Optimisation of composite corrugated skins for buckling in morphing aircraft

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A R T I C L E   I N F O

Article history:
Available online 15 September 2014

Keywords:
Morphing skins
Corrugation
Buckling

A B S T R A C T

Morphing aircraft aim to increase the performance of aircraft over multiple flight conditions, by enabling shape changes in flight in order to optimise their aerodynamic properties for the current conditions. The skin of a morphing aircraft is a critical component. It must be compliant in degrees of freedom that are required for actuation, to minimise the actuation loads. However, it must also carry structural loads, and therefore be stiff in load bearing degrees of freedom. This leads to a requirement for extremely anisotropic material systems. A common solution is the use of a corrugated panel. However, previous work on corrugations has not addressed the problem of compressive buckling loads. This work analyses the performance of corrugated panels under buckling loads, and optimises corrugation patterns for the objectives of weight, buckling performance, and actuation compliance. Simplified analytical models that derive properties equivalent to conventional plates are used to obtain approximate estimates of the buckling loads. Furthermore a new mode of buckling, that occurs entirely in-plane and is unique to panels with extreme anisotropy is analysed. The simple models allow optimisation to be performed, and both a single-objective and a multi-objective approach are demonstrated. The results are compared to Finite Element Analysis.

1. Introduction

Conventional aircraft wings are a compromise between multiple requirements; for example a high maximum lift coefficient is required for low-speed flight near take off and landing, whereas efficiency in cruise requires a high lift-to-drag ratio. A large body of literature is now dedicated to achieving a wing that can smoothly alter its shape to meet these conflicting requirements more completely over a whole flight, a concept generally referred to as a morphing wing [1,2]. Morphing wings technologies can alter planform properties, for example by altering sweep or aspect ratio, or change their aerofoil section, in terms of properties such as camber.

A critical component of a compliant morphing structure is the morphing skin, which has to be flexible in degrees of freedom required for actuation, to allow actuation loads to remain small, so that lightweight actuators may be used. However, typically it must also bear some load to keep the aircraft structural weight down, meaning that it must remain stiff in load bearing degrees of freedom. Therefore the skin must exhibit extreme anisotropy to meet these otherwise competing requirements [3,4].

Prior literature on morphing skins has addressed these requirements with three major design strategies; the first is the use of Flexible Matrix Composites (FMCs) [5,6], the second is the use of an elastomeric skin supported by a honeycomb-like substructure [7,8], and the third is the use of corrugation [9–12]. Often, these approaches overlap to some extent; for example corrugated skins will frequently require an elastomer covering to provide a smooth aerodynamic surface [10]. This work will focus on corrugated skins, for reasons that will become clear.

In this work the focus is on section morphing concepts, such as the FishBAC concept [13] shown in Fig. 1, which has been shown to achieve changes in lift coefficient with lower drag penalties than an equivalent conventional section with a hinged flap. In the skins of such sections, the principal actuation degree of freedom is in-plane strain in the chordwise direction; it can be seen from Fig. 1 that to achieve the deformation shown requires chordwise extension of the upper skin, and chordwise compression in the lower skin. In the context of a typical aircraft wing structure, the principal loading degree of freedom is spanwise in-plane strain. As shown in Fig. 2, in order to resist the bending loads on the wing,
the lower and upper skins must resist spanwise tension and compression respectively.

One aspect of the corrugated skin that has received relatively little attention with regard to morphing is that of buckling stability under compression in the loading direction. If the wing in Fig. 2 is a conventional (non-morphing) structure, the compact buckling of the upper skin will typically be a principal design driver [14]. If it is decided that some or all of the wing in Fig. 2 is to adopt a camber morphing concept, the upper morphing skin must bear a significant share of the compressive load if the structural weight is to be kept low. Flexible or elastomeric skins will be of little use in this situation, as they typically require to be in tension to withstand buckling in the upper skin. However, in order to achieve such a structure the buckling characteristics of the corrugation must be understood.

![Fig. 1. A FishBAC [13] camber-morphing aerofoil section. Red dashed box indicates the solid leading section, where bending loads are resisted. The green solid-line box indicates the approximate region of the thickest section, where bending loads may be resisted more efficiently; however this region overlaps with the morphing region, motivating the requirement for load-bearing morphing skins. Photograph used with the kind permission of B. K. S. Woods. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image1)

![Fig. 2. Origin of compressive loads in the upper wing skin.](image2)

An improvement to the section of Fig. 1 that allows the morphing skin to assist with structural loads is shown schematically in Fig. 3. This approach is a morphing equivalent to the use of ribs and stringers found in conventional aircraft structures [14]. The morphing skin has its area delimited into sections by ribs in the spanwise direction and stringers in the chordwise direction, that restrict its possible buckling modes to ensure a sufficient critical load. The corrugation poses problems for the boundary conditions that may be induced by the rib; for example if the rib rigidly supports the corrugation along the entire chord it will suppress its ability to extend. Therefore, the majority of the corrugation is supported by ‘minor fishbones’ that provide vertical support but not lateral restriction. Periodically along the chord, there are ‘major fishbones’ that have their lateral position rigidly controlled by the actuation system, and support stringers that run along the span to the next rib. This section provides a means of morphing whilst simultaneously providing a lightweight wing section that can withstand buckling in the upper skin. However, in order to achieve such a structure the buckling characteristics of the corrugation must be understood.

In the current work, the authors consider the design of an idealised panel representing a section of the upper surface of the camber-morphing wing shown in Fig. 3. The panel is subjected to compressive in-plane load in the span direction of the wing, with the requirements of having high in-plane compliance in the chordwise direction and a corrugation depth that is within a prescribed maximum. A trapezoidal corrugation is optimised to give the lowest weight, given certain choices of panel dimension. The analytical methods used make use of classical buckling models with homogenised plate properties to obtain approximate solutions, giving low cost solutions that are suitable for use in system level analyses and optimisation. Both global and local out-of-plane buckling modes are considered, and it is also shown that the extreme anisotropy of the morphing skin permits the existence of a further buckling mode that occurs entirely in-plane. This in-plane mode is unique to highly anisotropic plates; a simple analytical model is derived for the critical buckling load.

The models are used to perform an optimisation of a corrugated panel of fixed dimensions subject to compressive load, to understand the weight penalty that the compliance required for morphing may incur. It is also shown that using multiple laminates in the corrugation can lead to further weight savings compared to a constant-laminate corrugation. An FE analysis is used to evaluate the accuracy of the approximate methods used. Finally a multi-objective optimisation is performed, to give general insight into the optimal shape of a corrugation based on the operating conditions.

2. Problem definition

2.1. Idealised buckling panel

For this study, many of the complicated geometrical and manufacturing issues associated with a full morphing wing section design are neglected, and the problem is idealised as a periodic structure under compressive in-plane loads as shown in Fig. 4. In a similar manner to a classical non-morphing skin design [14], the panel is divided into subsections of dimensions $a \times b$ to provide buckling stability.

The subsections are delimited by stringers and ribs, which are assumed to provide pinned support in all translational degrees of freedom for the purpose of analysis. However, it is important to note that the ribs will not be conventional rigid ribs but camber-morphing ribs, for example similar to the concepts proposed in [13,15,16]. The use of concepts such as these will often lead to ribs...
that are far less rigid than an equivalent conventional aircraft rib. Furthermore, inspection of Fig. 5 reveals that pinning the corrugation profile all along its length could restrict its ability to deform along its length. Therefore the assumptions regarding boundary conditions will need to be considered carefully in actual design, however these issues are neglected for now.

It is assumed that failure occurs at the onset of buckling, and therefore no attempt is made to model post buckle strength.

2.2. Corrugation geometry

Fig. 5 shows the idealised corrugation geometry. The geometry considered is a trapezoidal corrugation, with the corner radii neglected, and is assumed to form an antisymmetric unit cell shape. This geometry is parametrized by the corrugation half-pitch $c$, half-depth $f$ and shape parameter $\zeta$.

The shape parameter $\zeta$ controls whether the corrugation is conventionally trapezoidal ($\zeta < 1$), square ($\zeta = 1$), or reentrant ($\zeta > 1$), by expressing the flange breadth $b_f$ as a proportion of the half pitch $c$. To prevent the corrugation profile overlapping itself, the shape parameter must be restricted to the range $0 < \zeta < 2$.

3. Analytical models for buckling

This section defines simple analytical models for the buckling modes of the corrugated panel. Much of the analysis concerning the global structure makes use of homogenised properties for the corrugation, and therefore Section 3.1 describes how these properties are calculated.

Two classical forms of buckling must be checked; local buckling and global buckling. Local buckling occurs when one of the discrete panels in the corrugation experiences panel buckling and is analysed in Section 3.2. Global buckling consists of a deformation of the entire corrugation panel between the stringers and ribs. As in the case of a conventional panel, this can occur out-of-plane and the analysis of this is given in Section 3.3. However, it was found that a further global mode occurred, and this mode is analysed in Section 3.4. Note that in practice further global buckling modes would need checking, where the stringers as well as the corrugation fail. To maintain focus on the optimal corrugation pattern, the current work ignores this and simply assumes that stiffeners are adequately designed to prevent this mode from occurring.

3.1. Homogenized plate properties

In order to study global buckling modes, properties of an equivalent specially orthotropic plate are taken from [17]. The bending stiffness and in-plane stiffness matrices may be written as:
where overbars are used to denote a property of the overall corruga-
tion. Expressions for the terms in the matrices are listed in Ta-
ble 1, for single material trapezoidal corrugations. The 1 direction
aligns with the loading (or spanwise) direction of the corruga-
tion, hence the 2 direction aligns with the actuation (or chordwise)
direction.

It is of interest to consider corrugations made from multiple
materials or laminates; for example in this work we consider the
classical orthotropic laminate. This leads to a buckling limit given by:

\[ N_c = 2 \sqrt{D_{11} D_{22}} \frac{\pi^2}{b^2} (1 + \beta) \]  \hspace{1cm} (2)

where \( \beta = \frac{D_{32} + 2D_{33}}{D_{11}} b \) is the width of the panel concerned and \( D_m \)
terms are the elements of the well-known laminate bending stiff-
ness matrix.

It should be noted that, unlike column buckling, the geometrical
term that dominates the response is the breadth \( b \), not the length \( a \).
The value of \( b \) to use is whichever is the larger of the flange breadth \( b_f \)
or the web breadth \( b_w \), as shown in Fig. 5. To prevent failure, the local critical load intensity \( N_c \) should be less than the global in-
plane load intensity, with a correction made for the increased path
length caused by corrugation:

\[ N_c < N_c^l < \epsilon \ell \]  \hspace{1cm} (3)

where \( \ell \) is half the path length of one unit cell of corrugation:

\[ \ell = b_f + b_w \]  \hspace{1cm} (4)

<table>
<thead>
<tr>
<th>( \bar{A} )</th>
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<tbody>
<tr>
<td>[ \bar{A}<em>{11} \quad \bar{A}</em>{12} \quad 0 ]</td>
<td>[ \bar{D}<em>{11} \quad \bar{D}</em>{12} \quad 0 ]</td>
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Table 1
Equivalent orthotropic plate properties for trapezoidal corrugated composite panels made from a single orthotropic material, as derived by Xia et al. [17]. The angle \( \alpha \) is shown on Fig. 5.

<table>
<thead>
<tr>
<th>( \bar{A} )</th>
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<tbody>
<tr>
<td>11 [ \bar{A}_{11} ]</td>
<td>[ \frac{1}{2} \int \bar{A}_{11} \left( \frac{a^2 - \alpha^2}{\alpha} \right) ds ]</td>
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<tr>
<td>12 [ \bar{A}_{12} ]</td>
<td>[ \frac{1}{2} \int \bar{A}_{12} \left( \frac{a^2 - \alpha^2}{\alpha} \right) ds ]</td>
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<tr>
<td>22 [ \bar{A}_{22} ]</td>
<td>[ \frac{1}{2} \int \bar{A}_{22} \left( \frac{a^2 - \alpha^2}{\alpha} \right) ds ]</td>
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<tr>
<td>66 [ \bar{A}_{66} ]</td>
<td>[ \frac{1}{2} \int \bar{A}_{66} \left( \frac{a^2 - \alpha^2}{\alpha} \right) ds ]</td>
</tr>
</tbody>
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Table 2
Equivalent orthotropic plate properties for corrugated composite panels made from orthotropic laminates with continuously varying properties.

\[ \bar{A}_{ij} \quad \bar{B}_{ij} \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>( j )</th>
<th>( \bar{A}_{ij} )</th>
<th>( \bar{B}_{ij} )</th>
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<td>1</td>
<td>[ \frac{1}{2} ] | \bar{A}<em>{11} | | \bar{D}</em>{11} | + \frac{1}{2} | \bar{A}<em>{11} | | \bar{D}</em>{11} | ]</td>
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<tr>
<td>12</td>
<td>2</td>
<td>[ \frac{1}{2} ] | \bar{A}<em>{12} | | \bar{D}</em>{12} | + \frac{1}{2} | \bar{A}<em>{12} | | \bar{D}</em>{12} | ]</td>
<td></td>
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<tr>
<td>22</td>
<td>2</td>
<td>[ \frac{1}{2} ] | \bar{A}<em>{22} | | \bar{D}</em>{22} | + \frac{1}{2} | \bar{A}<em>{22} | | \bar{D}</em>{22} | ]</td>
<td></td>
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<tr>
<td>66</td>
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<td>[ \frac{1}{2} ] | \bar{A}<em>{66} | | \bar{D}</em>{66} | + \frac{1}{2} | \bar{A}<em>{66} | | \bar{D}</em>{66} | ]</td>
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3.3. Global out-of-plane buckling model

In order to assess the global buckling performance of the corru-
gation, the equivalent orthotropic bending stiffness matrix is used in
the classical derivation of the simply supported plate. This results in
an expression for the global critical load intensity:

\[ N_c = \sqrt{D_{11} D_{22}} \pi^2 \left( \frac{xm^2 b^2}{\alpha^2} + 2b + \frac{\alpha^2}{x m^2 b^2} \right) \]  \hspace{1cm} (5)

where \( x = \frac{D_{22}}{\sqrt{D_{11} D_{22}}} \) and \( \beta = \frac{D_{32} + 2D_{33}}{D_{11}} \). The value of \( m \) that gives the lowest critical load intensity is used.

For the majority of corrugations, \( D_{22} \) is much greater than all
other constants, allowing Eq. (5) to be approximated by

\[ N_c \approx \sqrt{D_{22}} \frac{\pi^2 a^2}{b^2} \]  \hspace{1cm} (6)

where the second and third terms have been neglected and it has
been observed that the critical value will therefore occur for
\( m = 1 \). This approximation is valid as long as \( \frac{b}{a} \) is not too large. Eq.
(6) shows that in this case the different directional bending prop-
erties of the plate are largely decoupled; this gives insight as to why
modelling corrugated panels with beam like concepts, such as in
[10], can yield satisfactory results.

The form of Eq. (6) is analogous to Euler column buckling,
where buckling performance is dominated by the length \( a \) as opposed
to breadth \( b \). This suggests that the lateral support from stiffeners
does little to provide stability at the centre of a panel, unlike a
conventional plate. A corrugated architecture giving a large value
for \( \beta \) could be advantageous, as it would mean that the
corrugation would gain buckling stability from the second
bracketed term in Eq. (5).

3.4. Global in-plane buckling mode

Initial investigations revealed that the low in-plane stiffness of
the corrugation can cause a buckling mode that can occur with no
out-of-plane deformation. An example of this buckling mode is shown in part (c) of Fig. 13 and in Fig. 14. This mode can be modelled with the equivalent plate properties as follows.

Firstly we need to consider a plate element \( \Delta x \Delta y \) that has in-plane deformation on a scale more usually seen for out-of-plane deformation, as shown in Fig. 7. Summing the forces in the \( x \) direction leads to

\[
N_x \frac{\partial^2 v}{\partial x^2} + \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0
\]  

(7)

Note that the first term of (7) resembles the membrane force term in a classical out-of-plane buckling analysis, but instead acts in the plane of the plate. Now it is assumed that \( N_x \) is constant because it consists of structural loads that are much greater than other in-plane loads. The other in-plane loads are given by

\[
N_y = \bar{A}_{12} \epsilon_x + \bar{A}_{12} \epsilon_y
\]

\[
N_{xy} = \bar{A}_{66} \gamma_{xy}
\]  

(8)

The strains are related to displacements by

\[
\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\]  

(9)

The trial solution

\[
u = 0, \quad v = V \cos \left( \frac{n \pi y}{2b} \right) \sin \left( \frac{m \pi x}{2a} \right)
\]  

(10)

where \( m, n = 1, 2, 3 \). approximates the boundary conditions used in the FEA study in the next section. Substituting Eqs. (8)–(10) into Eq. (7) leads to

\[
N_x = \left( \frac{n}{m} \right) \bar{A}_{66} = \frac{a}{b} \bar{A}_{32} + \bar{A}_{66}
\]  

(11)

Noting that the first term on the right of Eq. (11) may be arbitrarily small if \( m \) is large (and also that inspection of the equations in Table 1 shows that typically \( \bar{A}_{32} \ll \bar{A}_{66} \)), a conservative assumption simplifies this to

\[
N_x = \bar{A}_{66}
\]  

(12)

This is a surprising result, because the critical buckling load is found to have no dependency on the dimensions of the panel. Note also that this mode of buckling is not due to the presence of the corrugation; the analysis suggest that it could potentially affect any form of extremely anisotropic material.

4. Finite element model

In order to validate analysis, and to check for potential buckling modes that are not analysed, an FEA model is used. The model is implemented in Abaqus 6.12 using S4R elements. The mesh density is driven by the requirements of local panel buckling; in these cases the panels will deform into half-cosine waves across the breadth of the panel, and it is assumed that the wavelength along the panel will be similar to this or greater. Therefore, in order to allow ten or more elements per buckling half-wave, so that local buckles are captured accurately, ten elements are located across each buckling panel and the spanwise element length is the smallest value of \( b/10 \) or \( b_w/10 \).

The model assumes a periodic buckling pattern and takes a section in the \( x \) direction, with a rib at its centre, with length equal to the rib pitch, as shown in Fig. 8. This approach allows the complexities of the rib boundary condition to be modelled without leaving free edges that would artificially reduce the reported buckling load. The \( x \)-facing ends of the model have symmetric boundary conditions \( u_w = u = 0 \) applied to all nodes. Note that instead of constraining \( u_1 \) to zero, an equation has been used to tie this degree of freedom for all nodes on the \( x \)-end; this allows the distributed force \( N_x - x \) to be applied without overconstraint. The central ‘rib’ boundary condition is applied at just the nodes where the webs intersect the midplane of the corrugation, and consists of a pin in the \( x \) and \( y \) directions \( (u_1 = u_3 = 0) \). This is in order to more accurately simulate a morphing rib, which will not support a corrugation continuously along its length as discussed in Section 2.1. Furthermore, the lack of lateral support in the \( y \) direction models the fact that the morphing rib may be quite compliant in this direction; for example the spines of the FishBAC concept \cite{13} are relatively soft cantilever springs along this axis. In the \( y \) direction a section between two stringers with width \( b \) is modelled, with pinned conditions \( (u_2 = u_3 = 0) \) at each stringer which also enforces antisymmetry in this direction. Again, this condition is applied only on the line where the web intersects the corrugation midplane. Clearly, these boundary conditions simply reflect conservative assumptions designed to give the modelling general relevance, and would have to be refined when studying actual morphing wing implementations.

Two load cases are applied to each model: firstly an eigenvalue extraction is performed to assess the linear buckling modes. Secondly, one stringer is displaced in the \( x \) direction to give a 2.5% strain to the corrugation. The reaction forces to this displacement are recorded and used to verify the in-plane compliance of the corrugation. Due to the actuation forces required being very low, they make little contribution to the overall buckling, so the coupling between actuation and critical load is not investigated here.

5. Single objective optimisation of weight for a given material and buckling panel dimension

5.1. Description of optimisation problem

In general, the optimal design will be the result of multi-objective optimisation between the competing objectives of weight, load bearing and compliance, as well as secondary issues such as manufacturing requirements and robustness. However, in this section a single-objective optimisation is performed on a case with many variables frozen to provide a simple illustration of the effect of different constraints. The case chosen is that of a buckling panel of fixed dimensions and loads, estimated to be roughly representative of the loads and allowable dimensions at the 3/4 wing span of a Global Hawk UAV; hence the study acts as a preliminary study towards establishing the feasibility of designing a camber morphing concept for the outboard section of this wing.
The load case parameters are listed in Table 4. The laminate was chosen after initial studies revealed that very thin laminates could meet the design criteria. Inspection of Table 1 shows that the theoretically ideal laminate would be a unidirectional laminate (i.e. \([0^\circ] / 90^\circ / 0^\circ / 90^\circ / 0^\circ / 90^\circ\]), in order to maximise \(A_{11}\) and \(D_{11}\) to maximise \(\bar{B}_{11}\), whilst minimising \(A_{22}\) and \(D_{22}\) to minimise \(\bar{A}_{22}\). However, since it is unlikely that such a laminate would be used in practice due to its susceptibility to matrix cracking, the crossply alternative is used instead.

The target of the optimisation is to minimise the average areal mass density of the corrugation given by:

\[
\bar{\rho}_A = \rho_A \frac{f}{\mathcal{C}}
\]

where \(\rho_A\) is the areal density of the laminate from which the corrugation is formed.

The variables to be optimised are the number of corrugations per stringer bay \(n\), shape parameter \(\zeta\), and half-depth \(f\). Since the panel dimensions are fixed, \(n\) dictates the corrugation half-pitch \(c\).

The constraints applied are local buckling in the flange or web:

\[
N_{cf} - N_{ch} \zeta / \ell < 0
\]

(14)

\[
N_{cw} - N_{ch} \zeta / \ell < 0
\]

(15)

global panel buckling:

\[
\mathcal{N}_c - N_{ch} \zeta < 0
\]

(16)

a maximum actuation force, expressed as a minimum required in-plane compliance:

\[
1 / \bar{A}_{22} - S_{	ext{min}} < 0
\]

(17)

and the geometrical constraints:

\[
0.2 < \zeta < 1.8
\]

(18)

\[
0 < f < f_{\text{max}}
\]

(19)

Note that the range of \(\zeta\) is slightly reduced, to prevent the thickness of the laminate causing clashes between corrugations. In all cases it was verified that the structural load never caused a direct strain of greater than 0.5%; although strain limits seldom become the critical design driver in conventional panels, it was considered that they could become relevant due to the thin laminates used.

Due to the presence of the discrete variable \(n\), which prevents the use of gradient approaches, a Genetic Algorithm is used to optimise the system, as implemented within Matlab [20]. The solution cost is low, allowing an increased population size of 100 to be used to give smoother results.

5.2. Optimisation results for a constant laminate corrugation

Fig. 9 shows the result of this process, with a range of different values for the minimum compliance. The associated values of \(f\) and \(\zeta\) are shown in Figs. 10 and 11 respectively. The results showed identical trends each time the optimisation was run, and it can be seen that at low values of \(S_{\text{min}}\), denoted by crosses, the design is driven by buckling, not compliance. These cases will represent approximately identical corrugations, and have greater compliance than that required by \(S_{\text{min}}\). The vertical dotted line shows that it is possible to achieve a significant strain at an actuation load.

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Note that strictly in-plane compliance should be found by inverting \(\mathcal{A}\), however the off diagonal terms are so small that the approximation used is generally accurate.
intensity 2 orders of magnitude below the structural load, with little effect on mass. As $S_{\text{min}}$ increases into the range driven by compliance, denoted by circles, the areal density increases slowly at first, and Fig. 10 shows that in this region $f$ rapidly increases to its maximum allowable value.

The weight steadily increases with $S_{\text{min}}$, as does $\zeta$ as shown by Fig. 11. However, after $S_{\text{min}} \geq 0.325$, the mass rises more sharply and increased numbers of corrugations are necessary for each increase in compliance. Figs. 10 and 11 show that both $f$ and $\zeta$ are at their maximum values in this region, and therefore compliance can only be increased by increasing $n$. As can be seen from the corrugation profiles, results for high compliance are likely to be infeasible when further issues such as manufacturing are considered.

Fig. 12 shows the results of the optimisation in terms of the reserve factor for each considered failure mode, compared with the FEA calculation of the first buckling mode. For each buckling constraint, a reserve factor $\left(\text{critical load divided by ultimate load}\right)$ has been calculated and plotted on Fig. 12. Typical mode shapes for each of the regions defined in Fig. 12 are plotted in Figs. 13 and in Fig. 14. As expected, the simple methods used give only approximate accuracy when compared to FEA, however the results give a sufficient accuracy for use in initial optimisation and system level studies.

The leftmost section of Fig. 12 shows that failure is predicted by the analysis to occur simultaneously in the global out-of-plane buckling modes and in local buckling of the web. However, FEA shows that the global mode occurs in this region. This explained by the assumption of simply supported panel edges in the local buckling model; this is a conservative assumption because in reality the rotation of the web edges is stiffened by the solid connection to the webs. Therefore, the critical load for web buckling is somewhat higher than that predicted, and therefore the global mode occurs first. It can be seen that the reserve factor for global out-of-plane buckling rises almost linearly with the required compliance, so for more compliant corrugations it ceases to become a relevant failure mode. In the ‘Local buckling’ region, the local buckling models show reasonably conservative agreement with FEA, with the lowest buckling mode changing depending on whether the flanges or webs have the greatest effective breadth.

In the global in-plane buckling region, it is seen that this mode decreases with increasing compliance, suggesting that this mode provides the ultimate limit to the achievable actuation compliance. The model shows reasonable agreement with FEA in terms of the critical load, although it not conservative in every case. However, the buckling mode shape nearly always shows a shape given by $m = n = 1$, as seen in Fig. 14; in just one case the critical mode was given by $m = 2, n = 1$. This disagrees with the prediction that $m$ will be large at the first buckling mode, according to (11). This would suggest that some unmodelled effects are influencing the buckling mode, and that these will have to be considered in further studies. However, if it is noted that the term containing $m$ will typically be dominated by the $A_{\text{fl}}$ term, it may be seen that this issue will have a relatively minor effect on the critical buckling load.

In all cases, the comparison of the compliance $\left(\frac{1}{A_{\text{fl}}}\right)$ with that calculated from FEA results was very good.

5.3. Effect of varying of laminate between flange and web

It is logical to direct greater stiffness to the flanges, where they are far from the neutral axis and promote high out-of-plane bending stiffness (a similar approach is taken in [11], where additional stiff rods are located away from the neutral axis). Similarly, it makes intuitive sense to assume that the webs are primarily responsible for in-plane compliance, and therefore make them thinner. Therefore, this section performs a similar optimisation to the previous section, but for a corrugation where the two outer plies of the laminate are removed for the webs, leaving just a single 90° layer.

Fig. 15 shows the results of this optimisation, and as expected the thin web corrugation achieves a lower mass density than the constant laminate corrugation; at $S_{\text{min}} = 0.5$ mm N$^{-1}$, the thin web corrugation is over 3 times lighter.

The trends for $f$ and $\zeta$ with increasing $S_{\text{min}}$ are far less clear for this case, as shown in Figs. 16 and 17. This is because now that the web is much thinner, the local web buckling mode has a greater influence on the optimisation; in fact Fig. 18 shows that the analysis considers this to be the first failure mode in nearly all cases. Therefore, increasing $f$ is not as beneficial as in the previous case, because it reduces the critical load for this buckling mode. All 3 design variables therefore show quite complex behaviour in the region $0.2$ mm N$^{-1} < S_{\text{min}} < 0.5$ mm N$^{-1}$, however the overall effect is still one of a smooth increase in mass with $S_{\text{min}}$.

Fig. 18 shows that the analysis predicts local web buckling to be critical in all except for the case at the far right of the graph (hence the dashed line being visible only for a short distance here, as elsewhere it is obscured by the heavy black line). However, the FEA consistently shows global failure modes, due to a combination of
Fig. 13. Typical mode shapes for each region of Fig. 12. (a) Out-of-plane global buckling. (b) Local buckling. (c) In-plane global buckling. Colouring indicates the relative magnitude of displacement.

Fig. 14. Plan view of the in-plane global buckling mode. Colouring indicates the relative magnitude of displacement.

Fig. 15. Optimised average areal mass density against minimum specified compliance, for corrugation with a reduced thickness web. Crosses (•) indicated cases limited by a buckling mode, circles (○) indicate cases limited by the compliance requirement. Vertical dashed line indicates compliance at which skin may be deformed in the y direction to a strain of 2.5%, at a load intensity $N_y$ that is two orders of magnitude below $N_x$. Numerals indicate the number of corrugation units in the panel. Images indicate the shape of the corrugation profile at selected data points.

Fig. 16. Values of half-depth $f$ for the cases shown in Fig. 15.

Fig. 17. Values of $\zeta$ for the cases shown in Fig. 15.
the analytical models being conservative in the case of the local mode and optimistic in the case of the global models. The local web buckling case is conservative because it assumes that the edges of the web are free to rotate, when in fact they are restrained by the presence of the flanges.

Fig. 18 shows that the out-of-plane global mode is non-conservative, by up to approximately 30% in some cases; despite this it mirrors the trend of the FEA curve reasonably well so still has utility in preliminary design work. Unlike the single-laminate case, the out-of-plane mode does not rise monotonically with increasing compliance. The in-plane global mode shows greater accuracy than the out-of-plane mode, however it still remains generally non-conservative.

It can be seen that towards the right end of the graph, the analysis shows the in-plane global mode to be close to being the critical failure mode. In fact it was found that designs that met the in-plane failure criteria could not be found at higher values of $S_{\text{min}}$ than shown in Fig. 18.

6. Multi-objective optimisation of a corrugation section with a constant laminate

6.1. Optimisation scheme

In Section 5 an optimisation of the corrugation for a set global panel dimension was performed. However, in design the optimal design of the global panel dimensions is often done concurrently with the design of the panel itself, in terms of the spacing of ribs and stringers and the sharing of the load between the skin and the stringers or other structural members. A full optimisation of this nature is far beyond the scope of this work, however in any design, the corrugation must be optimal for the requirements that are made of it. These requirements are generally summarised as follows:

1. Low mass.
2. High maximum load bearing ability.
3. High out-of-plane bending stiffness in the loading direction, this reduces the required number of ribs by allowing a higher unsupported length to be supported without buckling, and also reduces deflection under aerodynamic pressure.
4. Low in-plane stiffness in the actuation direction.

A multi objective optimisation with 4 objectives can be hard to interpret. Happily, in the case of a constant laminate corrugation, the number of objectives can be reduced to 3 by noting that items 3 and 4 above are always complimentary. This can be seen by noting that item 3 is achieved through a high value for $D_{11}$, while requirement 4 is met through a low value of $\bar{S}_{\text{wc}}$. It may be seen from Table 1 that promoting high values for the constants $I_1$ and $I_2$ will promote both objectives; in fact the expressions are dominated by $I_2$ so that a single geometric property largely determines both equivalent plate stiffness properties. This may also be seen in Fig. 12, where the global out-of-plane buckling reserve factor, which is determined by $D_{11}$, increases monotonically with the required compliance. Therefore a design that optimises $D_{11}$, will simultaneously minimise $\bar{S}_{\text{wc}}$. (This is not true of corrugations with varying laminates in each section, where the relevant stiffness properties given in Table 3 are more complicated, and as shown by the global out-of-plane buckling reserve factor in Fig. 18.)

So the targets that must be minimised are:

$$
\begin{align*}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix} &= \begin{bmatrix}
-\frac{D_{11}}{\rho A} \\
-\text{min}(N_{ct}; N_{cw}; \bar{A}_{\text{Web}}; \bar{A}_{11} e_{\text{max}})
\end{bmatrix} \\
\end{align*}
$$

Note that $y_2$ represents load bearing, with 4 possible failure cases which are local flange buckle, local web buckle, in-plane buckling limit, and a strain limit constraint. The strain limit is a typical conservative design constraint that the strain due to structural load should never exceed $e_{\text{max}} = 0.005$; whilst this constraint never proved critical, it was found that reserve factors could be low due to the thin laminates used, so this constraint was added as a precaution. The independent variables are $4 \text{ mm} < c < 25 \text{ mm}, 0.2 < \zeta < 1.8$, and $1 \text{ mm} < f < 10 \text{ mm}$. Again, a genetic algorithm was used due to the non smooth nature of $y_2$, as implemented within Matlab [20] by the function gamultiobj. The population was set to 10,000 to generate a reasonably solid surface without requiring excessive computing time. The initial population was supplied to the routine by noting that the bounds on the input variables form
a cubic space of acceptable values, so this design space was simply filled with equally spaced points.

6.2. Multi-objective optimisation results

Fig. 19 shows the Pareto surface for a corrugation consisting solely of the laminate [0,90,0]. As can be seen it consists of 3 regions, depending on whether input variables have reached their maximal values; a variable is defined as being saturated if it is within 5% of its maximum value.

The first region (nearest the plane $D_{11} = 0$) represents corrugations with relatively low requirements for stability (or actuation compliance). For all these corrugations, the laminate has less than the maximum depth permitted, and they appear in the noisy region of Fig. 20.

The second planar region of Fig. 19 shows the region where $f$ is at its maximum. In this region, any chosen value of $D_{11}$ shows a gentle trade off between maximum load bearing and weight.

Finally, there is a one dimensional region, where $\zeta$ has also reached its maximum, so now the only design variable remaining is $c$. In this region, the stiffness improves at the significant expense of weight, and at higher values of $D_{11}$, maximum load.

2D projections of the Pareto surface can give clearer insight into the performance characteristics of the corrugation. Fig. 21 shows the Pareto surface in terms of just stiffness and areal density, showing that the stiffness is the primary driver of the corrugation mass. Fig. 22 shows the trends of maximum load against out-of-plane stiffness, along with typical examples of the corrugation profiles for each region. It can be seen that as the required $D_{11}$ increases, firstly the corrugation depth increases, then the shape parameter $\zeta$ increases, until $c$ becomes the only non-saturated design parameter. In the middle region, increasing $\zeta$ or decreasing $c$ both have the effect of increasing $D_{11}$. However, increasing $\zeta$ on its own will result in a corrugation with large panel dimensions, and a consequent low maximum load due to local buckling. Increasing $c$ instead will initially increase the maximum load by reducing panel dimensions. However, at the right of the graph is the region where the maximum load is limited by in-plane buckling, and in this region there is direct competition between max load and $D_{11}$.

7. Conclusions

This work has begun to examine the optimisation of a corrugated skin in order to minimise weight whilst resisting a compressive buckling load, and allowing transverse strain with minimal resistance.

Multiple forms of buckling have been considered, including an unusual form of buckling that occurs with no out-of-plane displacement, that is unique to plates where anisotropy is so extreme as to mean that in-plane stiffness in the actuation direction falls to a value comparable with the out-of-plane stiffness of the plate.

Another effect of the extreme anisotropy is that the breadth of a panel made from corrugation is shown to have little effect on its critical global buckling loads, unlike a conventional panel where the breadth is critical to buckling loads.

The equivalent plate models used to model global buckling modes have been extended slightly to consider the effect of varying the laminate properties along the length of the corrugation.
A single-objective optimisation has been used to demonstrate the trade-off between in-plane actuator compliance, for low actuator weight, with the mass of the corrugation, for a given corrugation panel size and load. It has been shown that a high compliance can be achieved with a relatively low weight penalty. Furthermore, using a corrugation where a thinner laminate is used in the web has been shown to give significantly lighter corrugations.

The results of the buckling models using equivalent orthotropic plate properties are only approximate and generally non-conservative. This is unsurprising given the many assumptions and approximations involved; however the low cost of these models mean they are an invaluable tool for initial design and optimisation studies.

Finally, a multi-objective optimisation study was performed to give numerous insights into the design space of a constant laminate corrugation.

Further work should be done to gain similar understanding of a wider range of corrugation configurations. Furthermore, there may be benefits to refining the equivalent plate properties models used, perhaps by considering the effect of shear-like deformations on the corrugation.

Acknowledgements

The research leading to these results has received funding from the European Research Council under the European Union’s Seventh Framework Programme (FP/2007–2013)/ERC Grant Agreement no. [247045].

References


