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Damage detection using generic elements: Part I. Model updating

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Abstract

The authors propose the use of generic elements as a viable tool for parametric model based damage detection. The subject is divided into two related papers, and this paper is concerned with the requirements of the baseline model for damage detection, and the use of model updating to produce a validated finite element model of the undamaged structure. Two parameterisations are considered, based on the need to use the same parameterisation for subsequent damage detection. The novel aspect of this approach is the use of generic elements in damage detection where a preliminary updating exercise is required. The result is an updated model with physical meaning that may be used to detect and locate damage in a symmetric structure.

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1. Introduction

Damage detection methods that use mathematical models can be divided into non-parametric and parametric approaches. Doebling et al. [1] gave an extensive review of these methods. Friswell and Mottershead [2] provided a review of parametric identification techniques in the context of model updating. Farrar and Duffey [3] reviewed the subject of damage detection in rotating machinery. *Non-parametric methods* detect damage via the direct use of such quantities as natural frequencies and modes, stiffness and flexibility matrices. *Parametric methods* use a pre-selected set of parameters to define the model of the structure in question and the assumed damage mechanisms and their potential loca-

tion. These methods determine the damage state of the structure by means of changes in the values of the parameters. The choice of parameterisation and the type of damage can considerably influence the performance of these methods. A number of parameterisation techniques may be employed, among them are substructure, physical and geometric parameters, generic elements and equivalent models of inadequately idealised structural parts of the finite element models. The following presents a brief review of the recent use of different parameter types in damage detection.

Abdalla et al. [4] used substructure parameters along with the theory of linear matrix inequalities to formulate a convex optimisation problem for damage detection. The authors applied the same approach to a real cantilevered beam [5] and evaluated the approach for incomplete measurements [6]. Contursi et al. [7] proposed a procedure optimising the correlation between vectors of differences in the natural frequencies due to damage in the experiment and analytical model. Hassiotis [8] formulated damage detection as a quadratic programming

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problem with constraints in the form of linear equalities and inequalities, and damage is modelled by substructure parameters using element stiffness matrices. Santos et al. [9] formed a penalty function using the sensitivity method and damage was again modelled with substructure stiffness parameters. Zimmerman et al. [10] employed Ritz vectors in a modified minimum rank perturbation algorithm, and was compared with a method based on a genetic algorithm, again employing substructure parameters to represent the effect of damage. Sohn and Law [11] introduced Ritz vectors employing the concept of Bayesian probability using substructure stiffness parameters. Wang et al. [12] presented a two-stage damage detection algorithm, where a quadratic programming approach was applied at the second stage to determine the extent of the damage as proportional changes in the elemental matrices.

Friswell et al. [13] used physical parameters in a scheme combining a genetic algorithm and the sensitivity method for the location and quantification of damage, respectively. The parameters were chosen to be the flexural stiffness of all of the elements. Friswell et al. [14] used *subset selection* for damage location and evaluated the approach on a simulated cantilevered beam. The set of so called *candidate parameters* consisted of the mass and flexural stiffness of all of the elements, as well as discrete masses and grounded springs attached to all of the nodes. Papadimitriou et al. [15] presented damage detection as a quadratic programming problem with inequality constraints, which enabled the use of arbitrary physical parameter sets. Marwala and Heyns [16] presented a combined criteria based on minimising the Euclidean norm of the residual vector resulting from the underlying eigenvalue problem. The Young's moduli of each element were chosen to constitute the parameter vector. Yigui and Yichen [17] proposed the application of neural networks for damage detection, where the training samples were created by varying the parameter values, namely flexural stiffnesses. Sawyer and Rao [18] used physical parameters to parameterise a finite element model for subsequent training of a fuzzy logic based system. Damage was simulated by a reduction in structural stiffness by varying Young's modulus. Kim and Bartkiewicz [19] used a set of physical parameters consisting of cross-sectional areas, moments of inertia with respect to two different axes and polar moments of inertia corresponding to selected elements. The authors applied this strategy to a 10-story truss structure, reducing the overall number of parameters by using only the Young's modulus for each element of the model [20].

Most of the papers cited above used element parameters, although alternative parameterisations have been tried. Yun and Bahng [21] focused on structural joints and used an approach based on neural networks. The joints in the structure were modelled as connections with finite stiffness, i.e. flexible joints, using rotational

springs at the joint locations as an equivalent joint model. Wang et al. [22] also considered damage detection in structural joints, but used generic elements based solely on the translational degrees of freedom. Damage was assumed to occur only at joint locations and other parts of the structure were not parameterised. The approach was evaluated on a simulated model of a frame structure with L- and T-shaped joints. Law et al. [23] introduced a concept called *Damage Detection Oriented Modeling*, which is essentially the application of a generic element parameterisation to the model of the *Tsing Ma* (Hong Kong) bridge.

The goal of this and the subsequent paper is to propose the use of generic elements in the context of damage detection. The damage detection scheme consists of model updating of the baseline finite element model, followed by the use of the sensitivity matrix along with the vector of (relative) changes of the chosen dynamic properties for damage location. The approach is tested on a thin-walled H-shaped welded test article. This paper represents first part of the study dealing with the model updating of the baseline mathematical model created by the finite element method and it will be followed by second paper dealing with the damage detection aspects.

2. Theoretical considerations

2.1. General approach

The model updating and damage location approach proposed in this and the subsequent paper minimises the difference between modal quantities (usually natural frequencies and less often mode shapes) of the measured data and model predictions. This problem may be expressed as the minimisation of J where,

$$J(\mathbf{p}) = \|\mathbf{z}_m - \mathbf{z}(\mathbf{p})\|^2 = \boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon}, \quad \text{with } \boldsymbol{\varepsilon} = \mathbf{z}_m - \mathbf{z}(\mathbf{p}) \quad (1)$$

where $\mathbf{z}_m, \mathbf{z}(\mathbf{p}) \in \mathbb{R}^{n_F(n_M+1)}$ are the measured and computed modal parameter vectors, $\mathbf{p}$ is a vector of all parameters, $\boldsymbol{\varepsilon}$ is the modal residual vector and n_F, n_M are a number of identified natural frequencies and corresponding mode shape coordinates. The modal residual in Eq. (1) is a non-linear function of the parameters and the minimisation is solved using a truncated linear Taylor series and iteration. Thus the Taylor series is

$$\begin{aligned} \mathbf{z}_m &= \mathbf{z}_j + \mathbf{S}_j \delta \mathbf{p}_j + \text{higher order terms} \\ \mathbf{z}_j &= \mathbf{z}(\mathbf{p}_j), \quad \mathbf{S}_j = \mathbf{S}(\mathbf{p}_j), \quad \delta \mathbf{p}_j = \mathbf{p}_m - \mathbf{p}_j \end{aligned} \quad (2)$$

where the matrix $\mathbf{S}_j \in \mathbb{R}^{n_F(n_M+1) \times n_P}$ consists of the first derivatives of the modal quantities with respect to the model parameters, index j denotes the j th iteration and $\mathbf{p}_m$ is the parameter vector that gives the measured outputs. Friswell and Mottershead [2] gave more detail

on the algorithms available for model updating. In this paper mode shapes will not be used for the updating studies, except for pairing individual modes, and S_j will be of size $\mathbb{R}^{n_F \times n_p}$. This will not be the case for damage location presented in the companion paper. By neglecting higher order terms, an iterative scheme may be derived, using the linear approximation,

$$\delta \mathbf{z}_j = \mathbf{S}_j \delta \mathbf{p}_j \quad (3)$$

where $\delta \mathbf{z}_j = \mathbf{z}_m - \mathbf{z}_j$, $\delta \mathbf{p}_j = \mathbf{p}_{j+1} - \mathbf{p}_j$. This formula will be the basis of model updating and also for damage localisation in a slightly modified form.

A frequent problem that arises in *model-based vibration-based damage detection*, whether parametric or non-parametric, is the need for a very accurate mathematical model, so that it correctly captures the actual structural dynamic behaviour in some pre-determined frequency range. Often in structural health monitoring the changes in the measured quantities caused by structural damage are smaller than those observed between the healthy (i.e. undamaged) structure and the mathematical model. Consequently, it becomes almost impossible to discern between inadequate modelling and actual changes due to damage. There are two alternative approaches to this problem [14]. The first is to update the healthy model so that the correlation between the model and the measured data is improved. This approach requires that the errors that remain after updating are smaller in magnitude than the changes due to the damage. Furthermore the changes to the model should be physically meaningful, so that the updating process corrects actual model errors, and does not merely reproduce the measured data. The second approach is based on the use of (relative) differences between data measured on healthy and potentially damaged structure. Translated into mathematical terms, the following modified version of Eq. (3) is considered

$$\mathbf{S} \delta \mathbf{p} = \mathbf{z}_m - \mathbf{z}(0) = \mathbf{z}_d - \mathbf{z}_u \equiv \delta \mathbf{z} \quad (4)$$

where $\delta \mathbf{p}$ is a vector of parameter changes due to damage, $\mathbf{z}_d$, $\mathbf{z}_u$ are vectors of measured modal quantities of the damaged and undamaged structure, respectively. In this case, assuming that the *only* changes in the structure are due to damage, the problem may be reduced to finding those parameters that reproduce the measured changes. As the parameters in this case represent both the location and the type of damage, it should be possible through the minimisation of $\delta \mathbf{z}$ in Eq. (4) to identify the region as well as the form of the damage via the selection of an optimal subset of parameters from vector $\delta \mathbf{p}$. This approach leads to the problem of parameter subset selection from the full set of parameters included in $\delta \mathbf{p}$. The subject of parameter subset selection and its utilisation in damage location will be studied in the

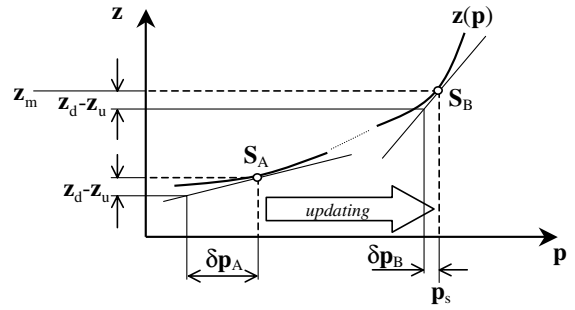


Fig. 1. The effect of model updating on sensitivity matrix and its use in damage location.

second paper. This paper will consider the need for accurate mathematical models and role of model updating.

The question that arises for this second approach is how accurate does the baseline model need to be? Titurus [24] successfully applied this technique using the original baseline model to a T-shaped structure and compared five different parameterisation approaches. However, if the baseline model is relatively inaccurate, so that the model predictions are significantly different from the measured data from the healthy structure, then model updating must be applied. This is demonstrated symbolically in Fig. 1. If the difference between the baseline mathematical model (represented by point A) and the data measured on the healthy structure (point B) is too large then model updating becomes necessary. In essence, this approach assumes availability of a model that is able to correctly estimate the actual parameter vector $\delta \mathbf{p}_B$ using $\mathbf{z}_d - \mathbf{z}_u$ and $\mathbf{S}_B$, i.e. a model close enough to point B. This is not the case for the baseline model represented by $\mathbf{S}_A$ and the same set of experimental data represented by $\mathbf{z}_d - \mathbf{z}_u$, producing an incorrect estimate $\delta \mathbf{p}_A$.

The above approach does assume that the damage is small, in the sense that the linear approximation given by Eq. (4) is accurate. In practice the change in the response is relatively small, although the change in a local parameter may be large. Although there is no guarantee that the approach will work when the linear approximation is inaccurate, usually the parameters with large changes will be identified. This is still a useful result, since the identification of damage location is often more important than accurately estimating damage extent.

2.2. Generic elements

Generic elements have been developed for use in model updating and may be considered as equivalent models of elements or substructures [25]. Law et al. [23] applied generic elements to the finite element model updating of the Tsing Ma bridge in Hong Kong. Wang et al. [22] used generic elements in damage detection,

dealing with the simulated problem of damage detection in a frame structure with flexible L-shaped and T-shaped structural joints.

Generic element parameterisation is based on allowing changes to the eigenvalues and eigenvectors of the stiffness matrices of structural elements or substructures. These changes are usually constrained so that properties such as the rigid body modes and the geometric symmetry are retained. The form of generic element parameterisation assumed in this paper is a modification of a standard formulation [2] where only changes in the stiffness matrices are allowed, assuming the correct modelling of mass properties and thus that the damage only influences the stiffness properties.

In the standard formulation, the eigenvalue problem for any selected substructure or element stiffness matrix can be written as

$$\begin{aligned} (\mathbf{K}^{\text{SUB}} - \lambda_i^{\text{SUB}} \mathbf{I}) \phi_i^{\text{SUB}} &= \mathbf{0} \\ (\Phi^{\text{SUB}})^T \mathbf{K}^{\text{SUB}} \Phi^{\text{SUB}} &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Lambda_s \end{bmatrix} \end{aligned} \quad (5)$$

where

$$\begin{aligned} \Phi^{\text{SUB}} &= [\phi_1^{\text{SUB}}, \dots, \phi_{n_R}^{\text{SUB}}, \phi_{n_R+1}^{\text{SUB}}, \dots, \phi_{n_{\text{SUB}}}^{\text{SUB}}] \\ &= [\Phi_R, \Phi_s] \in \mathbb{R}^{n_{\text{SUB}} \times n_{\text{SUB}}}, \quad n_R \leq 6 \end{aligned} \quad (6)$$

$\mathbf{K}^{\text{SUB}}$ is a substructure stiffness matrix, Φ^{SUB} is the eigenvector matrix of $\mathbf{K}^{\text{SUB}}$, λ_i^{SUB} and ϕ_i^{SUB} are the i th eigenvalue and eigenvector of matrix $\mathbf{K}^{\text{SUB}}$, respectively. Submatrix Λ_s is a diagonal matrix of non-zero eigenvalues of matrix $\mathbf{K}^{\text{SUB}}$. The dimensions of these matrices depend on the size of the chosen substructure, where n_{SUB} is a number of degrees of freedom of substructure and $n_R \leq 6$ is the number of rigid body modes, Φ_R , Φ_s are submatrices of Φ^{SUB} corresponding to the rigid and structural modes, respectively.

A modified set of substructure eigenvectors [2] may be obtained by a linear transformation, as

$$[\Phi_{0R}, \Phi_{0S}] = [\Phi_R, \Phi_s] \begin{bmatrix} \mathbf{S}_R & \mathbf{S}_{RS} \\ \mathbf{0} & \mathbf{S}_s \end{bmatrix} \quad (7)$$

where the index 0 denotes the original quantities and matrices *without* index 0 represent modified quantities. Notice that in Eq. (7) the modified rigid body modes do not contain any of the structural modes. By rearranging Eq. (5) and using Eq. (7), the modified substructure stiffness matrix may be written as

$$\begin{aligned} \mathbf{K}^{\text{SUB}} &= \Phi_{0S} \mathbf{S}_s^T \Lambda_s \mathbf{S}_s \Phi_{0S}^T \\ &= \Phi_{0S} \begin{bmatrix} \kappa_{1,1} & \cdots & \kappa_{1,(n_{\text{SUB}}-n_R)} \\ & \ddots & \vdots \\ \text{Sym} & & \kappa_{(n_{\text{SUB}}-n_R),(n_{\text{SUB}}-n_R)} \end{bmatrix} \Phi_{0S}^T \end{aligned} \quad (8)$$

Eq. (8) is a basis for generic element parameterisation for damage detection used in this paper. $\kappa_{1,1}, \dots, \kappa_{(n_{\text{SUB}}-n_R),(n_{\text{SUB}}-n_R)}$ are the most general parameters for this parameterisation. Employing additional assumptions related to the geometric symmetry or anti-symmetry of the corresponding eigenvectors may significantly reduce the total number of parameters. The sensitivity of natural frequencies with respect to these parameters is

$$\begin{aligned} \frac{\partial \lambda_i}{\partial p_j} &= \phi_i^T \frac{\partial}{\partial p_j} \left(\mathbf{K}_0 + \sum_{l=1}^{N_p} \mathbf{K}_l(\mathbf{p}_{\text{elem},l}) \right) \phi_i \\ &= \phi_i^T \frac{\partial \mathbf{K}_j(\mathbf{p}_{\text{elem},j})}{\partial p_j} \phi_i \end{aligned} \quad (9)$$

where N_p is a number of parameterised substructures or elements, $\mathbf{p}_{\text{elem},l}$ is a group of parameters corresponding to l th substructure or element, $\mathbf{p}$ is a vector of all parameters, $\mathbf{K}_0$ is non-parameterised part of the global stiffness matrix, λ_i is the i th eigenvalue and p_j is j th parameter of a chosen parameterisation.

Generic elements introduce flexibility into the joint in a controlled way. Other equivalent models, such as discrete rotational springs, offset parameters or changing the properties of elements adjacent to the joint may also be used, although generic parameters do have advantages [26]. In particular, all models pre-judge how the joint will operate within the full model of the structure, whereas the generic element approach automatically finds the likely low frequency motion of the joint. Consider a two dimensional ‘T’ joint constructed from three beam elements. Each node has three degrees of freedom and, since the substructure has four nodes, the substructure stiffness matrix has three rigid body eigenvectors and nine flexible eigenvectors. Fig. 2 shows the nine flexible eigenvectors for this substructure, where the circles and dots represent the nodes and the dotted line is the undeformed joint. The finite element shape functions have been used to produce smooth deformation shapes. The lower eigenvectors have much simpler deformation shapes that are more likely to represent the motion the substructure would undergo in many of the global modes of the structure. Thus reducing the eigenvalues corresponding to these eigenvectors makes the joint substructure more flexible in the frequency range of the global dynamics. Higher frequency eigenvectors of the substructure may also be included if the motion of the joint is more complex, however the first two eigenvectors of the ‘T’ joint were found to characterise the dynamics of the frame structure considered later. Fig. 3 shows the three flexible eigenvectors of a beam element, which has three degrees of freedom per node and hence three rigid body eigenvectors and three flexible eigenvectors. Again the deformation represented by the first eigenvector will most likely represent the motion the

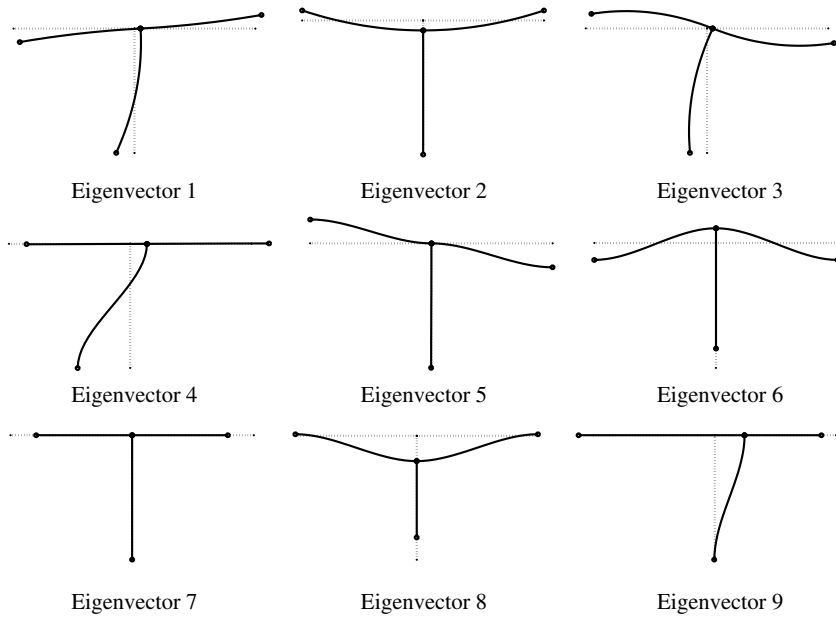


Fig. 2. Substructure eigenvectors for a 'T' joint.

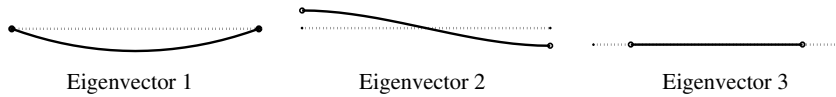


Fig. 3. Substructure eigenvectors for a beam element.

element would undergo in the global modes of the structure. Gladwell and Ahmadian [25] gave further explanation of the physical meaning of generic elements.

2.3. Model updating

This paper will use the eigensensitivity method for model updating, which requires the derivatives of the chosen response quantities with respect to the chosen model parameters [2]. The method is based on Eq. (3) and minimises the residual,

$$\boldsymbol{\varepsilon} = \delta \mathbf{z} - \mathbf{S} \delta \mathbf{p} \quad (10)$$

where $\boldsymbol{\varepsilon}$ is a modal residual vector and the matrix $\mathbf{S} \in \mathbb{R}^{n_F \times n_P}$ consists of the first derivatives of the response modal parameters with respect to the model parameters. Since Eq. (10) is a linear approximation the process is iterative, although the iteration index has been omitted for clarity. If natural frequencies alone are used, then n_F and n_P are the total number of identified natural frequencies and model parameters, respectively. The following cost function provides the basis for the iterative updating procedure, taking into account additional

regularisation constraints and the different relative importance of the measured data [2]. Thus,

$$\begin{aligned} J_{\alpha\beta}(\mathbf{p}) = & \boldsymbol{\varepsilon}^T \mathbf{W}_{ee} \boldsymbol{\varepsilon} + \alpha^j \{\mathbf{p} - \mathbf{p}_0\}^T \mathbf{W}_{pp} \{\mathbf{p} - \mathbf{p}_0\} \\ & + \beta^j \{\mathbf{Cp} - \mathbf{d}\}^T \{\mathbf{Cp} - \mathbf{d}\} \end{aligned} \quad (11)$$

$$\alpha \in (0, 1), \quad \mathbf{W}_{ee} \in \mathbb{R}^{n_F \times n_F}, \quad \mathbf{W}_{pp} \in \mathbb{R}^{n_P \times n_P}$$

$$\beta \in (0, 1), \quad \mathbf{C} \in \mathbb{R}^{n_{eq} \times n_P}, \quad \mathbf{d} \in \mathbb{R}^{n_{eq}}$$

where the diagonal weighting matrices $\mathbf{W}_{pp}$, $\mathbf{W}_{ee}$ represent the analyst's confidence in the initial model parameter values and the accuracy of measured data respectively, the parameter α controls the regularisation due to the initial parameter values, while the parameter β provides the same effect for the parameter constraints and $\mathbf{p}_0$ is an initial estimate of the parameter values. The regularisation conditions reduce the parameter change during the iteration process and assuming an exponential form ensures that the effect of the a priori information continually decreases. The magnitude of α and β affects the convergence rate of the parameter estimation process, but not the final parameter estimates. If the procedure converges to the local minimum closest to the initial parameter values, then since α and β are less than one, the regularisation terms become negligible. Thus,

on convergence, the cost function does not depend on α or β .

Eq. (11), without the constraint regularisation ($\beta = 0$) represents a modified version of a cost function given by Friswell and Mottershead [2], where here the regularisation parameter α reduces as the iteration progresses. The general form of Eq. (11), includes the third term which represents constraints given by properly chosen quantities $\mathbf{C}$ and $\mathbf{d}$ [26] and n_{eq} is the total number of these constraints. This term allows for additional a priori conditions where nominally equal parameters are approximately equal. The following example represents an equality condition for two arbitrary parameters resulting in one constraint equation, given by

$$\begin{aligned} p_i &= p_j \\ \mathbf{C} &= \begin{bmatrix} 0 & \dots & 0 & \underbrace{+1}_i & 0 & \dots & 0 & \underbrace{-1}_j & 0 & \dots & 0 \end{bmatrix}, \\ \mathbf{d} &= \mathbf{0} \\ \mathbf{C} &\in \mathbb{R}^{1 \times np}, \quad \mathbf{d} \in \mathbb{R} \end{aligned} \quad (12)$$

Matrices $\mathbf{W}_{pp}$ and $\mathbf{C}$ have to be chosen so that the extended Morozov's complementary condition,

$$\text{rank} \begin{pmatrix} \mathbf{S} \\ \mathbf{W}_{pp}^{1/2} \\ \mathbf{C} \end{pmatrix} = N_p \quad (13)$$

will be fulfilled [27].

Minimising the cost function (11) gives the following update of the model parameters (the iteration index has been omitted):

$$\begin{aligned} \delta \mathbf{p} &= [\mathbf{S}^T \mathbf{W}_{ee} \mathbf{S} + \alpha^j \mathbf{W}_{pp} + \beta^j \mathbf{C}^T \mathbf{C}]^{-1} \{ \mathbf{S}^T \mathbf{W}_{ee} \delta \mathbf{z} \\ &\quad - \alpha^j \mathbf{W}_{pp} (\mathbf{p}_j - \mathbf{p}_0) - \beta^j \mathbf{C}^T (\mathbf{C} \mathbf{p}_j - \mathbf{d}) \} \end{aligned} \quad (14)$$

where the sensitivity matrix $\mathbf{S}$ is computed at the j th parameter value, $\mathbf{p}_j$. Furthermore, because of possibly large differences in parameter values and the measured modal data, row and column scaling is employed to

prevent ill-conditioning problems, based on the initial parameter values and the measured data. Further information concerning the choice of the regularisation and weighting constants will be provided during the analysis of the experimental results.

3. Experimental structure

3.1. Geometry and experimental setup

The structure chosen to evaluate the strategy presented above consisted of four thin-walled tubes connected by four fillet welds. These joints were intentionally manipulated to produce one healthy and six damaged cases. This paper considers only the healthy case, that is the undamaged structure where the model requires updating. The second paper considers damage detection by comparing the selected damaged cases to the updated mathematical model of the undamaged structure.

Fig. 4 shows the experimental structure, together with the experimental (EMA) measurement locations. The finite element (FEM) nodes were placed at the measurement locations. Thus 32 degrees of freedom were measured, whereas the FE model contained 96 degrees of freedom (three degrees of freedom per node). The in-plane dynamics of the structure were measured, and the structure was supported in the free-free condition supported by elastic bands. The structure was excited by a roving impact hammer and the response was measured using two fixed accelerometers (Fig. 4(b)). The frequency range of interest was from 0 to 625 Hz and each time signal consisted of 2^{15} samples. The identification of the modal properties from the 64 frequency response functions was performed in the frequency domain using the Structural Dynamics Toolbox [28]. Model updating, data management and damage location were performed in MATLAB, although a detailed model of the structure was also created and evaluated in ANSYS.

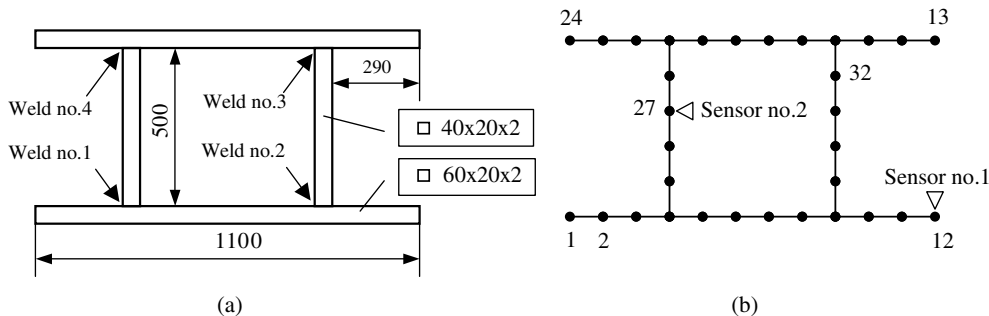


Fig. 4. The outline and the discretisation of the structure.

3.2. Modelling considerations: baseline FE model

The baseline finite element model of the structure shown in Fig. 4 was created using Euler–Bernoulli planar elements (EB2D) with three degrees of freedom (DOFs) per node. More advanced formulations, incorporating shear effects and rotary inertia [29,30], or more detailed modelling using the shell theory are not required. It will be shown that model updating of this simple beam model is able to accurately predict the structure's dynamics. Simple beam theory will break down near the joints, but here generic elements will be used to produce equivalent models that are sufficiently accurate for the lower frequency modes, and provide a good starting point for subsequent damage detection algorithms. The beam model results will be compared to a detailed ANSYS model using four-node linear shell elements, however the baseline beam model has 96 DOFs and the detailed shell model has 5040 DOFs.

Problem areas related to the use of EB2D (or EB3D) elements for the FE models are the structural joints. The

inevitable simplifications lead to systematic errors that have to be considered when a model is used for damage detection and should be addressed by model updating. Fig. 5 demonstrates the problem of the assumed idealisation that occurs when beam elements, such as EB2D, are employed in the modelling of joints. Determining the correct region for the attachment point of the horizontal and vertical parts is difficult and the assumption is made that these two parts always meet at a 90° , see Fig. 5b. While this approach is reasonable in the case of solid beams, Titurus [24] showed that the error in this assumption for thin-walled tubes appears to be significant, see Fig. 5c. Thus model updating, using so called equivalent joint models [31], may be used to produce a model that more accurately predicts the measured data. Fig. 6 illustrates the phenomenon described above, and shows a shell model of the structure (Fig. 6a), its first strain mode shape (Fig. 6b) and the detailed deformation near one of the welded joints (Fig. 6c). Clearly the region near the joint is more flexible than the beam model will predict, and highlights the regions requiring updating of the model stiffness.

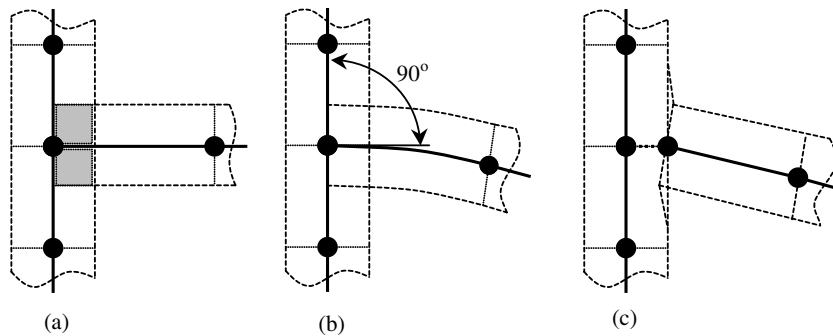


Fig. 5. The use of EB2D elements for T joint modelling.

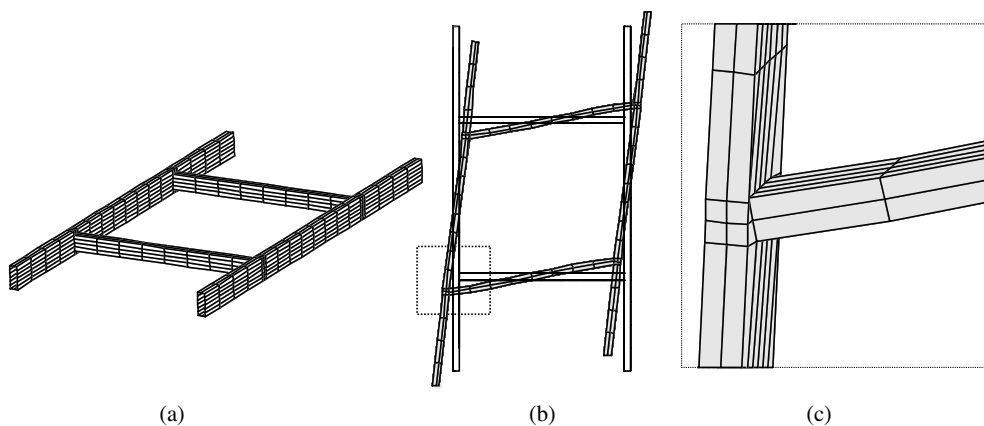


Fig. 6. The detailed shell model of the H structure.

3.3. The identification for the undamaged case

This section presents the results of the experimental modal analysis and compares this data with that from the different models with a view to increasing the understanding of the dynamics of the structure. Fig. 7 shows a typical FRF chosen from the 64 that were measured, where the structure is excited at node 2 and the response measured at node 12, in both cases perpendicular to the beams. The results of the identification correspond to the healthy or undamaged case, and the FRF fitted by the modal analysis procedure is also

shown. Fig. 8 shows the corresponding identified mode shapes for the healthy structure. The identified natural frequencies and mode shapes constitute the measured data for model updating.

The experimental results may be compared to the two finite element models available, namely the beam model created using EB2D elements with 96 DOFs and the detailed shell model with 5040 DOFs and created using ANSYS. Table 1 compares the natural frequencies obtained from both models with the measured data, and Fig. 9 gives the modal assurance criteria (MAC) matrix, which compares the mode shapes obtained from the

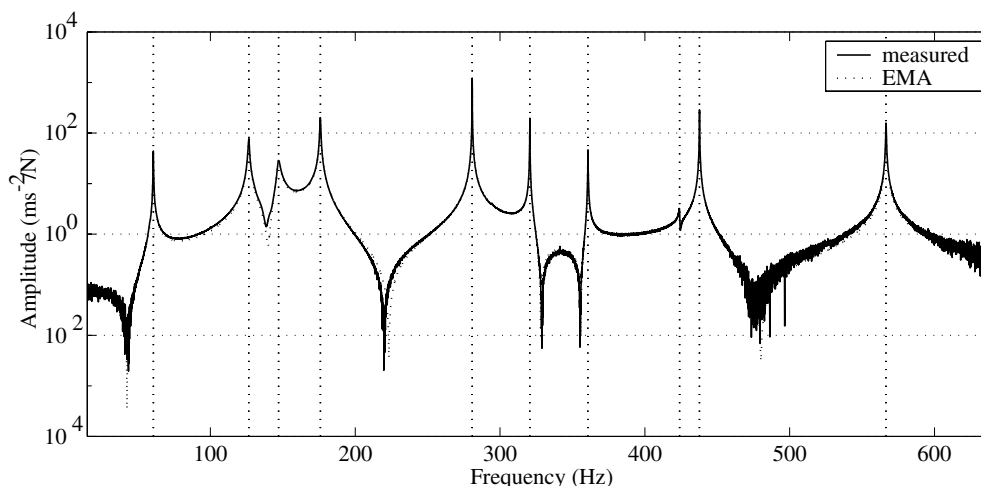


Fig. 7. Frequency response function between DOFs 2y and 12y.

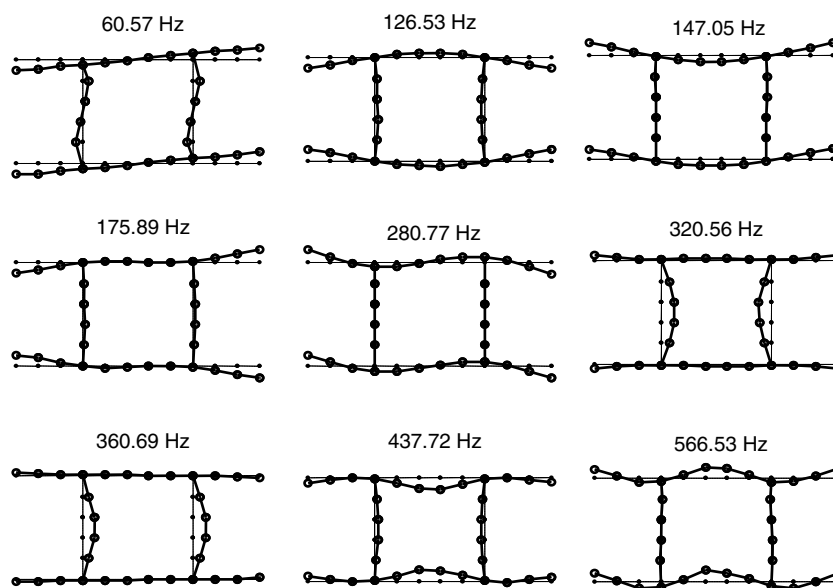


Fig. 8. Identified mode shapes and natural frequencies of the undamaged H structure.

Table 1

The natural frequencies corresponding to the baseline FE model, the detailed shell model and the experimental data

	EMA [Hz]	EB2D [Hz]	Shell [Hz]	Δf [%]	
				EMA vs EB2D	EMA vs shell
1	60.57	98.15	49.99	62.06	–17.47
2	126.53	135.95	138.95	7.44	9.82
3	147.05	182.65	151.60	24.21	3.09
4	175.89	184.93	194.12	5.14	10.36
5	280.76	293.68	310.70	4.60	10.66
6	320.56	405.33	326.48	26.44	1.84
7	360.70	510.47	366.54	41.52	1.62
8	437.72	561.67	458.51	28.32	4.75
9	566.52	608.82	613.75	7.47	8.34

EMA denotes experimental modal analysis, EB2D uses Euler–Bernoulli planar beam element (96 DOFs), and Shell uses detailed shell model (5040 DOFs).

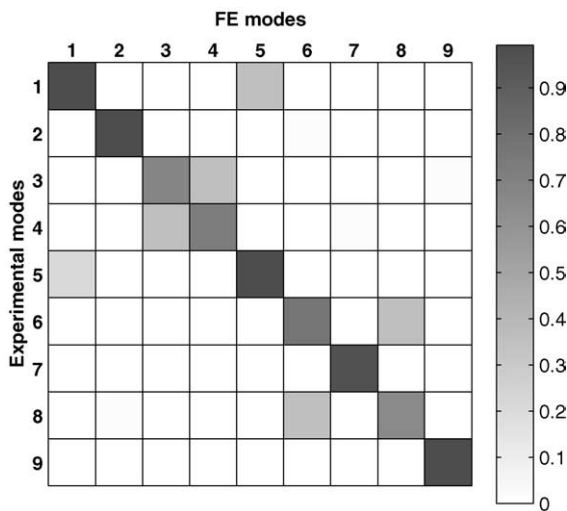


Fig. 9. The MAC matrix showing the correlation between the modes of the experimental and beam model for the undamaged H structure.

beam model with those obtained experimentally. There are clearly significant differences between the experimental data and the baseline (beam) model, most likely originating from the model of the joints. Table 1 shows the natural frequencies predicted by the shell model, and clearly demonstrates that modelling the deformation near the joints is able to significantly reduce the errors in the predicted natural frequencies. The biggest discrepancy in the beam model is 62.18% for the first natural frequency, and all of the frequencies corresponding to this model are overestimated. Possible causes of this are:

- *Stiffening*: The beam model is stiff because rigid joints are assumed, created by merging coincident nodes belonging to the horizontal and vertical parts. Of

course the shell model does not suffer from this problem. Fig. 6 is a demonstration of this for the first mode shape.

- *Material properties*: A common shift in all of the observed natural frequencies is usually due to incorrect material constants or thickness. This problem will occur in both the beam and shell models. A linear isotropic material is characterised by the Young's modulus E , Poisson's ratio μ and the density ρ . Changing the material constants can also allow for small changes in wall thickness. These parameters will change only slightly from their initial values.
- *Weld uncertainty*: In the shell model the conditions in the region of the fillet weld will be highly uncertain, due mainly to two effects with opposite influences on the structural dynamics. A local stiffening of this region will occur due to presence of additional-welding material and a local softening is possible (and actually observed) due to a reduction in plate thickness due to the welding process.

Titurus [24] analysed these possible factors and concluded that due to the difficulty in assessing their relative influence on the predicted results, that model updating of the beam model should be used to improve the correlation of the model and measurements.

4. Model updating

4.1. Parameterisation

Two different parameterisations are chosen, since two alternative approaches will be proposed for the subsequent damage detection [32]. Both approaches use parameters based on generic elements or substructures. *Parameterisation A* for the thin-walled structure will be used for *partial* damage localisation, while *parameterisation B* will be used for *complete* damage localisation

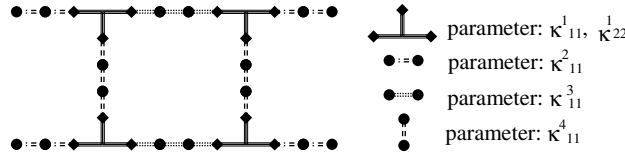


Fig. 10. Parameterisation *A* of the baseline model of the thin-walled H structure.

and is able to handle structures that are symmetric. The nominal values of the geometric dimensions are given in Fig. 4.

Parameterisation *A* consists of the five parameters shown in Fig. 10. The first two parameters are based on the first two eigenvalues of the ‘*T*’ joint substructures containing the fillet welds. The remaining parameters are from the generic elements of the straight beams. The beams are split into three groups, and each group has a different generic parameter, namely the first eigenvalue. The reasons for choosing these parameters was highlighted in Section 2.2 and Figs. 2 and 3 give the substructure eigenvectors. Thus the parameter vector $\mathbf{p}$ is

$$\mathbf{p} = [p_1, p_2, p_3, p_4, p_5]^T = [\kappa_{11}^1, \kappa_{22}^1, \kappa_{11}^2, \kappa_{11}^3, \kappa_{11}^4]^T \quad (15)$$

where κ_{jk}^i denotes the (j, k) element of matrix given in Eq. (8) for the i th element or substructure. Initial values of these parameters are determined from the nominal geometric and material parameters of the structure. Note that all elements or substructures within a group have the same element matrix. This parameterisation allows partial localisation to be performed where the type of element containing the damage may be identified.

Parameterisation *B* extends this parameterisation by allowing the generic parameters related to similar parts of the structure, whether elements or substructures, to be independent. This allows complete damage localisation by using 28 parameters and is shown in Fig. 11. This parameterisation will be employed primarily for damage detection and localisation. The parameters are ordered as,

$$\begin{aligned} p_{2i-1} &= \kappa_{11}^i, & p_{2i} &= \kappa_{22}^i, & i &= 1, 2, 3, 4 \\ p_{j+4} &= \kappa_{11}^j, & j &= 5, 6, \dots, 24 \end{aligned} \quad (16)$$

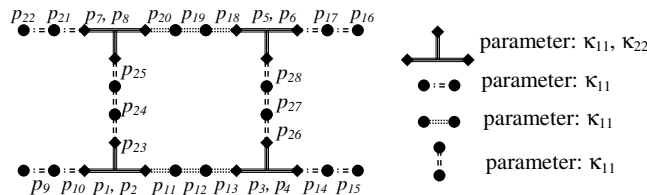


Fig. 11. Parameterisation *B* of the baseline model of the thin-walled H structure.

In fact 28 parameters are too many for a well conditioned model updating problem, although the complete set of parameters will be needed for damage location. For model updating the physical understanding of the nature of the structure reduces the number of parameters to 11, and also introduces regularisation terms, as explained in the next section.

4.2. Parameter estimation

The finite element model used for updating consisted of EB2D elements, as shown in Fig. 4b. The presence of the accelerometers is taken into account by adding discrete mass elements. The initial material parameters were chosen as $E = 210$ GPa and $\rho = 7850$ kg m⁻³. In both cases the first seven natural frequencies were used for model updating (see Table 1). The eighth and ninth natural frequencies, not used for model updating, were used to check the quality of the updated model outside the frequency range considered. The weighting matrices, $\mathbf{W}_{ee}$ and $\mathbf{W}_{pp}$, were taken to be diagonal with the reciprocal values of the variance of the natural frequencies and parameters along the diagonals. The variances of these quantities were chosen as a constant percentage of the relevant quantities; 0.8% for the natural frequencies and 2% for the parameters. For both parameterisations, the updated results showed little sensitivity to the choice of the parameter α , except for the speed of convergence, and so its value was simply set equal to 0.5, without any need to optimise the regularisation parameters. Fig. 12. shows the convergence of the parameter estimates and modal predictions for parameterisation *A*.

For updating the baseline model, the number of parameters in parameterisation *B* was reduced to help avoid possible ill-conditioning. Parameters related to the welded joints (that is parameters p_1 to p_8) were left independent, although additional regularisation conditions

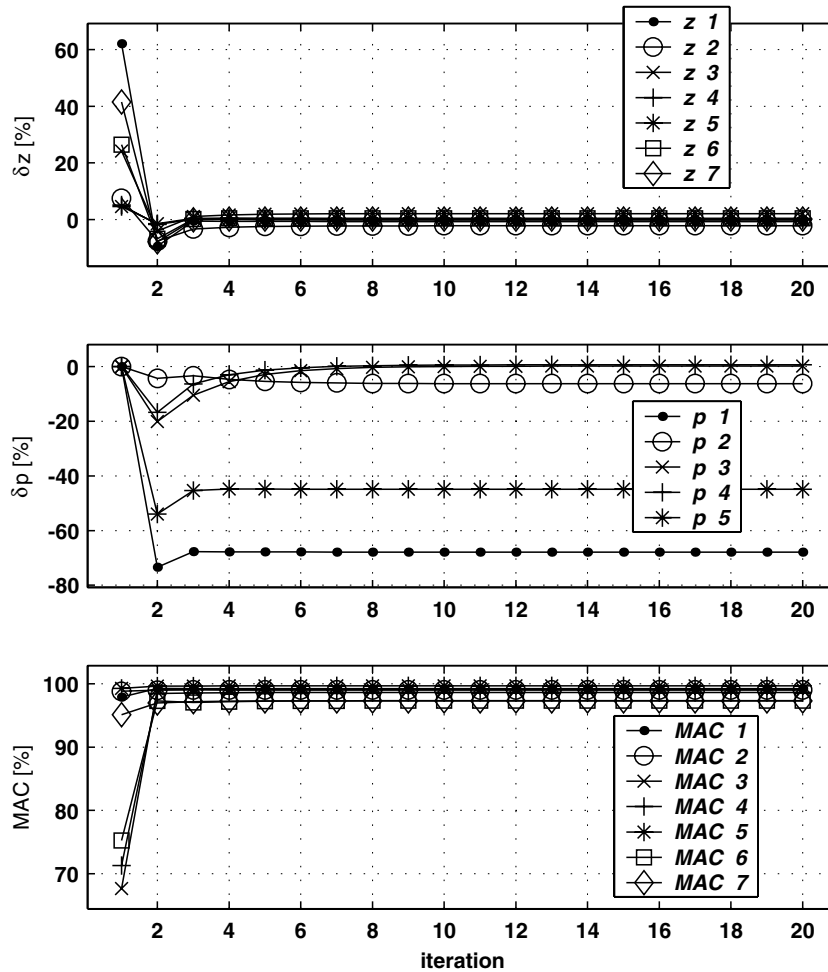


Fig. 12. Model updating of the H structure for parameterisation A.

were employed so that related parameters in the different welds had similar stiffness. Thus, parameters p_1, p_3, p_5, p_7 were considered to be nominally identical, as were parameters p_2, p_4, p_6, p_8 . Parameters $\{p_9, p_{10}, p_{14}, p_{15}, p_{16}, p_{17}, p_{21}, p_{22}\}$ were replaced by a single global parameter, as were $\{p_{11}, p_{12}, p_{13}, p_{18}, p_{19}, p_{20}\}$ and $\{p_i, i = 23, \dots, 28$. A further constraint was introduced, as the first two of sets of global parameters for the uniform beam elements are nominally identical (those sets containing p_9 and p_{11} , see Fig. 11). Note that the cross beams have a different cross section to the long beams (see Fig. 4). This approach may be justified on physical grounds as the beam cross section should be very consistent, whereas the four welds, although nominally identical will be different due to manufacturing tolerances. The global regularisation parameter β was chosen using engineering judgement to ensure that the model converged consistently to the nearest local minimum, and its value was set to 5×10^{-7} . In total, 11 parameters were updated and seven addi-

tional regularisation equations were employed. Fig. 13 shows the convergence of the parameter estimates and modal predictions for parameterisation B.

Table 2 compares the predictions from the initial and updated models to the experimental natural frequencies. The MAC values are not shown due to space limitations, although it is clear from Figs. 12 and 13 that the MAC values are all above 95% on convergence. The worst correlation in case of parameterisation A is 97.3% (originally 95.1%) corresponding to the seventh mode, whereas initially the lowest correlation belonged to the third mode, which changed its MAC value from 67.6% to 99.1%. In the case of parameterisation B, the lowest MAC value was 97.2% (originally 95.1%) for the sixth mode, whereas initially the lowest correlation belonged to the third mode, whose MAC value changed from 67.6% to 99.0%. These values were used to pair the computed and experimental mode shapes throughout the model updating exercise. The MAC values or mode

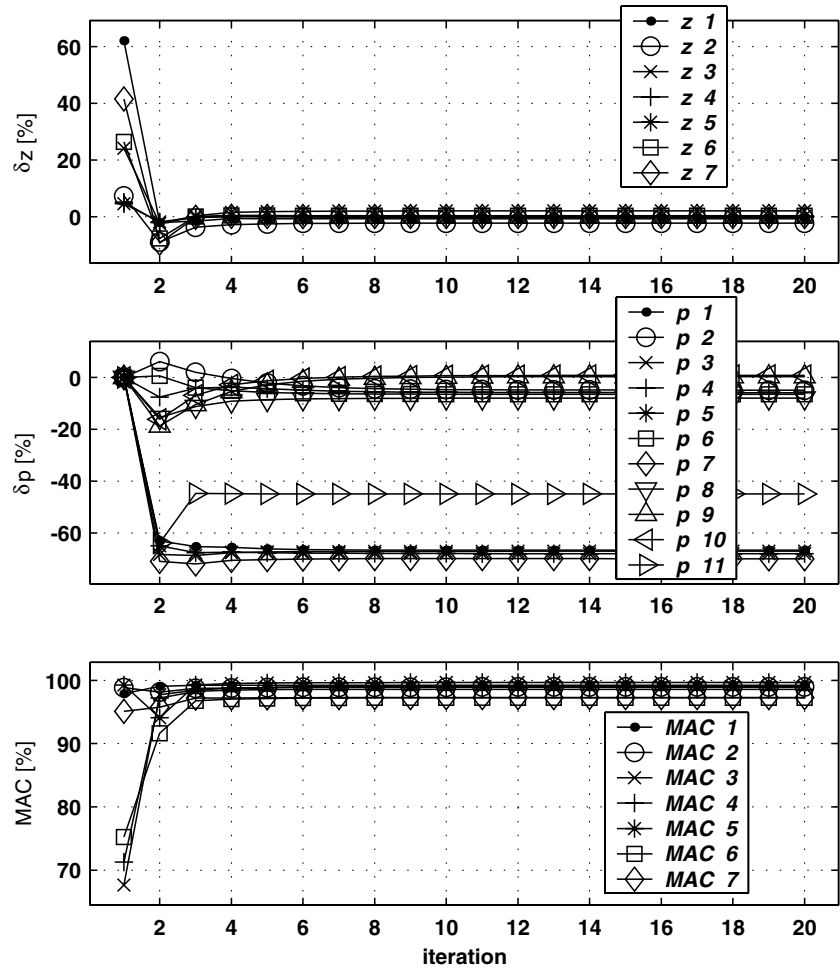


Fig. 13. Model updating of the H structure for parameterisation B.

Table 2
The natural frequencies of the baseline model, updated model and experiment

	FEM [Hz]	PAR A [Hz]	PAR B [Hz]	EMA [Hz]	FEM vs EMA [%]	PAR A vs EMA [%]	PAR B vs EMA [%]
1	98.15	60.13	60.23	60.57	62.06	−0.71	−0.55
2	135.95	123.74	123.76	126.53	7.44	−2.21	−2.19
3	182.66	150.15	150.15	147.05	24.21	2.11	2.11
4	184.93	175.49	175.54	175.89	5.14	−0.23	−0.20
5	293.68	281.53	281.42	280.77	4.60	0.27	0.23
6	405.33	321.95	321.73	320.56	26.44	0.43	0.36
7	510.48	361.15	361.03	360.69	41.52	0.13	0.09
8	561.67	485.35	485.58	437.72	28.32	10.88	10.93
9	608.82	594.05	594.06	566.53	7.47	4.86	4.86

FEM denotes the baseline FE model, PAR A denotes the model with parameterisation A after updating, PAR B denotes the model with parameterisation B after updating, EMA denotes the experimental data.

shapes were not used in the objective function. Figs. 12 and 13 show the relative differences in the predicted and

measured natural frequencies, and also the changes in the parameters from their initial values.

4.3. Summary of updating results

Model updating based on generic elements resulted in greatly improved mathematical representations of the real structure. The predictions of natural frequencies improved greatly, as demonstrated in Table 2. The initial error in the first natural frequency was reduced from 62.06% to -0.71% (parameterisation *A*), and -0.55% (parameterisation *B*). The maximum errors after updating were in the second and third natural frequencies, which were about $\pm 2\%$, whereas the other natural frequencies were predicted to within $\pm 0.5\%$. The most likely cause of the larger errors on the second and third natural frequencies is the difficulty in exciting the ends of the longerons of the structure. These problems were observed in the identification of the modal parameters and in the comparison of measured and synthesised FRFs. Almost identical results were achieved for both parameterisations and only slightly better results were achieved for parameterisation *B*. This suggests that the extra freedom in this parameterisation does not provide any improvement for the given set of experimental data, and it is likely that the quality of the welding was high. It is also important to note that the eighth and ninth natural frequencies, which were not used for updating, also improved significantly.

5. Conclusions

This paper has presented the use of generic elements in the context of finite element model updating, where the model will be subsequently used for damage detection. An updated model, that retains physical meaning, is vital. Furthermore this model should retain a large number of parameters so that the damage location may be determined, for example by using subspace angles. The proposed approach was verified on an H-shaped frame structure made of thin-walled beams and containing four fillet welds. After model updating, the finite element model produced using only Euler–Bernoulli 2D beam elements accurately predicted the real behaviour of the structure represented by the experimental modal model. Confidence in the physical meaning of the updated model is further enhanced due to an improved correlation between the experimental and predicted mode shapes, as well as a significant improvement in the natural frequency prediction outside frequency range used for updating.

References

[1] Doebling SW, Farrar CR, Prime MB. A summary review of vibration-based damage identification methods. *Shock Vibr Dig* 1998;30(2):91–105.

- [2] Friswell MI, Mottershead JE. Finite element model updating in structural dynamics. Dordrecht: Kluwer Academic Publishers; 1995.
- [3] Farrar CR, Duffley TA. Vibration-based damage detection in rotating machinery. In: Proceedings of DAMAS'99, Dublin, Ireland, 1999. p. 224–35.
- [4] Abdalla M, Grigoriadis KM, Zimmerman DC. Enhanced damage detection using linear matrix inequalities. In: Proceedings of the 16th International Modal Analysis Conference (IMAC), Santa Barbara, California, 1998. p. 144–9.
- [5] Abdalla M, Grigoriadis KM, Zimmerman DC. Experimental validation of the LMI methods for structural damage detection. In: Proceedings of the 17th International Modal Analysis Conference (IMAC), Orlando, Florida, 1999. p. 1373–8.
- [6] Abdalla M, Grigoriadis KM, Zimmerman D. LMI based hybrid expansion reduction damage detection method. In: Proceedings of the 18th International Modal Analysis Conference (IMAC), San Antonio, Texas, 2000. p. 1537–43.
- [7] Contursi T, Messina A, Williams EJ. Detection of damage at multiple sites with a correlation and iterative scheme. In: Proceedings of the 17th International Modal Analysis Conference (IMAC), Orlando, Florida, 1999. p. 1725–31.
- [8] Hassiotis S. Identification of damage using natural frequencies and markov parameters. *Comp Struct* 2000;74:365–73.
- [9] Santos JVA, Soares CMM, Soares CAM, Pina HLG. A damage identification numerical model based on the sensitivity of orthogonality conditions and least squares techniques. *Comp Struct* 2000;78:283–91.
- [10] Zimmerman DC, James III GH, Cao TT. An experimental study of damage detection using modal, strain, and Ritz properties. In: Proceedings of the 17th International Modal Analysis Conference (IMAC), Orlando, Florida, 1999. p. 586–92.
- [11] Sohn H, Law KH. Application of load-dependent Ritz vectors to Bayesian probabilistic damage detection. *Probab Eng Mech* 2000;15:139–53.
- [12] Wang X, Hu N, Fukunaga H, Yao ZH. Structural damage identification using static test data and changes in frequencies. *Eng Struct* 2001;23:610–21.
- [13] Friswell MI, Mottershead JE, Ahmadian H. Combining subset selection and parameter constraints in model updating. In: 16th ASME Biennial Conference on Mechanical Vibration and Noise, Sacramento, California, 1997, paper VIB4147.
- [14] Friswell MI, Penny JET, Garvey SD. Parameter subset selection in damage location. *Inverse Prob Eng* 1997;5(3): 189–215.
- [15] Papadimitriou C, Levin-West M, Milman M. A methodology for finite model updating using modal data. In: Proceedings of the 11th VPI Symposium on Structural Dynamics and Control, Blacksburg, Virginia, 1997. p. 91–100.
- [16] Marwala T, Heyns PS. A multiple-criterion method for determining structural damage. In: Proceedings of the 16th International Modal Analysis Conference (IMAC), Santa Barbara, California, 1998. p. 1682–7.

- [17] Yigui X, Yichen Z. Structural damages detection based on dynamic model intelligent updating. In: Proceedings of the 17th International Modal Analysis Conference (IMAC), Orlando, Florida, 1999. p. 1873–9.
- [18] Sawyer JP, Rao SS. Structural damage detection and identification using fuzzy logic. *AIAA J* 2000;38(12):2328–35.
- [19] Kim HM, Bartkiewicz TJ. A two-step structural damage detection approach with limited instrumentation. *J Vibr Acoust* 1997;119(2):258–64.
- [20] Kim HM, Bartkiewicz TJ. An experimental study for damage detection using a hexagonal truss. *Comp Struct* 2001;79(2):173–82.
- [21] Yun CB, Bahng EY. Substructural identification using neural networks. *Comp Struct* 2000;77(1):41–52.
- [22] Wang D, Friswell MI, Nikraves PE, Kuo EY. Damage detection in structural joints using generic joint elements. In: Proceedings of the 17th International Modal Analysis Conference (IMAC), Orlando, Florida, 1999. p. 792–8.
- [23] Law SS, Chan THT, Wu D. Efficient numerical model for the damage detection of a large scale complex structure. *Eng Struct* 2001;23(5):436–51.
- [24] Titurus B. Damage detection of vibrating mechanical systems, PhD thesis, Dept. of Technical Mechanics, Slovak University of Technology, Bratislava (*in Slovak*), 2002.
- [25] Gladwell GML, Ahmadian H. Generic element matrices suitable for finite element model updating. *Mech Syst Signal Process* 1995;9(6):601–14.
- [26] Friswell MI, Mottershead JE, Ahmadian H. Finite element model updating using experimental test data parameterisation and regularisation. *Trans Royal Soc London Ser A (special issue on Experimental Modal Analysis)* 2001; 359:169–86.
- [27] Ahmadian H, Mottershead JE, Friswell MI. Regularisation methods for finite element model updating. *Mech Syst Signal Process* 1998;12(1):47–64.
- [28] Balmès E. Structural Dynamics Toolbox, Users Guide, Version 3.1. Scientific Software Group, Sèvres Cedex, France, 1997.
- [29] Timoshenko SP, Gere JM. Theory of elastic stability. New York: McGraw-Hill; 1921.
- [30] Vlasov VZ. Thin-walled elastic beams. Israel Program for Scientific Translations, Jerusalem, 1961.
- [31] Horton B, Gurgenci H, Veidt M, Friswell MI. Finite element model updating of the welded joints in a tabular H-frame. In: Proceedings of the 17th International Modal Analysis Conference (IMAC), Orlando, Florida, 1999. p. 1556–62.
- [32] Titurus B, Friswell MI, Starek L. Damage detection using generic elements: Part II. Damage detection. *Comp Struct* 2003;81(24–25):2287–99.