigned to the updated joint parameters will be investigated.

2. MODEL UPDATING USING EIGENVALUE SENSITIVITIES

It is well known [15] that the sensitivity of the ith eigenvalue, λ_i, to changes in the jth parameter, θ_j, can be calculated from knowledge of λ_i and the ith eigenvector, $\boldsymbol{\phi}_i$. Thus,

$$\frac{\partial \lambda_i}{\partial \theta_j} = \boldsymbol{\phi}_i^{\mathrm{T}} \left[\frac{\partial \mathbf{K}}{\partial \theta_j} - \lambda_i \frac{\partial \mathbf{M}}{\partial \theta_j} \right] \boldsymbol{\phi}_i \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass stiffness matrices respectively. The uncertain parameters may be updated by minimising the cost function,

$$J = (\delta\boldsymbol{\lambda} - \mathbf{S}\delta\boldsymbol{\theta})^{\mathrm{T}} \mathbf{W}_{\lambda} (\delta\boldsymbol{\lambda} - \mathbf{S}\delta\boldsymbol{\theta}) + \delta\boldsymbol{\theta}^{\mathrm{T}} \mathbf{W}_{\theta} \delta\boldsymbol{\theta}. \tag{2}$$

In equation (2), $\delta\boldsymbol{\lambda}$ is a vector containing the differences between the m measured and modeled eigenvalues, $\delta\boldsymbol{\theta}$ is the vector of small corrections to the n updating parameters, $\boldsymbol{\theta}$, and $\mathbf{S}$ is the $m \times n$ matrix of eigenvalue sensitivities. In this article we restrict our attention to the overdetermined problem, when $m > n$. The weighing matrices, $\mathbf{W}_{\lambda}$ and $\mathbf{W}_{\theta}$, reflect the confidence in the measurements and parameters respectively. Typically these are diagonal matrices, containing the reciprocal of the estimated variance of the measurements or parameters across the diagonal.

In this paper only eigenvalue measurements will be used for updating, and the mode shapes will only be used to pair the modes. There are three reasons for only using eigenvalues: the modes shapes contain considerably more measurement error than the eigenvalues; the mode shapes are very insensitive to changes in the parameters; and we have reduced the number of parameters such that an overdetermined problem may be obtained using eigenvalues only.

The mass and stiffness matrices may be given, at each iteration, in terms of the updating parameters by the first order Taylor series

$$\mathbf{M} = \mathbf{M}_0 + \sum_{j=1}^{n} \delta\theta_j \frac{\partial \mathbf{M}}{\partial \theta_j} \tag{3}$$

$$\mathbf{K} = \mathbf{K}_0 + \sum_{j=1}^{n} \delta\theta_j \frac{\partial \mathbf{K}}{\partial \theta_j}. \tag{4}$$

Certain choices of θ_j result in $\partial\mathbf{M}/\partial\theta_j$ and $\partial\mathbf{K}/\partial\theta_j$ being submatrices of $\mathbf{M}$ and $\mathbf{K}$ respectively. When $\mathbf{M}$ and $\mathbf{K}$ are non-linear functions of $\boldsymbol{\theta}$, then the matrices $\partial\mathbf{M}/\partial\theta_j$ and $\partial\mathbf{K}/\partial\theta_j$ will themselves be functions of $\boldsymbol{\theta}$, and the derivatives are evaluated using the parameter values estimated at the previous iteration.

3. THE WELDED JOINT

Two beams, each with a welded flange as shown in Fig. 1, were tested. The two beams were of unequal lengths (0.6 and 0.4 m) but had identical cross-sections (70 × 12 mm). The flange area (in both cases) was 110 × 70 mm and the thickness of both flanges was 6 mm. Pairs of bolt holes, of diameter 12 mm and 40 mm apart, were drilled 25 mm from the edge of the longer part of the flange and 15 mm from the edge of the shorter part of the flange, as shown in Fig. 1. Only vibration in a single plane was considered. The two beams were nominally identical, apart from their lengths. The beams were excited using burst random excitation and the natural frequencies identified using the complex mode indicator function. The resonances of the structures were lightly damped and well separated, making the natural frequency identification and mode shape pairing straightforward.

3.1. MODELING THE WELDED JOINT

The beams and flanges were modeled by using cubic beam elements. The nodes, at the ends, posess an axial and transverse translational degree-of-freedom together with a rotation in the same plane. The longer beam was represented by 12 elements and eight elements were used for the shorter beam. Each of the flanges was represented by five elements, and nodes were located to coincide with the bolt holes. The precise modeling of the welded joint is shown in Fig. 1. The shaded area is considered to be rigid and a

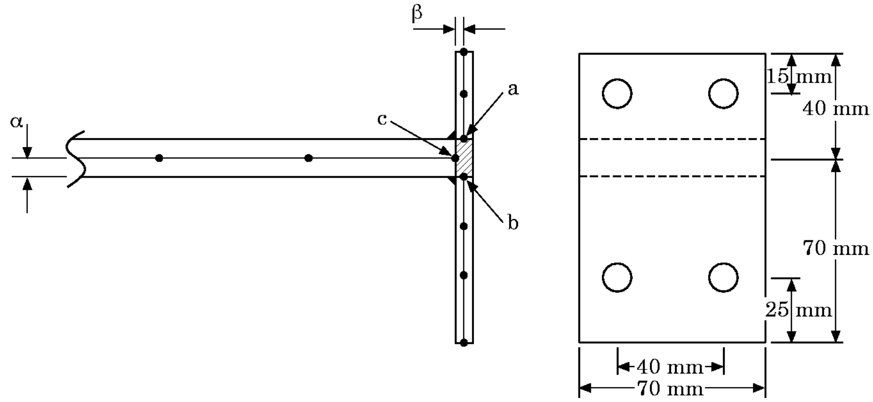


Figure 1. Modeling of the welded joint.

connection matrix linking nodes a, b and c can be written as,

$$\mathbf{C} = \left[\begin{array}{c|ccc|c} \mathbf{I} & & & & \\ \hline & 1 & & -\alpha & \\ & & 1 & \beta & \\ & & & 1 & \\ \hline & 1 & & \alpha & \\ & & 1 & \beta & \\ & & & 1 & \\ \hline & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ \hline & & & & \mathbf{I} \end{array}\right] \begin{array}{l} \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{a} \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{b} \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{c} \\ \\ \end{array} \tag{5}$$

The application of equation (5) results in nodes a and b being offset to coincide with node c. The mass and rotary inertia of the shaded (rigid) area were lumped at node c.

3.2. UPDATING PARAMETERS AND SENSITIVITIES

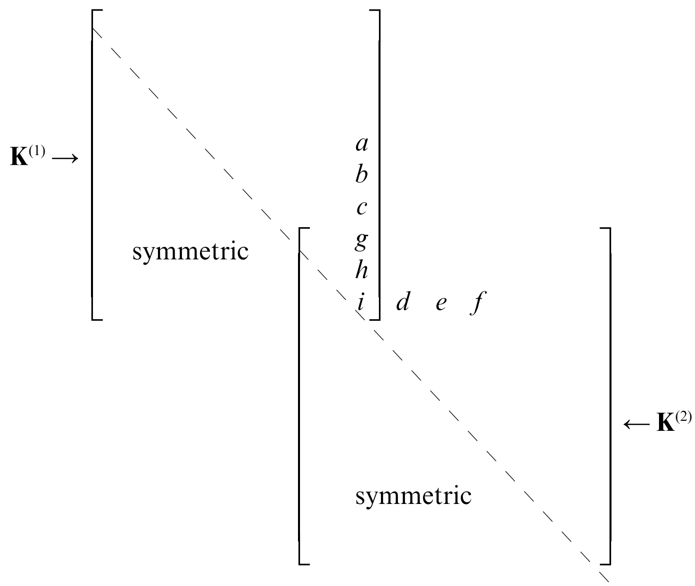
There are a number of ways in which the welded joint can be parameterised for updating. We will consider two alternatives: (i) the insertion of a translational spring, k_x, and a rotational spring, k_θ, between node c and the offset nodes (a and b); and (ii) updating of the offset dimensions α and β.

In the first of the two alternatives k_x and k_θ will be assumed to be uncoupled. They will also take values which are several orders of magnitude greater than any other terms in the stiffness matrix. This is because there is no distance separating node c from the offset nodes. It is straightfoward, from equation (1), to find that the sensitivity of the eigenvalues to a small change in k_x is,

$$\frac{\partial \lambda_i}{\partial k_x} = \boldsymbol{\phi}_i^T \begin{bmatrix} 0 & & & & & & & \\ & \ddots & & & & & & \\ & & 0 & & & & & \\ & & & 1 & -1 & & & \\ & & & -1 & 1 & & & \\ & & & & & 0 & & \\ & & & & & & \ddots & \\ & & & & & & & 0 \end{bmatrix} \boldsymbol{\phi}_i \quad \text{for } i = 1, \dots, m. \tag{6}$$

Since k_x is very large, the mode shape terms corresponding to the two ends of spring k_x will be almost identical. In that case equation (6) returns extremely low sensitivities for the eigenvalues, λ_i, for $i = 1, \dots, m$. The same result is found for eigenvalue sensitivity to k_θ.

When the offset dimensions are used as updating parameters then α and β appear, to the second order of products, as coefficients on the original finite element stiffness terms. The stiffness matrix may be obtained by using equation (5) to express the co-ordinates of nodes a and b in terms of the co-ordinates of c. Suppose that $\mathbf{K}^{(1)}$ is the 9×9 stiffness matrix of the substructure consisting of the shorter part of the flange joined to node a, where the last three co-ordinates are those of node a. Similarly let $\mathbf{K}^{(2)}$ be the 12×12 stiffness matrix of the longer part of the flange joined to node b, where the first three co-ordinates are those of node b. The contributions of these substructures to the stiffness matrix of the full structure may be given in terms of the generalised co-ordinates in the following order: the co-ordinates of the two nodes on the shorter flange (not node a), the co-ordinates of node c, and the co-ordinates of the three nodes on the longer flange (not node b). The stiffness terms including the co-ordinates of nodes a and b are written in terms of the co-ordinates of node c to give the following form for the corresponding stiffness submatrix



(7)

where

$$a = -\alpha k_{47}^{(1)} + \beta k_{48}^{(1)} + k_{49}^{(1)}$$

$$b = -\alpha k_{57}^{(1)} + \beta k_{58}^{(1)} + k_{59}^{(1)}$$

$$c = -\alpha k_{67}^{(1)} + \beta k_{68}^{(1)} + k_{69}^{(1)}$$

$$d = \alpha k_{14}^{(2)} + \beta k_{24}^{(2)} + k_{34}^{(2)}$$

$$e = \alpha k_{15}^{(2)} + \beta k_{25}^{(2)} + k_{35}^{(2)}$$

$$f = \alpha k_{16}^{(2)} + \beta k_{26}^{(2)} + k_{36}^{(2)}$$

$$g = -\alpha k_{77}^{(1)} + \beta k_{78}^{(1)} + k_{79}^{(1)} + \alpha k_{11}^{(2)} + \beta k_{12}^{(2)} + k_{13}^{(2)}$$

$$h = -\alpha k_{78}^{(1)} + \beta k_{88}^{(1)} + k_{89}^{(1)} + \alpha k_{12}^{(2)} + \beta k_{22}^{(2)} + k_{23}^{(2)}$$

$$i = -\alpha(-\alpha k_{77}^{(1)} + \beta k_{78}^{(1)} + k_{79}^{(1)}) + \beta(-\alpha k_{78}^{(1)} + \beta k_{88}^{(1)} + k_{89}^{(1)}) - \alpha k_{79}^{(1)} + \beta k_{89}^{(1)} + k_{99}^{(1)}$$
$$+ \alpha(\alpha k_{11}^{(2)} + \beta k_{12}^{(2)} + k_{13}^{(2)}) + \beta(\alpha k_{12}^{(2)} + \beta k_{22}^{(2)} + k_{23}^{(2)}) + \alpha k_{13}^{(2)} + \beta k_{23}^{(2)} + k_{33}^{(2)}$$

and $k_{jl}^{(1)}$ is the (j, l)th element of $\mathbf{K}^{(1)}$. Only the terms that contain α and β, and therefore affect the eigenvalue sensitivities, are given. It is important to note that the eigenvalue sensitivities are of the same order as the finite element stiffness terms, and are therefore

TABLE 1

The eigenvalue sensitivity to the welded joint parameters—long beam (θ_5 is the beam and flange thickness relative to the initial values)

Eigenvalue	$\frac{\partial\lambda_i}{\partial k_\theta}$	$\frac{\partial\lambda_i}{\partial\theta_5}$	$\frac{\partial\lambda_i}{\partial\alpha}$	$\frac{\partial\lambda_i}{\partial\beta}$
		Free–free		
1	1.2	-3.9×10^6	-4.4×10^6	-5.4×10^6
2	1.9×10^2	-2.7×10^6	-1.4×10^8	-5.4×10^7
3	7.3×10^3	-5.9×10^7	-9.9×10^8	-1.5×10^8
4	4.1×10^4	-1.5×10^8	-1.2×10^9	8.9×10^7
5	2.2×10^4	-3.6×10^8	1.1×10^9	-5.0×10^8
6	7.9×10^2	-6.0×10^8	-3.8×10^8	-1.7×10^9
		Clamped–free		
1	3.1×10^1	-2.6×10^4	1.6×10^6	-4.7×10^4
2	9.5×10^2	-1.0×10^6	5.2×10^7	-5.8×10^6
3	5.7×10^3	-7.9×10^6	3.3×10^8	-6.9×10^7
4	1.3×10^4	-3.0×10^7	7.3×10^8	-3.6×10^8
5	1.5×10^4	-5.5×10^7	-4.7×10^9	4.9×10^7

powerful in updating. The offset parameters have a physical meaning with regard to stiffness updating: the shaded (rigid) region in Fig. 1 can be considered to expand or contract depending upon whether the dimensions α and β are extended or reduced by updating. The mass matrix was not updated by using α and β.

The large magnitude of the sensitivity of the offset parameters may be easily demonstrated in this example. The sensitivity of the eigenvalues of the model of the longer beam to the offset parameters α and β, are given in Table 1. Also given in Table 1 are the sensitivity of the eigenvalues to the change in thickness of the beam and flange. To enable an accurate comparison of the sensitivities all the physical parameters are normalised so that their initial analytical estimates correspond to an updating parameter value of unity. Table 1 demonstrates that the eigenvalues are more sensitive to the offset parameters than to change in thickness. Table 1 also shows the sensitivity of the eigenvalues to a discrete boundary stiffness, k_θ, when $k_\theta = 10$ MNm/rad, and highlights the problem of low sensitivity associated with updating stiffness parameters.

3.3. UPDATING OF TWO WELDED JOINTS

The offset dimensions will now be used as updating parameters. The offset parameters have a physical meaning with regard to stiffness updating: the shaded (rigid) region in Fig. 1 can be considered to expand or contract depending upon whether the offset dimensions are extended or reduced by updating. The offset dimensions were assumed to affect only the stiffness matrix and the mass matrix is unaffected.

Updating of the mass matrix was performed by means of two parameters: the loss of mass at the two bolt holes; and the mass of the weld. A fifth parameter, the variation in the thickness of the beam and flange, was used to allow for a global shift in all the modeled natural frequencies. Table 2 summarises the parameters used for updating.

The beams were tested twice: under free suspension, and clamped at the flanges. Updating of the welds was carried out by using both sets of test data and a combined sensitivity matrix. Using two sets of boundary conditions increases the data, that is the number of natural frequencies, available for use in the updating procedure, providing that the change in boundary conditions does not introduce systematic errors. Further the free–free measurements are likely to give the most information on the inertia properties

TABLE 2

The parameters used in the updating of the welded joints

Parameter	Description	Initial value
θ_1	Offset dimension, α	6 mm
θ_2	Offset dimension, β	3 mm
θ_3	Loss of mass at two bolt holes	0 g
θ_4	Mass of the weld	0 g
θ_5	Change in beam thickness	0%

of the joint, and the clamped measurements are likely to give the most information on the stiffness properties.

Three sets of updating parameters, based on the parameters of Table 2, were tried: all five parameters; the three parameters θ_1, θ_2 and θ_5; and just the two offset parameters. The parameter weighting matrix was given by

$$\mathbf{W}_\theta = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 10^{11} & & \\ & & & 10^{11} & \\ & & & & 10^{-4} \end{bmatrix}.$$

These weighting matrices reflect our assumption that the loss of mass at the bolt holes and the mass of the welds was going to be small, and this was reflected by a weight of 10^{11} in both cases. The first two natural frequencies in both the free–free and the clamped tests were given a weighting of 10, and the remaining frequencies were given a unity weighting.

The converged values of the updating parameters are given in Tables 3 and 4, for the short and long beams respectively. Also given in Tables 3 and 4 are the experimental, analytical and updated natural frequencies. The results for all cases show small changes in the offset dimensions due to updating. However, the two beams with similar welded joints (but dissimilar in some other respects) provide converged offset dimensions which are quite similar to each other. This result appears to confirm the physical validity of the updated joint model. In case 1, the loss in mass at the bolt holes is compensated by a comparable gain in mass at the weld. This fact, together with the high weighting assigned to these parameters, gave the motivation for case 2, where these parameters are not updated. The quality of the updated model is not compromised by keeping these parameters at their initial zero values. If the thickness of the beams is also kept constant, case 3, then the quality of the updated model is degraded, confirming that a global parameter should be updated. In cases 1 and 2, the thickness only changes by about 3%, which is within the tolerance that the thickness of the beams was measured.

4. THE CANTILEVER PLATE

The updating of a welded joint in a beam has shown the advantages of using geometrical parameters for updating. The use of geometric parameters will now be tested on the updating of a cantilever plate. A steel plate, of width 0.23 m and thickness 2.5 mm, is supported along one side so that 0.3 m is free to vibrate, as shown in Fig. 2. The plate

TABLE 3

Natural frequencies and updated parameters for the welded joint in the short beam

	Natural frequencies (Hz)										Parameters				
	Free–free						Clamped–free				θ_1 (mm)	θ_2 (mm)	θ_3 (g)	θ_4 (g)	θ_5 (%)
Measured	324	823	1243	1975	3022	3898	56	354	986	1523					
Initial	318	813	1212	1940	2976	3833	55	349	972	1504	6	3	0	0	
Updated—case 1	325	827	1234	1975	3022	3898	56	355	987	1524	6.3	3.0	−10.9	10.5	−2.9
Updated—case 2	325	827	1235	1978	3023	3897	56	356	989	1525	6.4	3.0			−3.2
Updated—case 3	322	831	1217	1940	3024	3892	53	340	956	1559	1.8	3.6			

TABLE 4

Natural frequencies and updated parameters for the welded joint in the long beam

	Natural frequencies (Hz)											Parameters				
	Free–free						Clamped–free					θ_1 (mm)	θ_2 (mm)	θ_3 (g)	θ_4 (g)	θ_5 (%)
Measured	152	423	792	1120	1537	2183	26	160	451	875	1185					
Initial	150	415	780	1101	1513	2155	25	160	448	870	1175	6	3	0	0	
Updated—case 1	152	421	791	1119	1535	2185	26	162	454	881	1190	5.8	3.6	−10.0	9.98	−3.0
Updated—case 2	152	421	791	1119	1535	2185	26	162	454	881	1190	5.8	3.6			−3.0
Updated—case 3	152	421	791	1102	1524	2186	25	160	450	879	1189	4.2	3.9			

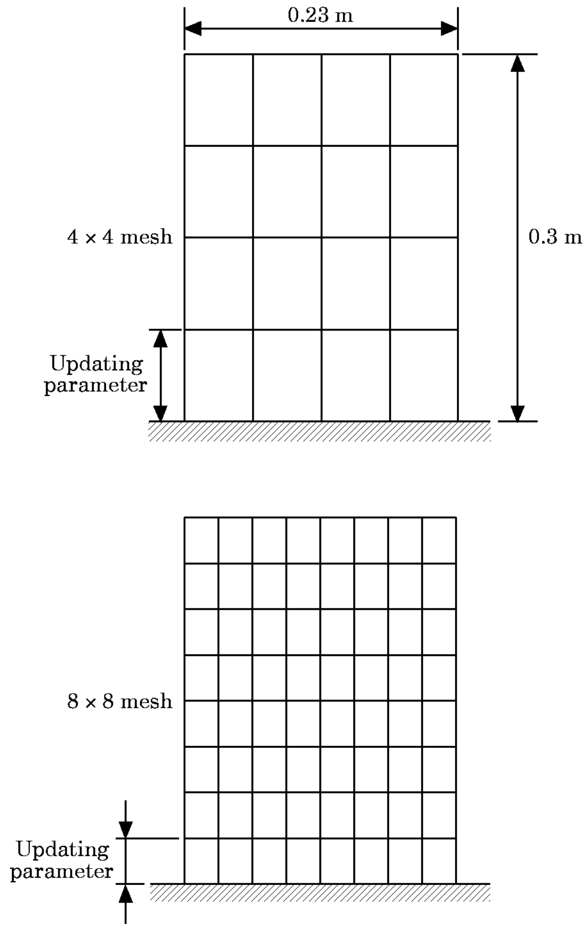


Figure 2. The cantilever plate example.

was supported between two rigid steel blocks and clamped using two bolts. The bolts were gradually tightened, which should increase the stiffness of the plate support. The natural frequencies of the plate were measured at four levels of bolt tightness, given in terms of bolt torque.

4.1. MODELING THE BOUNDARY

The plate is modeled using finite element analysis with either 16 or 64 elements, as shown in Fig. 2. A suitable parameter for the modeling of the joint using a geometric parameter is the plate length. Although this parameter has physical meaning, it may be thought that the dimensions of the plate could be measured accurately. In practice the 'rigid' support has some flexibility and our model allows for this by increasing the effective length of the plate. The alternative is to model the joint as an elastic foundation, although the eigenvalues will be insensitive to the associated stiffness parameters. A practical implementation of this foundation consists of a pinned boundary with a rotational stiffness. To demonstrate the improved sensitivity of the eigenvalues to the plate length consider three parameters: the plate length, obtained by varying the size of the row of elements nearest to the clamped boundary; the plate thickness; and an elastic foundation with rotational springs of magnitude 10 kNm/rad at the edge nodes, and 20 kNm/rad at

TABLE 5

Sensitivity of the eigenvalues to the cantilever plate parameters

Eigenvalue no.	Eigenvalue sensitivity to		
	Rotational stiffness	Plate thickness	Length of first row of elements
1	2.9	4.6×10^4	-2.3×10^4
2	4.6×10^1	4.2×10^5	-1.3×10^5
3	1.4×10^2	1.8×10^6	-8.9×10^5
4	5.4×10^2	4.9×10^6	-1.7×10^6
5	3.6×10^1	6.7×10^6	-4.0×10^5
6	9.8×10^2	1.5×10^7	-7.3×10^6

the internal nodes. Table 5 shows the sensitivity of the first six eigenvalues of the 16 element plate model to these parameters. To enable an accurate comparison, all the parameters have been normalised so that their initial value is 1. The sensitivity to the plate length is clearly of a similar magnitude as the sensitivity to the plate thickness, whereas the eigenvalues are very insensitive to the rotational stiffness of an elastic foundation.

4.2. UPDATING THE BOUNDARY

Only one parameter in each model will be updated, namely the length of the row of elements nearest the fixed support. Figure 2 shows the two meshes and highlights the geometric parameter to be updated. The natural frequencies were weighted equally in the updating cost function. Tables 6–9 show the measured natural frequencies, the initial finite element predictions and the updated frequencies. Also shown are the updated element lengths.

The updated finite element models accurately predict the measured frequencies. There is some finite element discretisation error, although this is small and has little effect on the updated plate dimensions. As the bolts are tightened the support becomes stiffer, the measured natural frequencies increase, and the updated plate length becomes smaller, as expected. Given that only a single parameter was updated the correspondence between the six measured and updated natural frequencies is remarkable, and gives confidence that the updated model accurately reflects the dynamics of the physical system.

5. CONCLUSIONS

Welded joints have been updated by using geometric parameters. A problem in the updating of welded joints is that they are stiff. The use of offset nodes, and updating of offset dimensions has been applied successfully. An updated joint with "physical" meaning is considered to have been achieved in this case.

The use of geometric parameters in updating was applied to the "rigid" boundary of a cantilever plate. The dimensions of the plate were successfully updated and produced an updated model which accurately reflected the measurements. As the support stiffness increased the effective plate length decreased, as expected.

The parameterisation of joints and boundary conditions is difficult. Measurements tend to be insensitive to stiffness parameters. Geometric parameters have considerable potential in updating. They have physical meaning and the measurements are sensitive to changes in them.

TABLE 6

Cantilever plate example for bolt torque 1 *Nm*

		Natural frequencies (Hz)						Element length
Experimental		21	65	131	229	289	379	
4 × 4	Initial	24	73	151	251	292	432	0.075
Mesh	Updated	21	67	132	229	287	379	0.0957
8 × 8	Initial	24	73	150	250	293	426	0.0375
Mesh	Updated	21	67	133	230	288	379	0.0562

TABLE 7

Cantilever plate example for bolt torque 13 *Nm*

		Natural frequencies (Hz)						Element length
Experimental		22	66	136	234	290	389	
4 × 4	Initial	24	73	151	251	292	432	0.075
Mesh	Updated	22	68	135	233	288	389	0.0914
8 × 8	Initial	24	73	150	250	293	426	0.0375
Mesh	Updated	22	69	136	235	289	389	0.0519

TABLE 8

Cantilever plate example for bolt torque 27 *Nm*

		Natural frequencies (Hz)						Element length
Experimental		22	66	136	235	290	390	
4 × 4	Initial	24	73	151	251	292	432	0.075
Mesh	Updated	22	68	136	234	288	390	0.0909
8 × 8	Initial	24	73	150	250	293	426	0.0375
Mesh	Updated	22	68	137	235	290	390	0.0514

TABLE 9

Cantilever plate example for bolt torque 81 *Nm*

		Natural frequencies (Hz)						Element length
Experimental		22	67	137	237	290	391	
4 × 4	Initial	24	73	151	251	292	432	0.075
Mesh	Updated	22	68	136	234	288	392	0.0904
8 × 8	Initial	24	73	150	250	293	426	0.0375
Mesh	Updated	22	68	137	236	290	391	0.0509

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