Damage detection using generic elements: Part II. Damage detection

Branislav Titurus a, Michael I. Friswell b,*, Ladislav Starek a

a Department of Technical Mechanics, Slovak University of Technology, Nam. slobody 17, Bratislava 81231, Slovakia
b Department of Aerospace Engineering, Queen’s Building, University of Bristol, University Walk, Bristol BS8 1TR, UK

Received 25 November 2002; accepted 3 July 2003

Abstract

This paper proposes the use of generic elements in damage detection, based on the use of an updated baseline finite element model, modal sensitivities and changes in the measured modal quantities arising from structural damage. A companion paper presented the fundamental theory of model updating and generic element or substructure parameterisation. The experimental structure was chosen so that a considerable difference existed between the baseline finite element model and the undamaged (healthy) structure. However the structure was successfully updated so that the model can be used effectively for damage detection and location. The main aim of this paper is to propose a method for the estimation of damage location and apply it to an experimental case study. The novel aspects of this approach are the type of parameterisation employed for the model updating and damage detection studies and the use of measured mode shapes to handle geometrical symmetry of the structure.

© 2003 Elsevier Ltd. All rights reserved.

Keywords: Generic elements; Parameterisation; Experimental modal analysis; Finite element method; Damage detection; Damage location

1. Introduction

Four distinctive features are addressed by the current pair of articles, namely the updating of imprecise models for subsequent use in damage detection/location, finite element model parameterisation by means of generic elements/substructures, damage location by means of parameter subset selection and the experimental demonstration of the approach. While the first paper [1] improved the baseline model using model updating so that it more accurately reflected the structure’s dynamics, this one applies that model and its parameterisations for the purposes of damage detection and location. Gladwell and Ahmadian [2] first proposed generic elements as a parameterisation in model updating, where the eigenvalues and/or eigenvectors of individual substructure stiffness matrices are modified. They also demonstrated that this parametrisation incorporates many standard parametrisations (for example, substructure parameters, physical parameters). Law et al. [3] applied generic elements in a simulated study of the Tsing Ma bridge in Hong Kong, by representing parts of the structure by super-elements. An eigensensitivity approach was used to update the model for the simulated case, and the influence of noise and modeshape incompleteness was investigated. However, improvements are needed because of the influence of model reduction for the super-elements. Wang et al. [4] gave probably the first application of generic elements to damage detection to date, and considered the simulated problem of damage detection in a frame structure with flexible L-shaped
and T-shaped structural joints. Damage was assumed to only occur at the joints, and the generic parameters were determined using translational degrees of freedom only. The parameters were estimated from the measured FRFs corresponding to the damaged and undamaged cases.

Parameter subset selection is a technique that selects the best subset of parameters from a candidate set, utilising some application dependent cost function that provides a measure of goodness of each subset. Often, these techniques only obtain a sub-optimal estimate of the best subsets in some sense due to the excessive computational burden posed by the original problem. These techniques are firmly rooted in statistics and related fields [5], although recently applications in structural mechanics have appeared. Friswell et al. [6] gave an overview of subset selection and also proposed the use of this technique for damage detection. The approach presented in this paper is based on forward parameter subset selection, which is especially suited to local damage. Friswell et al. [6] tested forward subset selection on a simulated cantilever beam example with physical parameters corresponding to either element or node properties. Different selection and iteration strategies were evaluated, and the case where multiple measurement sets are available was handled by computing the principal angles between two vector subspaces.

A wide range of experimental examples of vibration-based damage detection has been studied. Titurus [7] and Doebling et al. [8] gave comprehensive reviews of this area, including the techniques that may be applied and the different types of experiments performed. The example presented here is a welded frame structure, manufactured from thin-walled tubes of two different cross-section dimensions. The type of damage introduced mimics the real situation when damage occurs in the welded joints, although only single damage location cases will be considered. Throughout the paper it is assumed that the damage only influences the stiffness properties of the structure, represented by the global stiffness matrix, and that the mass properties will be unaffected by the damage.

2. Theoretical considerations

2.1. Computational equations

Generic element parameterisation is based on allowing changes to the eigenvalues and eigenvectors of the stiffness matrix of specific elements or substructures. The basis for this parameterisation is an eigenvalue decomposition of the selected element or substructure stiffness matrices of the form [1],

\[
\mathbf{K}^{\text{SUB}} = \mathbf{\Phi}_{05} \mathbf{S}_k^T \mathbf{A}_k \mathbf{S}_k \mathbf{\Phi}_{05}^T
\]

\[
= \mathbf{\Phi}_{05} \begin{bmatrix}
K_{1,1} & \cdots & K_{1,\text{nSUB-nS}} \\
\vdots & \ddots & \vdots \\
K_{\text{nSUB-nS},1} & \cdots & K_{\text{nSUB-nS},\text{nSUB-nS}}
\end{bmatrix}
\mathbf{\Phi}_{05}^T
\in \mathbb{R}^{\text{nSUB} \times \text{nSUB}}
\tag{1}
\]

where \( \mathbf{K}^{\text{SUB}} \) is element or substructure stiffness matrix, \( \mathbf{\Phi}_{05} \) is the eigenvector matrix corresponding to non-zero eigenvalues of the original \( \mathbf{K}^{\text{SUB}} \) matrix, \( \mathbf{A}_k \) is a diagonal matrix of the non-zero eigenvalues of the matrix \( \mathbf{K}^{\text{SUB}} \) and the matrix \( \mathbf{S}_k \) represents that part of the orthogonal transformation matrix between the modified and original sets of eigenvectors corresponding to the non-zero eigenvalues. \( k_{1,1}, \ldots, k_{(\text{nSUB-nS}), (\text{nSUB-nS})} \) are the parameters that may be used for model updating or damage detection. The total number of parameters may be reduced by assuming geometric symmetry or anti-symmetry of the relevant eigenvectors, or by choosing to change only particular eigenvalues. The dimensions of these matrices depend on the size of the chosen substructure, where \( n_{\text{SUB}} \) is the number of degrees of freedom of the substructure and \( n_S < 6 \) is the number of rigid body modes of the substructure.

The parameters from all of the generic elements or substructures may be assembled into a vector of parameters, having the form,

\[
\mathbf{p} = [p_1, p_2, \ldots, p_{n_p}]^T \in \mathbb{R}^{n_p}
\tag{2}
\]

where \( p_j \) is \( j \)th model parameter. Then, assuming a stationary and linear mechanical system, a class or family of mathematical models can be defined by the global mass and stiffness matrices \( \mathbf{M} \) and \( \mathbf{K} \). The contributions from the individual elements are added together to form the global matrices as,

\[
\mathbf{M} = \sum_{j=1}^{n_e} \mathbf{M}^{\text{exp},j} \in \mathbb{R}^{N \times N},
\]

\[
\mathbf{K} = \mathbf{K}^{\text{exp}} = \sum_{j=1}^{n_e} \mathbf{K}^{\text{exp},j}(\mathbf{p}) \in \mathbb{R}^{N \times N}
\tag{3}
\]

where \( \mathbf{M}^{\text{exp},j} \) and \( \mathbf{K}^{\text{exp},j} \) are the element mass and stiffness matrices in the global coordinate system, \( n_e \) is the total number of elements and \( N \) is a number of degrees of freedom for the whole structure. It should be mentioned that each \( \mathbf{K}^{\text{exp},j} \) is likely to be a function of only a few parameters, or possibly none at all.

The problem of vibration-based damage detection is to determine the health of the structure using identified dynamic quantities obtained from measurements. This characterisation places the problem of damage detection into the much broader context of inverse problems in engineering. One of the standard approaches for this
kind of problem is the inverse eigensensitivity algorithm based on a truncated, first-order Taylor series expansion of the chosen modal quantities [9]. This approach leads to the following sensitivity equation [6]

$$\mathbf{S}_h \delta \mathbf{p} = \delta \mathbf{z}$$  \hspace{1cm} (4)

where $\delta \mathbf{z} = \mathbf{z}_d - \mathbf{z}_h$, $\delta \mathbf{p} = \mathbf{p}_d - \mathbf{p}_h$, the subscripts h, d and u denote healthy (i.e. undamaged), damaged and updated model data, respectively, $\mathbf{S}_h = \mathbf{S}(\mathbf{p}_h)$ is the sensitivity matrix and $\mathbf{z}$ is the vector of identified modal quantities. Note we will assumed that the updated model is a good representation of the healthy structure, so that $\mathbf{p}_h \approx \mathbf{p}_u$, $\mathbf{S}_h \approx \mathbf{S}_u$, etc. Eq. (4) is the basis for damage location using the approach given in Section 2.2. In general, the vector $\mathbf{z}$ will have the following form:

$$\mathbf{z} = [\hat{\zeta}_1, \hat{\zeta}_2, \ldots, \hat{\zeta}_{n_p}, \phi_1^T, \phi_2^T, \ldots, \phi_{n_p}^T] \in \mathbb{R}^{2n_p(nu + 1)}$$  \hspace{1cm} (5)

where $\hat{\zeta}_j = \omega_j^2 \in \mathbb{R}$ and $\phi_j \in \mathbb{R}^{nu}$ are the $j$th eigenvalue and corresponding normal mode shape. Thus, the sensitivity matrix $\mathbf{S}_h$ is

$$\mathbf{S}_h = \begin{bmatrix} S_{1,1} & \cdots & S_{1,n_p} \\ \vdots & \ddots & \vdots \\ S_{n_p,1} & \cdots & S_{n_p,n_p} \\ S_{1,1} & \cdots & S_{1,n_p} \\ \vdots & \ddots & \vdots \\ S_{n_p,1} & \cdots & S_{n_p,n_p} \end{bmatrix} \in [\mathbb{R}^{n_p(nu + 1) \times (nu)}]$$  \hspace{1cm} (6)

where $n_p$, $n_M$ and $n_p$ represent the number of identified natural frequencies, measured degrees of freedom and model parameters, respectively. The sensitivities may be calculated as

$$S_{k,k,j} = \frac{\partial \hat{\zeta}_k}{\partial p_j} \bigg|_{\mathbf{p} = \mathbf{p}_h}, \quad S_{p,k,j} = \frac{\partial \phi_k}{\partial p_j} \bigg|_{\mathbf{p} = \mathbf{p}_h}$$  \hspace{1cm} (7)

and standard techniques are available for their computation [10,11].

2.2. Parameter subset selection

The need for an extremely reliable and precise mathematical model in the chosen frequency range is a characteristic feature of many model-based damage detection techniques. Often, however, the changes in the observed quantities between the healthy and damaged states are smaller than the differences between the predictions (from the original, detailed or updated models) and the measurements for the healthy structure. Consequently, it is almost impossible to differentiate the changes due to a mechanical fault from those due to modelling error. One solution to this problem is to use differences between the healthy and damaged states in Eq. (4). In this case, assuming that damage is the only reason for the change in output from $\mathbf{z}_h$ to $\mathbf{z}_d$, then the damage location may be formulated as a problem of parameter subset selection, where those parameters responsible for the measured changes are sought. It should be highlighted that the standard approach to subset selection is not iterative, but only uses Eq. (4), evaluated at the initial parameter values. It would be possible to update each candidate parameter set until convergence, and then compare the performance of the different subsets, although in practice the computational cost is prohibitive. As the model parameters are usually local in nature and may also allow for different damage mechanisms, parameter subset selection selects parameters from $\mathbf{p}$ that identify both the damage location and mechanism. This formulation requires the selection of the optimum parameter subset from $\mathbf{p}$. The most straightforward approach is to use an exhaustive search where all $(2^n - 1)$ possible cases have to be searched. The number of cases renders this approach computational intensive and thus impractical in many real situations. Consequently sub-optimal schemes have to be used. An additional problem is that the addition of a parameter to a previously selected subset inevitably reduces the residual in Eq. (4). Thus there is a trade off between the number of parameters selected and the magnitude of the residual.

Eq. (4) can be viewed as a set of linear equations

$$\mathbf{A} \mathbf{x} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_{n_p}] \mathbf{x} = \mathbf{b}$$  \hspace{1cm} (8)

This paper concentrates on the case of a single damage location, which leads to a simplified version of the above philosophy. When only one parameter is selected, the optimum parameter is that which best fits the changes due to damage characterised by the vector $\mathbf{b}$ in Eq. (8). Thus, the goal is to find the column $\mathbf{a}_j$ of matrix $\mathbf{A}$ that minimises

$$J = ||\mathbf{b} - a_j \hat{\mathbf{p}}||^2$$  \hspace{1cm} (9)

where $\hat{\mathbf{p}}_j$ is the least squares estimate of the $j$th parameter in $\mathbf{p}$. Friswell et al. [6] showed that minimising Eq. (9) is equivalent to finding the column of $\mathbf{A}$ that minimising the angle with $\mathbf{b}$. Hence the best parameter is the $j$th and found by

$$\min\{\psi_1, \psi_2, \ldots, \psi_{n_p}\} = \hat{\mathbf{p}}_j, \mathbf{a}_j$$  \hspace{1cm} (10)

and $\psi_i$ is the angle between vectors $\mathbf{a}_i$ and $\mathbf{b}$. This step is part of a general technique used in damage detection [6] and is called forward parameter subset selection. This is a sub-optimal technique of subset selection, starting with the above step and continuing by additional parameter searches where the already selected parameters are
The modified problem defined by Eq. (12), where a second parameter may now be selected by means of this cost function is also employed in Efroymson algorithm is thus created to search for the best parameter subset, denoted by \( \mathbf{J}_m \), and residual \( \varepsilon \) are

\[
\hat{\mathbf{p}}_{j_i} = \frac{\mathbf{a}_{j_i}^T \mathbf{b}}{\mathbf{a}_{j_i}^T \mathbf{a}_{j_i}} \Rightarrow \varepsilon = \mathbf{b} - \hat{\mathbf{p}}_{j_i} \mathbf{a}_{j_i}. \tag{11}
\]

Note that \( \varepsilon \) is orthogonal to \( \mathbf{a}_{j_i} \). A new parameter is then sought by considering the subspace defined by columns of \( \mathbf{A} \), but orthogonal to \( \mathbf{a}_{j_i} \). The modified problem is defined as [6],

\[
\mathbf{a}_j \rightarrow \mathbf{a}_j - \frac{\mathbf{z}_j \mathbf{a}_j}{\mathbf{a}_j^T \mathbf{a}_j} \quad \mathbf{b} \rightarrow \mathbf{b} - \hat{\mathbf{p}}_{j_i} \mathbf{a}_{j_i} \tag{12}
\]

where \( \mathbf{z}_j = (\mathbf{a}_{j_i}^T \mathbf{a}_{j_i})/(\mathbf{a}_{j_i}^T \mathbf{a}_{j_i}) \).

A second parameter may now be selected by means of the modified problem defined by Eq. (12), where \( j \neq j_i \). Further parameters may be selected in the same way. An algorithm is thus created to search for the best parameter subset, denoted by \( \{\mathbf{p}_{j_i}, \mathbf{p}_{j_2}, \ldots, \mathbf{p}_{j_m}\} \), minimises the cost function,

\[
\mathbf{J}_m = \left\| \mathbf{b} - \sum_{i=1}^{m} \hat{\mathbf{p}}_{j_i} \mathbf{a}_{j_i} \right\|^2 \tag{13}
\]

This cost function is also employed in Efroymson’s algorithm for forward subset selection, which focuses on adding or removing parameter selections from chosen subsets. Thus the number of candidate damage locations may be controlled based on the expected reduction in the residual [5,6]. Since only single damage location cases will be examined in detail here this subject will not be considered further.

### 2.3. Weighting

The final theoretical aspect is the need for weighting when Eq. (4) is used for damage location, and two types of weighting will be considered. First, weighting is needed to handle the different numerical values corresponding to the different modal quantities. Thus, only relative, or percentage changes in the modal quantities, due to damage, will be employed. The second type of weighting arises as a result of combining two different entities in the sensitivity matrix, namely natural frequencies and mode shapes for complete damage location. The experimental origin of the measurements means that the errors in mode shape estimation are usually greater than the errors in natural frequency estimation. This weighting is employed in the calculation of the subspace angles between the vector \( \delta \mathbf{z} \) and the columns of the matrix \( \mathbf{A} \), Eq. (10), and is based on the weighted scalar product [12]. The procedure is also called the scalar \( A \)-based product and has its origins in statistics (note that the \( A \) in the name of this product has nothing to do with the matrix \( \mathbf{A} \) in Eq. (8)). Knyazev and Argentati [12] studied this scalar product in the context of the numerically stable computation of principal angles between two linear subspaces. The scalar \( A \)-based inner product is defined as

\[
(\mathbf{x}, \mathbf{y})_A = (\mathbf{x}, \mathbf{A}_w \mathbf{y}) = \mathbf{y}^T \mathbf{A}_w \mathbf{x} \tag{14}
\]

where \( \mathbf{x}, \mathbf{y} \in \mathbb{R}^n \) are vectors, and \( \mathbf{A}_w \in \mathbb{R}^{n \times n} \) is a symmetric, positive definite matrix. \( A \)-based vector and matrix norms, \( \| \ldots \|_A \), may be defined as

\[
\| \mathbf{x} \|_A = (\mathbf{x}, \mathbf{x})_A = \| \mathbf{A}^{1/2} \mathbf{x} \|, \quad \| \mathbf{B} \|_A = \| \mathbf{A}^{1/2} \mathbf{B} \mathbf{A}^{-1/2} \|
\]

where \( \mathbf{B} \in \mathbb{R}^{n \times n} \) is an arbitrary matrix.

The applicability of this type of product for damage location based on the additional use of mode shape sensitivities in \( \mathbf{S} \) and mode shape differences due to damage will be studied for an experimental, geometrically symmetric structure in a later section. Since \( \delta \mathbf{z} \) is derived from experimental data, and assuming that the mass distribution does not change with damage, no additional scaling of individual mode shapes, with respect to other modes, will be employed. Since the natural frequencies are measured much more accurately than the mode shapes, the natural frequencies should be used to determine the candidate damage locations. The \( A \) weighting on the mode shapes is then used for geometrically symmetric structures to ensure that the most likely damage location from among the candidate locations identified from the natural frequencies is chosen. The weighting of the mode shapes is increased until a perceptible difference occurs between these candidate locations, but is kept as low as possible to reduce the effect of the noise on the mode shapes.

### 3. Experimental structure

#### 3.1. Geometry and experimental setup

The structure chosen to evaluate the proposed strategy consisted of four thin-walled tubes connected to each other by four fillet welds. These joints were intentionally manipulated to produce one healthy and six damage cases. Titrus et al. [1] gave a detailed discussion of the identification results for the healthy/undamaged structure. This paper describes both the damage cases, as well as the results obtained by experimental modal analysis performed on the damaged structure. Fig. 1 shows the experimental structure, together with the experimental (EMA) measurement locations. The finite element (FEM) nodes were placed at the measurement locations. Thus 32 degrees of freedom were measured, whereas the FE model contained 96 degrees of freedom.
(three degrees of freedom per node). The in-plane dynamics of the structure were measured, and the structure was supported in the free–free condition by elastic bands.

3.2. Damage cases

Table 1 gives a detailed description of all of the damage cases. These cases were produced by the intentional incompleteness of one or more of the fillet welds used to interconnect the four tubular parts of the structure. A distinctive feature of this structure is its geometrical symmetry, which is likely to cause problems for damage location based on measured natural frequencies alone. Two different approaches will be evaluated; the first assumes that the influence of the transducer mass will be sufficient to break the symmetry, whilst the second uses the measured mode shapes and their associated sensitivities. However, partial damage location will be tried, based on the use of natural frequencies alone.

These damage cases were selected to give a reasonable coverage of all possible combinations of damage cases, within practical constraints. This paper concentrates on the single location damage case, and so only state VII, VI and V from Table 1 will be studied in detail. The remaining cases will be used occasionally for demonstration purposes.

3.3. Identification results for the damage cases

A full modal test was performed for each of the damage cases shown in Table 1, however as the number of results is large only a selection will be considered here. The measurements were performed in the frequency range from 0 to 625 Hz and Fig. 2 shows an example of the natural frequency shifts due to damage, for all of the damage cases. Here, the shift in the fourth natural frequency using the point receptance at node 12 is shown.

Table 2 gives the first nine measured natural frequencies for all of the damage cases. The fifth and sixth modes swap order between damage states III and IV. The last column corresponds to the undamaged/healthy structure, that is the structure with fully welded joints. Generally, the natural frequencies decrease with increasing level of damage, as a result of the decreasing stiffness of the structure. However, some small increases were observed in some natural frequencies from one case to another. One possible reason might be a small decrease in mass due to the absence of some weld material. Alternatively, taking the structure from the free–free suspension to undertake the welding may give small frequency changes due to slightly different suspension conditions.

Fig. 3 shows the modal assurance criteria (MAC) matrices between the reference mode shapes of the healthy structure and mode shapes corresponding to the
damaged cases. It is clear that the fifth and sixth modes interact and swap over for damage states I, II and III. Another interesting feature shown by the MAC matrices is the relative insensitivity of the mode shapes to increasing damage, despite large changes in the natural frequencies. Indeed the mode shapes for state I correlate better with the healthy mode shapes than those belonging to some of the intermediate damage cases, most likely due to the symmetry of the damage inflicted for state I (see Table 1). Titurus et al. [1] gave other experimental results, in particular the mode shapes corresponding to the healthy structure and further discussion of modelling issues.

4. The analysis of an experimental model

4.1. Parameterisation overview

Titurus et al. [1] provided a detailed explanation of parameterisations to be used for damage location, however, for the sake of completeness, a summary is provided here. Parameterisation A is expressed in terms of two groups of generic elements. The first group consists of one generic substructure that models the parts of the structure containing the fillet welds, and two parameters are required for this substructure, as shown in Fig. 4. The other group consists of three different generic elements.

### Table 2

<table>
<thead>
<tr>
<th>State</th>
<th>State I</th>
<th>State II</th>
<th>State III</th>
<th>State IV</th>
<th>State V</th>
<th>State VI</th>
<th>State VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.63</td>
<td>33.57</td>
<td>34.18</td>
<td>48.60</td>
<td>50.26</td>
<td>60.06</td>
<td>60.57</td>
</tr>
<tr>
<td></td>
<td>(-54.38)</td>
<td>(-44.58)</td>
<td>(-43.57)</td>
<td>(-19.76)</td>
<td>(-17.02)</td>
<td>(-0.83)</td>
<td>(0)</td>
</tr>
<tr>
<td>2</td>
<td>118.64</td>
<td>120.94</td>
<td>120.74</td>
<td>125.04</td>
<td>124.82</td>
<td>126.60</td>
<td>126.53</td>
</tr>
<tr>
<td></td>
<td>(-6.24 )</td>
<td>(-4.42)</td>
<td>(-4.57)</td>
<td>(-1.18)</td>
<td>(-1.35)</td>
<td>(0.06)</td>
<td>(0)</td>
</tr>
<tr>
<td>3</td>
<td>126.38</td>
<td>129.92</td>
<td>130.64</td>
<td>138.97</td>
<td>139.63</td>
<td>147.86</td>
<td>147.05</td>
</tr>
<tr>
<td></td>
<td>(-14.05)</td>
<td>(-11.65)</td>
<td>(-11.16)</td>
<td>(-5.49)</td>
<td>(-5.05)</td>
<td>(0.44)</td>
<td>(0)</td>
</tr>
<tr>
<td>4</td>
<td>169.77</td>
<td>172.28</td>
<td>172.32</td>
<td>174.77</td>
<td>175.42</td>
<td>175.86</td>
<td>175.89</td>
</tr>
<tr>
<td></td>
<td>(-3.48 )</td>
<td>(-2.05)</td>
<td>(-2.03)</td>
<td>(-0.64)</td>
<td>(-0.27)</td>
<td>(-0.02)</td>
<td>(0)</td>
</tr>
<tr>
<td>5</td>
<td>264.77</td>
<td>275.73</td>
<td>275.42</td>
<td>280.09</td>
<td>280.33</td>
<td>280.81</td>
<td>280.76</td>
</tr>
<tr>
<td></td>
<td>(-17.40)</td>
<td>(-13.98)</td>
<td>(-14.08)</td>
<td>(-0.24)</td>
<td>(-0.15)</td>
<td>(0.01)</td>
<td>(0)</td>
</tr>
<tr>
<td>6</td>
<td>279.64</td>
<td>280.38</td>
<td>280.24</td>
<td>300.44</td>
<td>301.72</td>
<td>319.60</td>
<td>320.56</td>
</tr>
<tr>
<td></td>
<td>(-0.40 )</td>
<td>(-0.14)</td>
<td>(-0.19)</td>
<td>(-6.28)</td>
<td>(-5.88)</td>
<td>(-0.30)</td>
<td>(0)</td>
</tr>
<tr>
<td>7</td>
<td>298.91</td>
<td>312.58</td>
<td>317.15</td>
<td>342.10</td>
<td>347.97</td>
<td>359.55</td>
<td>360.70</td>
</tr>
<tr>
<td></td>
<td>(-17.13)</td>
<td>(-13.34)</td>
<td>(-12.07)</td>
<td>(-5.16)</td>
<td>(-3.53)</td>
<td>(-0.32)</td>
<td>(0)</td>
</tr>
<tr>
<td>8</td>
<td>393.61</td>
<td>396.78</td>
<td>399.06</td>
<td>418.68</td>
<td>420.05</td>
<td>436.24</td>
<td>437.72</td>
</tr>
<tr>
<td></td>
<td>(-10.08)</td>
<td>(-9.35)</td>
<td>(-8.83)</td>
<td>(-4.35)</td>
<td>(-4.04)</td>
<td>(-0.34)</td>
<td>(0)</td>
</tr>
<tr>
<td>9</td>
<td>550.55</td>
<td>551.78</td>
<td>552.71</td>
<td>560.06</td>
<td>560.63</td>
<td>565.93</td>
<td>566.52</td>
</tr>
<tr>
<td></td>
<td>(-2.82 )</td>
<td>(-2.60)</td>
<td>(-2.44)</td>
<td>(-1.14)</td>
<td>(-1.04)</td>
<td>(-0.10)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

The quantities in brackets represent percentage differences compared to state VII, allowing for the frequency swap due to damage between modes 5 and 6.

Fig. 2. The shift in the fourth natural frequency due to damage (FRF: 12y/12y).
elements, each requiring one parameter, as shown in Fig. 4. Thus, parameterisation $A$ requires the parameter vector $p$ given by

$$p = [p_1, p_2, p_3, p_4, p_5]^T = [\kappa_{11}^1, \kappa_{22}^1, \kappa_{11}^2, \kappa_{11}^3, \kappa_{11}^4]^T$$

(16)

where $\kappa_{ij}^k$ denotes the $(j,k)$ element of the matrix of the $i$th element/substructure, based on generic elements as detailed in Eq. (1). The values of these parameters may be determined by model updating [1]. This parameterisation allows partial localisation to the type of region where damage has occurred.

Fig. 3. MAC criterion between state VII and damage states I to VI. The rows of each MAC matrix correspond to mode shapes of state VII while columns correspond to mode shapes of states I–VI. (a) State VII and state VII (auto MAC), (b) state VII and state VI, (c) state VII and state V, (d) state VII and state IV, (e) state VII and state III, (f) state VII and state II, (g) state VII and state I.

Fig. 4. Parameterisation $A$ of the baseline model of the thin-walled $H$ structure.
Parameterisation $B$ allows similar elements or sub-structures to have independent values of the corresponding generic parameters, to enable complete damage localisation. Parameterisation $B$ requires 28 parameters for the $H$-shaped structure, as shown in Fig. 5, and these are defined as

\begin{equation}
\begin{aligned}
p_{2i-1} &= k_{11}^i, & p_{2i} &= k_{22}^i, & i &= 1, 2, 3, 4 \\
p_{j+4} &= k_{11}^j, & j &= 5, 6, \ldots, 24
\end{aligned}
\end{equation}

**4.2. Partial damage location—natural frequencies alone**

In this section, parameterisation $A$ will be used for partial damage localisation, using only the measured natural frequencies. This simplified form of damage localisation is chosen as a first step in the damage detection of a geometrically symmetric structure. The first seven natural frequencies corresponding to the healthy and damaged structures, as well as the sensitivity matrix $S$, determined at the updated parameters values [1], were used to test the approach proposed in Section 2.2. Both the sensitivity matrix and the measured differences $\delta z_j = z_{d,j} - z_0$ for the considered damage cases, were normalised by the corresponding measured natural frequencies. Since single-location damage states are of primary interest, state VI and state V will be used, as they represent two levels of damage in one fillet weld. State IV and II will also be considered, as these are multi-location damage states with different levels of damage (see Table 1).

The results of damage location, in the form of subspace angles, are shown in Fig. 6. Individual groups of columns correspond to particular model parameters. Within each group, corresponding to each parameter, the columns represent different comparisons of the damage cases (states II, IV, V, VI) with the healthy structure (state VII). Fig. 6 suggests that the damage corresponds to parameter $p_1 = k_{11}^1$, which corresponds to the generic substructures containing the welded joints. Thus the damage is correctly localised to the welded joint. An important feature of this study is that even relatively small damage corresponding to state VI is readily observable and clearly identifiable. An increasing level of damage, represented here by state V, leads to improved and clearer identification of damage location or damage type.

The results disagree with the expectation that the increasing level of damage should lead to increasing subspace angles and consequently to a deteriorating quality of damage localisation, due to the increasing error in the linearisation of the original non-linear problem. However the level of damage for state VI is relatively small (the maximum difference between state VII and state VI is $0.83\%$ for the first natural frequency, see Table 2) and therefore susceptible to measurement error. State V is characterised by a larger extent of damage, and therefore large differences in the natural frequencies (the maximum difference for this combination is $17.02\%$ for the first natural frequency), produces smaller subspace angles. Fig. 6 also gives the subspace angles for state II and state IV, and the angles corresponding to parameter 1 are smaller still. Although
these are multi-damage cases, the damage still lies in the joints. Another noticeable and beneficial feature of the results is the insensitivity of the subspace angles from other parameters due to damage in the welded joints, reducing the possibility of false alarms.

It was observed throughout the EMA that some natural frequencies were influenced by experimental limitations more significantly than others, in particular modes 2, 3 and 7. If these problem natural frequencies are not used for the damage detection then the results should improve. Fig. 7 shows the subspace angles when only natural frequencies 1, 4, 5 and 6 are used, and this demonstrates a significant improvement. Now the subspace angles corresponding to parameter $p_1$ decreases as the level of damage increases to state VI, due the influence of measurement noise. However, the subspace angles increase slightly for states IV and II due to the errors from the linearisation.

Figs. 8–10 provide additional information in terms of selection trees. A selection tree is a representation of the forward parameter subset selection where each node of the tree corresponds to a selected subset and its colour represents the numerical value of the residual. The root of the tree represents the initial system, Eq. (8). The branching factor and the depth of the tree are decided in

Fig. 7. Subspace angles for partial localisation computed using selected natural frequencies, parameterisation $A$.

Fig. 9. Binary tree representing forward selection, state VII vs. state V.

Fig. 10. Binary tree representing forward selection, state VII vs. state IV.
advance and in our case results in binary selection trees with three levels corresponding to the selected parameter subsets. All three figures are determined using the first seven natural frequencies. The second best single parameter would be $p_5$, a parameter that also effectively monitors the stiffness in the regions connected with welded joints, as will be shown later. The important relative indicators of damage level for a given situation are the absolute values of the residuals provided by the colour bars on the left of the Figs. 8–10, as the ability to reproduce the vector $\delta z$ decreases with increasing damage level and consequently the magnitude of residuals also increases.

4.3. Complete damage detection—natural frequencies alone

The parameter values of parameterisation $B$ were determined by model updating for damage state VI and state V (compared to the healthy state VII), using the first seven non-zero natural frequencies. Note that the model of the healthy structure was first updated by assuming constraints between the parameters, such that only 11 of the 28 parameters were independent [1]. Fig. 11 shows the resulting subspace angles for the 28 parameters. It is clear that the damage location has not been estimated uniquely due to the geometric symmetry of the structure. For a purely symmetric structure, the same change to any of the four welded joints would result in the same change in natural frequencies. The ability to localise damage correctly, i.e. attribute it uniquely to parameter $p_7$ (the actual damage location), has been lost even in the case of a relatively large level of damage represented by state V and a slight asymmetry introduced by the presence of the accelerometers.

To further investigate the effect of the accelerometer mass and measurement noise, a simulated example of the structure, using parameterisation $B$, was considered. The damage was simulated by a reduction of 5% in parameter $p_7$ in the finite element model. The first seven natural frequencies were used for the damage localisation. Two cases were considered, namely with and without the presence of the accelerometer mass. Fig. 12 shows the resulting subspace angles, and it is clear that the asymmetry introduced by the accelerometer mass is sufficient (with no measurement noise) to enable unique damage location, although the localisation is somewhat limited. Measurement noise is always present in real experiments. This noise was modelled as a random signal with a normal distribution, having zero mean and a 0.1% standard deviation, and was added to the computed first seven natural frequencies. The 0.1% level of noise was chosen to be representative of the discretisation of the frequency domain in the experiments. The noise was added to the natural frequencies of both the damaged and healthy structures. The statistics of the resulting subspace angles are represented by their means and standard deviations and these were computed from

![Fig. 11. Subspace angles computed by means of the first seven non-zero natural frequencies, real experiment, parameterisation $B$.](image1)

![Fig. 12. The influence of the accelerometer mass on the subspace angles for the simulated experiment, 5% damage in parameter $p_7$, 0% noise, parameterisation $B$.](image2)
1000 samples using Monte Carlo simulation. The results are shown in Fig. 13, where the mean values are represented by vertical bars and the error lines represent ±1 standard deviation. Clearly even with a very small level of measurement noise the damage cannot be localised uniquely. Fig. 14 is similar to Fig. 13, except that the level of damage has been increased to 20%. Even in this case of substantial damage and little measurement noise, damage location is barely possible. This means that the asymmetry due to the presence of sensors in an actual experiment is not sufficient for unique localisation of damage.

4.4. Complete damage detection—natural frequencies and mode shapes

The only way to deal with damage localisation for geometrically symmetric structures is to use spatial information in the form of mode shapes. Once again the analysis was limited to the single location damage case, i.e. state VII (healthy), state VI (level 1 damage at welded joint 4, see Fig. 1a) and state V (level 2 damage at welded joint 4). The subspace angles corresponding to the individual parameters (parameterisation B) were computed by the techniques presented in Section 2.2. However, since the vector \( dz \) and the sensitivity matrix \( S \) contain elements corresponding to both the natural frequencies and the mode shapes, additional weighting must be included to represent the relative importance of the natural frequency and mode shape information, as presented in Section 2.3.

There are problems in using mode shape information, particularly since the accuracy of their estimation from measured data is worse than for natural frequencies. This is compounded since the proposed approach uses the differences between the measured damaged and measured undamaged mode shapes. Thus only mode shapes that are sensitive to the candidate damage sites should be chosen. Table 2 shows the changes in the natural frequencies for the different damage states, and gives a good indication of this sensitivity. However the table shows the sensitivity of the natural frequencies, which is not necessarily the same as the sensitivity of the mode shapes. Certainly, if the natural frequencies change very little with damage, then the corresponding mode shapes will not be sensitive. Thus, of the first seven modes, modes 2, 3, 4 and 5 are unlikely to give useful spatial information (note that mode 3 has been excluded because of the slight increase in the natural frequency in state VI). The sensitivity of the mode shapes to damage also increases with mode number, as the mode shapes corresponding to higher frequencies, contain more local deformation. Mode 1 is a global mode and therefore its shape is insensitive to damage.

The proposed approach using mode shapes will be demonstrated using spatial information from mode 6. Relative errors in the first seven natural frequencies and the difference in the mode shape elements were used. No further weighting was included, as similar results were obtained with other weighting values. Figs. 15 and 16 show the subspace angles corresponding to damage state VI and state V, respectively. Both figures provide the
correct indication of damage location, corresponding to parameter $p_7$. This parameter belongs to the fourth generic $T$ substructure, representing fillet weld number 4 (see Fig. 5). The damage location is more clearly identified in the case of higher damage, in accordance with earlier observations. Other significant parameters indicated by the subspace angles are parameters $p_{23}$ to $p_{28}$, which are located on the crossbar neighbouring the damaged region. However the results from the frequency only estimation, Fig. 11, clearly indicates the damage to be located in the joints. Thus damage in welded joint 4 may be confidently predicted.

4.5. Summary of the results

Two approaches have been considered for damage localisation. Partial localisation only identifies the type of damage location due to the geometric symmetry of the structure. The application of this approach to a real structure, even with small damage, was shown to be successful. It was shown in Section 4.3, that this technique cannot fully localise damage or to distinguish between multiple damage regions. However this technique can lead to a considerable reduction in candidate damage locations for further investigation. The second proposed approach is concerned with complete damage localisation and is based on the use of spatial information in the form of mode shapes. The use of the weighted scalar product to determine the subspace angles was proposed in this case, to improve the conditioning of the experimental data. It was shown that even data significantly corrupted by experimental noise on the elements of the measured mode shapes was able to produce useful localisation, especially in case of geometrically symmetric structures. A good correspondence between the real experiment data and simulated examples of the proposed techniques was shown. An important conclusion is that despite an increasing linearisation error corresponding to an increasing level of damage, the relative quality of damage location does increase in a real environment influenced by inevitable experimental noise.

5. Conclusion

This paper has provided an approach to handle damage location in a structure with an imprecise baseline mathematical model, using the sensitivity matrix and subspace angles between its columns and a vector of modal residuals. Two novel approaches proposed for achieving these goals are the use of generic parameters and mode shape residuals. Generic elements were also effectively employed in the modal updating study where the major inconsistencies were discovered to be in the region of the welded joints. In addition, generic elements provide an effective tool for assigning parameters to important, damage-sensitive regions, as the experimental study demonstrated. The use of mode shape sensitivities along with the corresponding residuals was shown to be promising. The impetus for their use was provided by the inability of natural frequencies alone to be used in a symmetric structure. Further work is needed in the case of multi-mode and multi-location damage...
cases. Also methods to provide a more rigorous determination of relative weights and the selection of natural frequencies and mode shape elements to be used for damage localisation, would be welcome. However, the use of the weighted scalar product was shown to be a promising approach.

References