An equivalent model of corrugated panels with axial and bending coupling

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ABSTRACT

In this paper, modification of an existing equivalent model of the corrugated panel is investigated. The axial and bending coupling of the corrugated panel supplements the previous equivalent properties when the corrugated panel has a fixed boundary condition. The analytical expressions of the coupling vertical deflections are obtained and verified by the finite element method. A method to eliminate the vertical deflection is proposed, and the importance of the coupling effect is demonstrated by the application of the modified model in a compliant structure.

1. Introduction

Corrugated panels are made of plate with periodic profiles and are often used as cores in sandwich structures. Although the finite element method can be used to evaluate the stiffness of the corrugated structures, the detailed modelling of the profiles will lead to a high computational cost, which motivates the study of equivalent models [1–3].

In recent years the study of morphing aircraft becomes another motivation of the research into corrugated structures [4–6]. The low axial stiffness of the corrugated structure allows for a large deformation with limited actuation force while the high anisotropy of the corrugated structure still provides a large out-of-plane stiffness to carry aerodynamic loads.

The study of equivalent models of corrugated panels plays a significant role in research and applications. Yokozeki et al. [4] developed the analytical solution of the equivalent axial and transverse tensile and flexural moduli with the round corrugation. The analytical solution provided a reasonable accuracy compared to the experimental results. Samanta and Mukhopadhay [7] derived analytical solutions of the axial and transverse equivalent moduli of the trapezoidal corrugated panel. The experimental investigation of Thill et al. [8] showed a three-stage stress-strain relationship of the trapezoidal corrugated panel, in which the first linear stage had a relatively good agreement with the analytical solution modified from [7]. Xia et al. [9] developed a more complete approach of the corrugated panel, which gives the equivalent model of round and trapezoidal corrugations. The method provides the closed forms of the six effective components in the stiffness matrix of the equivalent orthotropic plate. Comparisons to other analytical results and the finite element method have shown the accuracy of the equivalent model. In this method, the equivalent orthotropic plate is a classical Kirchhoff plate, which does not consider the transverse shear. A recent publication from Mohammadi et al. [10] derived the transverse shear modulus, which few researchers have studied. The equivalent models of corrugated cores with elastomeric coatings were also derived by Dayyani et al. [11].

Although the equivalent properties of the corrugated panel have been studied extensively, this paper investigates another aspect, namely the axial and bending coupling when the corrugated panel has a fixed boundary condition, to which little attention has been paid in the literature. The corrugated panel will have a vertical deflection caused by the pure extension load when it has a fixed boundary condition. If the corrugated panel has a pinned boundary condition, no vertical deflection will occur. This deflection-extension coupling effect could be negligible in some cases. But it can also have significant influence on the entire structure, especially for the morphing applications where the corrugated structures are often required to have a relatively large deformation. For example, if the corrugated structure is used as the morphing skin, the tendency of the out-of-plane deflection could change the wing shape or increase the actuation energy. The current investigation can also provide guidelines for installing the corrugated panels in real-world applications. In the previous
references, the symmetry of the structure is usually applied and only half \[7\] or quarter \[4,10\] of the corrugation unit is analysed. The fixed boundary condition is applied to calculate the strain energy and internal moment. For the homogeneous method \[9\] the boundary condition will not affect the equivalent properties of the entire structure since the boundary conditions are constrained when calculating the strain energy.

In this paper, with the fixed boundary condition, vertical deflections under pure extension load will be analysed together with axial deflections. The axial deflections are applied to calculate the equivalent axial modulii, which are compared to those from the existing models. The coupling effect can be represented as an effect of the boundary condition if the entire corrugated panel is separated into two segments. As a supplement to the previous equivalent model, a modification to the equivalent model proposed in Ref. \[9\] is made by introducing a coupling component. After representing this deflection, the influence of considering this coupling effect is shown on its potential application to a compliant structure. The method to eliminate this coupling is also proposed by using an offset boundary condition.

2. Deflections caused by extension loads

Fig. 1 shows the shape of the trapezoidal and round corrugation panels, which are the subject of this study. As a periodic structure in the xz plane, the entire shape and size of a corrugated structure is determined by the shape of a basic single unit and the number of units.

Fig. 2 shows the unit geometry and the internal bending moment for the corrugated shapes. The corrugation unit is fixed at one end and under extension load \(T\) at the other end. As shown in the figure, the trapezoidal unit consists of the straight beams \(AB, BC, CD, DE, EF\) and \(FG\), and the round unit is made of straight beams \(AB, CD, EF\) and curved beams \(BC, DE\).

The deflection of each separate beam is calculated using classical mechanics and Castigliano’s second theorem. The deflection of the unit corrugation is then calculated by accumulating the local deflections of the beams and considering the rigid translation caused by the rotation angles of the beam cross sections. Fig. 3 shows the schematic of the deflection components. For a beam with the inclined angle \(\theta\), the deflection of the beam \(p\), which is...
perpendicular to the beam, can be decomposed to the axial deflections \( u \) and the vertical deflection \( w \). According to the inclined angle and direction definition, we have \( u = -p \sin \theta \), and \( w = p \cos \theta \). Also, the rotation angle of the beam cross section will lead to a rigid translation. The rotation angle \( \alpha \) of an axial beam will cause a vertical deflection \( w \) and an axial deflection \( u \) can be obtained if the rotation angle occurs in a vertical beam. The axial and vertical deflections of the trapezoidal and round corrugation unit are both calculated.

As shown in Fig. 2(a), \( l_1, l_2 \) and \( \theta \) are the length of beams \( AB \) and \( BC \), and the inclined angle respectively in the trapezoidal corrugation. The shape of the trapezoid can also be represented by the length \( c \) and the height \( f \). These beams are rigidly connected to the previous part and thus the global vertical deflection of a point depends on the deformation on the previous beams. For example the deflection of point \( C \) includes three parts: the global vertical deflection of point \( B \), the local vertical deflection of beam \( BC \) and the rigid translation of point \( B \) caused by the rotation angle of the beam cross section at point \( B \). The rotation angle of the beam cross section will also be accumulated by the local rotation angle. Then, the global vertical deflection of point \( C \), \( w_C \), can be calculated as

$$w_c = w_B + w_{BC} + \alpha_B \cdot l_2$$  \hspace{1cm} (1)

where \( p, u \) and \( w \) are the deflection value perpendicular to the beam, in the axial or vertical direction respectively. \( \alpha \) is the rotation angle of the beam cross section. The subscript ‘\( AB \)’ or ‘\( BC \)’ means the local deflection or beam cross section rotation angle of that beam, the subscript of a single point ‘\( B \)’ or ‘\( C \)’ means the global value at that point.

Thus, \( w_{BC} \) is the local vertical deflection of beam \( BC \), and \( w_B \) and \( \alpha_B \) are the global deflection and rotation angle of the cross section of point \( B \). Also for local deformation of beams \( AB \) and \( BC \), we have

$$p_{AB} = \frac{T l_1^2 \sin \theta}{6EI}, \quad \alpha_{AB} = \frac{T l_1^2 \sin \theta}{2EI} \hspace{1cm} (2)$$
$$w_B = p_{AB} \cos \theta = \frac{T l_1^2 \sin \theta \cos \theta}{6EI} \hspace{1cm} (3)$$
$$u_B = -p_{AB} \sin \theta = -\frac{T l_1^2 \sin \theta \sin \theta}{3EI} \hspace{1cm} (4)$$
$$p_{BC} = \frac{T l_2^2 \sin \theta}{2EI}, \quad \alpha_{BC} = \frac{T l_2^2 \sin \theta}{2EI} \hspace{1cm} (5)$$

where \( E \) is the Young’s modulus, \( l \) is the second moment of area and \( T \) is extension force in the x direction. Repeating the above steps for the other beams, the deflections and rotation angle at point \( G \) can be calculated, which are equal to the deflections and rotation angle of the unit as

$$u_{unit} = u_G = \frac{2}{3} \frac{T l_1^2 (\sin \theta)^2 (2l_1 + 3l_2)}{EI} \hspace{1cm} (7)$$
$$w_{unit} = w_G = \frac{T l_1 \sin \theta (l_1 + l_2) (l_2 + 2l_1 \cos \theta)}{EI}$$
$$\alpha_{unit} = \alpha_G = 0$$

For the round corrugation, the same approach can be applied to obtain the vertical deflection and rotation angle of a single unit. The geometry and internal bending moment of the round corrugation unit is shown in Fig. 2(b). The curved beams are assumed to be thin enough compared to the radius \( R \), that the beam theory for a straight beam may be applied. The local vertical deflection of the curved beam \( BC \) is obtained by the Castigliano’s second theorem as

$$w_{BC} = \int_0^x M \phi M \frac{EI}{(\phi M)'} \cdot Rd\phi$$

(8)

where \( M \) is the bending moment in the curved beam \( BC \), and \( \phi \) is the current angle with respect to the x axis. \( M_a \) and \( F_p \) is virtual vertical force applied at point \( C \) to obtain the vertical deflection.

The deflections and rotation angle of a round corrugation unit can be obtained using the same approach to the trapezoidal corrugation as

$$u_{unit} = u_F = \frac{1}{3} \frac{1}{EI} \left[ T(4l_1^2 + 6n l_2^2 + 24l_2 R + 3\pi R^2) \right]$$
$$w_{unit} = w_F = \frac{2TR(l_1^2 + 2R^2 + \pi LR)}{EI}$$
$$\alpha_{unit} = \alpha_F = 0$$

From the above equation, we can find the rotation angle of the beam cross section will be zero again after a whole unit, which means for the next unit the previous unit will not affect its global deflection by the rigid translation. It is caused by the symmetrical internal bending moment in the whole corrugation unit. Thus the global deflections of \( n \) units can be expressed as

$$u_n = n \cdot u_{unit} \hspace{1cm} (9)$$
$$w_n = n \cdot w_{unit}$$

The deflections and beam cross section rotation angle of the different points in the trapezoidal and round corrugation unit are listed in Tables 1–3. In the tables we can find the beam cross section rotation angles are symmetrical about the centre line of the corrugation unit.

Table 1
<table>
<thead>
<tr>
<th>Axial deflections in the corrugation unit.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point</strong></td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td><strong>Trapezoidal corrugation unit</strong></td>
</tr>
<tr>
<td>( B )</td>
</tr>
<tr>
<td>( C )</td>
</tr>
<tr>
<td>( D )</td>
</tr>
<tr>
<td>( E )</td>
</tr>
<tr>
<td>( F )</td>
</tr>
<tr>
<td>( G )</td>
</tr>
</tbody>
</table>
Table 2
Rotation of the beam cross section and vertical deflections in the trapezoidal corrugation unit.

<table>
<thead>
<tr>
<th>Point</th>
<th>Rotation angle of the beam cross section</th>
<th>Vertical deflection $w_i = A, B, C, \ldots G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$\frac{1}{2} \pi \sin \theta$</td>
<td>$\frac{1}{2} \pi \sin \theta \cos \theta$</td>
</tr>
<tr>
<td>$C$</td>
<td>$2$</td>
<td>$\frac{1}{2} T \sin \theta (l_1 + 2 l_2)$</td>
</tr>
<tr>
<td>$D$</td>
<td>$\frac{1}{2} T \sin \theta (l_1 + 2 l_2)$</td>
<td>$\frac{1}{2} T \sin \theta \cos \theta + 3 l_1 l_2 + 3 l_2^2$</td>
</tr>
<tr>
<td>$E$</td>
<td>$2$</td>
<td>$\frac{1}{2} T \sin \theta (l_1 + 2 l_2)$</td>
</tr>
<tr>
<td>$F$</td>
<td>$0$</td>
<td>$\frac{1}{2} T \sin \theta (l_1 + 2 l_2)$</td>
</tr>
</tbody>
</table>

Table 3
Rotation of the beam cross section and vertical deflections in the round corrugation unit.

<table>
<thead>
<tr>
<th>Point</th>
<th>Rotation angle of the beam cross section</th>
<th>Vertical deflection $w_i, i = A, B, C, \ldots F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$\frac{1}{2} \pi \sin \theta$</td>
<td>$0$</td>
</tr>
<tr>
<td>$C$</td>
<td>$2$</td>
<td>$T \sin (\theta + 2 \pi L R)$</td>
</tr>
<tr>
<td>$D$</td>
<td>$\frac{1}{2} T \sin \theta (l_1 + 2 l_2)$</td>
<td>$T \sin (\theta + 2 \pi L R)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$\frac{1}{2} T \sin \theta (l_1 + 2 l_2)$</td>
<td>$2 T \sin (\theta + 2 \pi L R)$</td>
</tr>
<tr>
<td>$F$</td>
<td>$0$</td>
<td>$2 T \sin (\theta + 2 \pi L R)$</td>
</tr>
</tbody>
</table>

3. Verification of the closed Form moduli and the equivalent model

The axial deflections are used to calculate the equivalent modulus in the axial direction. The equivalent modulus in the axial direction can be expressed for the trapezoidal corrugation as

$$E_x = \frac{T}{\text{wunit}/(2c)} = \frac{3EI(2l_1 \cos \theta + l_2)}{S(l_1 \sin \theta)(2l_1 + 3l_2)}$$

(11)

where $S$ is the area of the cross section of the corrugation.

For the round corrugation we have

$$E_x = \frac{12EI}{S(4L^3 + 6 \pi L R^2 + 24L R^2 + 3 \pi R^3)}$$

(12)

The axial equivalent moduli are then compared to those from existing methods and the finite element method. The finite element models are built in Abaqus [12]. To ensure accuracy, a mesh convergence study is first performed, which gives a mesh size 0.001 m. Both the Euler beam element B33 and the Timoshenko beam element B31 are used for the verification, which are labelled as ‘FEM (A)’ and ‘FEM (B)’ in Figs. 4 and 5 respectively. The equivalent methods from [4,7,9] are applied for the comparisons. The corrugated panel is assumed to be made of Aluminum sheet with a width 0.01 m. Fig. 4 shows the equivalent axial modulus of the trapezoidal corrugation panel with different corrugation shapes. The proposed method has results identical to those from [7] since in the reference the equivalent modulus was also calculated by dividing the deflection under extension loads, although only half of the corrugation was considered. The homogenous method has some differences to the proposed method. Compared to the finite element method, all the analytical methods have some errors. The error is relatively large when the plate becomes thick compared to the size of the trapezoid. Also the Euler beam element has closer results to the proposed method since the method uses the classical beam theory and no transverse shear in the beam is considered. If the corrugated panel is supposed to be applied in the field of morphing structures, the beam will be relatively thin since the large thickness of the beams will increase the stress in the structures, which constrains the deformation limit of the morphing structure. The beams with large thicknesses (e.g. 6 mm) are impractical, since the accuracy of the beam models would be poor and the maximum stresses would be high, but these results are included because the errors are so small for the thicknesses that would be used in practice (e.g. less than 2 mm).

The axial equivalent modulus of the round corrugation panel is shown in Fig. 5. The homogenous method [9] provides very close results to the method proposed by Yokozeki et al. [4]. And the proposed method generally has a smaller error compared to the finite element than the trapezoidal corrugation panel, especially for the Euler beam element. The differences are due to the beam extension. The proposed method only considers the deflections caused by the bending of the beams. As shown in Fig. 6, if the beam extension is included, the errors can be reduced significantly. Detailed analytical derivation also points out the extension has very small influence on the vertical deflections of trapezoidal corrugation panels and both axial and vertical deflections of round corrugation panels.

Also we can represent the coupling coefficient $C_{ew}$ between the extension force and the vertical deflection of the corrugated panel as

$$C_{ew} = \frac{W_n}{T} = \frac{n - \text{wunit}}{T}$$

(13)

For the trapezoidal corrugation

$$C_{ew} = \frac{l_1 \sin(\theta)(l_1 + l_2)(l_2 + 2l_1 \cos(\theta))}{EI}$$

(14)

For the round corrugation

$$C_{ew} = \frac{2R(L^2 + \pi LR + 2R^2)}{EI}$$

(15)

Then the vertical deflections of the corrugation panels are verified by the finite element method. Each finite element model has 10 units. The Euler beam element is used in the finite element model and the extension load is 1 N/mm. Fig. 7 shows a linear relationship between the vertical deflection and the unit number. Different combinations of the shape of the corrugation unit are tested with the variable $t$ representing the thickness of the corrugation panel. As shown in the figure, the analytical solution has a good agreement with results obtained by the finite element model, which verifies the analytical expressions of the vertical deflection and coupling coefficient.

A general equation is used to express the axial and bending coupling effect. As shown in Fig. 8, the corrugated panel is represented by two segments: the first corrugation unit which takes the boundary condition into account, and the second segment represents the rest of the corrugation units. Then the vertical deflection of the entire structure $w_n$ can be expressed as

$$w_n = \tan \gamma_{xz} \cdot L_n$$

(16)

$$\tan \gamma_{xz} \equiv \frac{W_1}{2c}$$

where $L_n$ is the length of the corrugated panel, and $w_n = n - 2c$, and the vertical deflection of the first segment is denoted by $w_1$. By dividing the entire structure into two segments, the vertical deflection due to the fixed boundary condition can be represented.
by the rotation of the first segment. If the corrugated panel has a pinned boundary condition, no rotation angle will be generated. If the corrugated panel has a fixed boundary condition, the rotation angle \( \gamma_{xz} \) of the first corrugated unit will be constant in the entire structure. This shows how the boundary condition can affect the coupling terms, which is considered a supplementary to the previous model.

A modification can be made to include the vertical deflection into the equivalent model. Another row (and column) is added to the global flexibility matrix on the basis of Ref. [9], in which \( S_{44} \) is the transverse component in the plane \( xz \) and \( S_{14} \) is the coupling component between the loading in the \( x \) direction and the shear strain in the \( xz \) plane.

When only extension load is applied, the load factor is \([N_x \ 0 \ 0 \ 0 \ 0 \ 0]^{T}\). The shear strain \( \gamma_{xz} \) is

\[
\gamma_{xz} = N_x S_{14}
\]

Thus,

\[
S_{14} = \frac{\gamma_{xz}}{N_x} = \frac{a \tan(w_n/(n \cdot 2c))}{N_x}
\]

where \( N_x \) is the force per unit width of the corrugated panel.

4. Method to eliminate the vertical deflection from extension

Section 3 investigates the vertical deflection caused by the extension loads when the corrugated panel has a fixed boundary condition. However this deflection is not wanted in some cases, such as span morphing or camber morphing applications, since the vertical deflection may change the aerodynamic shape or increase the required actuation force. If pinned boundary conditions can be used in these cases, the vertical deflections can be eliminated. But in some cases it is necessary to apply fixed boundary conditions for corrugated panels. The following section provides a solution to eliminate the deflection. The concept is to
use the opposite rotation angle of the beam cross section, which is able to introduce an opposite vertical deflection. The opposite beam rotation is obtained by providing an offset to the initial boundary condition. As shown in Fig. 9, the offset of the corrugated panel \( L_0 \) generates the vertical deflection \( w_0 \) and rotation angle \( \alpha_0 \).

The rotation angle \( \alpha_0 \) will lead to the vertical deflection \( w_r \) in the opposite direction to \( w_n \). Then the total vertical deflection with the offset boundary condition can be represented as

\[
W_{\text{offset}} = w_0 + w_r + w_n
\]

\[
= w_0 + \alpha_0 \cdot L_0 + w_n
\]
For the trapezoidal corrugation,

\[ w_0 = \frac{Tl_1 \sin \theta - \cos \theta}{3EI} \]

\[ z_0 = -\frac{Tl_2 \sin \theta}{2EI} \cdot nl \]

Then,

\[ w_{offset} = \frac{Tl_1 \sin \theta}{3EI} \cos(\theta) - \frac{Tl_2 \sin \theta}{2EI} (n \cdot 2c) + n \left( Tl_1 \sin \theta (l_1 + l_2) (l_2 + 2l_1 \cos \theta) \right) \]
Making the above equation zero will give a cubic equation of $l_0$. For the round corrugation, we have

$$w_0 = 0$$

$$x_0 = \frac{\pi l_0^2}{2EI}$$

$$w_{offset} = -\frac{\pi l_0^2}{2EI} (n \cdot 4R) + n \frac{2TR(L^2 + 2R^2 + \pi LR)}{EI}$$

Making the above equation zero, we can obtain the offset as

$$l_0 = \sqrt{L^2 + 2R^2 + \pi LR}$$

(25)

The offset boundary condition will also affect the axial deflection, which changes the equivalent axial modulus. For the round corrugationpanel, the axial deflection of the offset part is

$$u_0 = \frac{\pi l_0^2}{3EI}$$

(26)

Thus, the change of the equivalent axial modulus is

$$\frac{du}{u_0} = \frac{u_0 - u_{non}}{n - u_{non}}$$

(27)

5. Application to a morphing structure

A novel concept of compliant structure has been proposed by the authors [13,14] and the proposed method is applied to calculate the deformation of the compliant structure when round corrugated panels are used. As shown in Fig. 10, the compliant structure consists of upper, lower and middle beams, which are represented as $AB$, $DC$ and $BC$.

As shown by the red dashed lines in Fig. 10, if the upper and lower beams have unsymmetrical stiffness ($E_1 A_1 < E_2 A_2$), rotation of the structure can be induced by linear actuation. Analytical equations have been derived, and details of the derivation can be found in Appendix A. But the coupling effect of the extension force and the vertical deflection is not included, which causes errors between the analytical results and the finite element method. According to the study in the literature [14], the axial and vertical deflection $u_x$ and $v_x$ of the compliant structure can be expressed as

$$u_x = -\frac{F}{E} \frac{2a E_1 l_1 + 2E_2 l_2 + b^2}{K_x} (E_1 A_1 + E_2 A_2)$$

$$v_x = \frac{F b a^2}{2} \frac{E_2 A_2 - E_1 A_1}{K_x}$$

(28)

where $K_x$ is the combination of the axial and bending stiffness of both upper and lower beams, which can be expressed as

$$K_x = E_1 A_1 E_1 l_1 + E_2 A_2 E_2 l_2 + E_1 A_1 E_2 l_2 + E_1 l_1 E_2 A_2$$

$$+ E_1 A_2 E_2 b^2$$

(29)

$E_1$, $A_1$, $l_1$, $E_2$, $A_2$, $l_2$, $E_3$, $A_3$, $l_3$ are the modulus, area and second moment of area of the upper, lower and middle beams, respectively. The length and height of the structure are denoted $a$ and $b$ respectively, $F$ is the actuation force. In the study the round corrugation panel is used as the upper and lower parts of the compliant structure. And the equivalent value of the corrugation panel should be applied to substitute the corresponding term.

According to Eq. (28), no vertical deflection should be obtained when the upper and lower beams have the same extension stiffness ($E_1 A_1 = E_2 A_2$). However results from the detailed finite element models have shown differences. Modification is made to include the vertical deflection caused by the extension force.

The new prediction of the vertical deflection of compliant structure consists of two components, namely the deflection calculated from Eq. (28) and the vertical deflection due to the extension force. As shown in Fig. 10, the vertical deflections of point $B$ and $C$, i.e., $v_B$ and $v_C$, satisfy the relationship as

$$z_{BC} = b' \cos \beta$$

$$z_{BC} = z_{BC} - B' B' = b + C' C - B' B' = b + v_C - v_B$$

(30)

Then we have

$$v_B - v_B = b' \cos \beta - b$$

(31)

where $b'$ is the deformed length of $BC$, $\beta$ is the inclined angle of the middle beam $BC$. If the middle beam $BC$ is assumed to be rigid enough, we can obtain

$$v_B = v_C$$

(32)

From the early study, we determined that the original vertical deflections of points $B$ and $C$ are the same, and thus the vertical deflections of points $B$ and $C$ due to its extension force $v_{BM}$ and $v_{CM}$ are also the same. According to Eq. (13), we have

$$v_{BM} = C_{xcw}^{up} A_1$$

$$v_{CM} = C_{xcw}^{low} X_1$$

(33)

where $C_{xcw}$ is the coupling coefficient of the corrugated panel. The superscript 'up' and 'low' correspond to the upper and lower corrugated panels. $A_1$ and $X_1$ are the extension forces in the two corrugated panels. Since $A_1 + X_1 = F$, and $v_{BM} = v_{CM}$, we can deduce the vertical deflection of the compliant structure due to the extension force $F$ is

$$v_{BM} = \frac{v_{BM} + v_{CM}}{2} = \frac{C_{xcw}^{up} C_{xcw}^{low}}{C_{xcw}^{up} + C_{xcw}^{low}} F$$

(34)

The modified analytical method is then compared to the finite element method, as well as the original analytical method. The finite element analysis is performed in Abaqus [12] and a rigid body constraint is applied to the vertical beam to simulate its rigidity. Mesh convergence has also been verified before the analysis and the general purpose shell element S4R in Abaqus is used since the element is able to simulate both thin and thick plates. The modified analyt-

![Fig. 10. A compliant structure making use of the unsymmetrical stiffness of a corrugation panel.](image-url)
ical solution is labelled as ‘Method A’ and the analytical solution, which does not take the coupling effect into account, is labelled as ‘Method B’ in Fig. 11.

Fig. 11 shows four different cases with different combinations of shapes and number of units. The corrugated panel is supposed to be made of Aluminum and $t_1$ and $t_2$ are the thicknesses of the upper and lower corrugated panels. The compliant structure is actuated by the force $F = 100 \, \text{N}$ and $t_1$ is equal to 0.002 m. The width of the structure is fixed at 0.01 m. The x axis represents the change of the ratio of the lower and upper panel thickness, which indicates the change of the stiffness asymmetry. We can conclude the Method A is able to predict the vertical deflections more accurately than the Method B, especially when the upper and lower corrugation panel has the same stiffness. It is found that the vertical deflection due to extension force can be even larger than the maximum deduced deformation from stiffness asymmetry in some cases, when the height of the compliant structure is 0.2 m, which highlights the importance of taking the effect into account in this application.

6. Conclusion

In this paper, the vertical deflection under the pure extension load has been investigated when the corrugated panel has the fixed boundary condition, which is called axial and bending coupling of the corrugated panel. Trapezoidal and round corrugation panels are analysed. The corrugated panel is simplified as a two dimensional frame, which is made of classical beams. Detailed analysis of the internal bending moment shows the rotation angle of the beam cross sections and deflections of different points in the corrugated panel.

Analytical expressions of the vertical deflections are obtained by accumulating the local deflections and rigid translation due to beam rotation. The axial deflections are also obtained, which are applied to calculate the equivalent modulus in the axial direction. The deflections are verified by the existing methods in the literature and the finite element method. It is also found that the beam extension needs to be considered for the prediction of the equivalent axial modulus of trapezoidal panels.

The vertical deflection is found to be linear in terms of the number of corrugation units if the corrugated panel is assumed to be linear. In this case, the coupling between the vertical deflection and the pure extension load can be represented as an effect of the boundary condition of the corrugated panel. To include the coupling effect into the previous equivalent model, a coupling component is added to the equivalent flexibility matrix of the corrugated panel. The inverse coupling effect between the vertical force and the axial deflection can also be derived using a similar method, although this is not the focus of the current paper.

With the help of the modified model, the application of corrugated panels in morphing structures is shown. The importance of the coupling effect between the extension force and vertical deflection is highlighted in the study of a compliant structure. The differences between the analytical solution and the detailed finite element models can be reduced significantly if the coupling effect is taken into account. The approach to eliminate the vertical deflections is also introduced by providing an offset to the fixed boundary condition.
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Appendix A

This section shows the process of calculating the displacements of the proposed compliant structure. Since the upper and lower beams are fixed, there are six unknown reaction forces as shown in Fig. A1, which are related to only three independent equations of equilibrium, thus the structure is statically indeterminate. The reaction forces applied at point D are chosen as redundant reactions. Then the structure becomes statically determinate with $X_1$, $X_2$, $X_3$ applied at point D.

To satisfy the original boundary condition, the redundant reactions will make the horizontal displacement $u$, vertical displacement $v$ and rotation $\alpha$ at point D zero. Using the principle of superposition, $u$ can be expressed as

$$u = u_F + u_{X_1} + u_{X_2} + u_{X_3}$$  \hspace{1cm} (1)

where $u_F$ is horizontal displacement caused by the actuation force $F$ alone, and $u_{X_i}$ ($i = 1, 2, 3$) is the horizontal displacement caused by the reaction $X_i$ alone.

According to the principle of virtual work, the displacement under an external load can be expressed as

$$u = \int m \frac{M}{ET} dx + \int n \frac{N}{EA} dx$$  \hspace{1cm} (2)

where $M$ and $N$ are the bending moment and axial force under the external load, and $m$ and $n$ are the bending moment and axial force under the unit load applied in the same direction as the required displacement. Fig. A2 shows the internal bending moment and axial force caused by $F$ and the unit load, which is in the same direction to $X_i$ ($i = 1, 2, 3$). Thus, using the above equation, we obtain

$$u_F = \int_0^a -b \cdot \left( \frac{1}{2} F \frac{b}{E I_1} \right) dx + \int_0^a \frac{1 - F}{E I_1} dx$$

$$u_{X_1} = \int_0^a 1 \cdot \frac{X_1}{E A} dx + \int_0^a \frac{b}{E A} \frac{b X_1}{E I_1} dx$$

The same method can be used to get the other components of $u$, as well as the vertical displacement $v$ and rotation $\alpha$. Finally, at point D we obtain

$$u = \frac{Fa}{E I_1} - \frac{Fab}{2 E I_1} + \left( \frac{ab^2}{E I_1} + \frac{a^2 b}{E I_2} + \frac{a}{E I_3} \right) X_1 + \frac{a^2 b}{E I_1} X_2 - \frac{ab}{E I_1} X_3$$

$$v = \frac{a^2 b}{4 E I_1} F + \frac{a^2 b}{2 E I_1} X_1 + \frac{a^2}{3 E I_1} X_2 - \left( \frac{a^2}{2 E I_1} + \frac{a^2}{2 E I_2} \right) X_3$$

$$\alpha = \frac{Fab}{2 E I_1} \frac{ab}{E I_1} X_1 - \left( \frac{a^2}{2 E I_1} + \frac{a^2}{2 E I_2} \right) X_2 + \left( \frac{a}{E I_1} + \frac{a}{E I_2} \right) X_3$$  \hspace{1cm} (5)

Fig. A1. Redundant forces and the geometry relationship of point B, C, and E.

Fig. A2. Reactions and internal loads.
Making the displacement and rotation at point D zero, the redundant reactions can be solved to give

\[
X_1 = \frac{E_2 A_2 (E_1 A_1 b^2 + 2E_1 I_1 + 2E_2 I_2)}{(E_1 A_1 E_1 I_1 + E_1 A_2 E_2 I_2 + E_1 A_1 E_2 A_2 + E_1 A_2 E_1 A_2 b^2)} \frac{F}{2}
\]

\[X_2 = 0\]

\[
X_3 = \frac{E_2 J_2 (E_1 A_1 - E_2 A_2)}{(E_1 A_1 E_1 I_1 + E_2 A_2 E_2 I_2 + E_1 A_1 E_2 A_2 + E_1 A_2 E_2 A_2 b^2)} \frac{Fb}{2}
\]

Then, we can obtain the reactions at point A from the equations of equilibrium as shown in Fig. A3.

According to the equilibrium of the whole structure,

\[F - AX_1 - X_3 = 0\]

\[AX_2 - X_2 = 0\]

\[AX_3 + X_1 b - \frac{1}{2} Fb - X_3 = 0\]

Thus,

\[AX_1 = \frac{E_1 A_1 (E_2 A_2 b^2 + 2E_1 I_1 + 2E_2 I_2)}{(E_1 A_1 E_1 I_1 + E_2 A_2 E_2 I_2 + E_1 A_1 E_2 A_2 + E_1 A_2 E_1 A_2 b^2)} \frac{F}{2}\]

\[AX_3 = \frac{E_1 I_1 (E_1 A_1 - E_2 A_2)}{(E_1 A_1 E_1 I_1 + E_2 A_2 E_2 I_2 + E_1 A_1 E_2 A_2 + E_1 A_2 E_2 A_2 b^2)} \frac{Fb}{2}\]

Applying the equations of equilibrium to the upper and lower beams separately gives the internal forces and moments at point B and C, after which the displacements of point B and C can be obtained.

For the upper beam AB:

\[BX_1 - AX_1 = 0\]

\[BX_2 - AX_3 = 0\]

\[AX_3 - BX_3 = BX_3 a = 0\]

\[u_b = BX_3 a / (E_1 A_1)\]

\[v_b = -BX_3 a^2 / (2E_1 I_1) - BX_3 a^3 / (3E_1 I_1)\]

So,

\[a = \frac{F - E_2 A_2 (E_1 A_1 b^2 + 2E_1 I_1 + 2E_2 I_2)}{2K_t E_1 A_1}\]

\[u_b = \frac{F - E_2 A_2 (E_1 A_1 b^2 + 2E_1 I_1 + 2E_2 I_2)}{2K_t E_1 A_1}\]

\[v_b = \frac{-E_2 A_2 (E_1 A_1 b^2 + 2E_1 I_1 + 2E_2 I_2)}{2K_t E_1 A_1}\]

\[u_c = \frac{CX_1 a}{E_2 A_2}\]

\[v_c = \frac{CX_3 a^2 + CX_3 a^3}{2E_2 I_2}\]

Thus

\[u_c = \frac{F}{2} \frac{2E_1 I_1 + 2E_2 I_2 + E_1 A_1 b^2}{K_t}\]

Point E is the midpoint of beam BC. As shown in Fig. A1, BB', CC', EE' are the horizontal displacements of points B, C and E respectively, and B'B', E'E', C'C' are the vertical displacements of points B, C and E. According to the geometry relationship,

\[u_l = \frac{u_b + u_c}{2}\]

\[v_l = \frac{v_b + v_c}{2}\]

\[u_l = \frac{F - 2E_1 I_1 + 2E_2 I_2 + E_1 A_1 b^2}{2K_t}\]

\[v_l = \frac{v_b + v_c}{2}\]

\[v_l = \frac{Fb a^3 E_2 A_2 - E_1 A_1}{2K_t}\]

References


ABAQUS 6.13, 2013, Dassault Systèmes.
