Modelling the effect of ‘heel to toe’ roll-over contact on the walking dynamics of passive biped robots

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Abstract

The ‘heel to toe’ rolling contact has a great influence on the dynamics of biped robots. Here this contact is modelled using the roll-over shape defined in the local co-ordinate system aligned with the stance leg. The roll-over shape is characterised by six constants: forefoot, midfoot and hindfoot gains and length values. A piecewise parabolic polynomial constructed from these six values is able to match the realistic roll-over shape with continuous slope and variable curvature. The effect of these constants and the roll-over shape arc length has been studied on various gait descriptors such as average velocity, step period, inter-leg angle (and hence step length), mechanical energy. The bifurcation diagrams have been plotted for point feet and different gain values. The insight gained by studying the bifurcation diagrams for different gain and length values is not only useful in understanding the stability of the biped walking process but also in the design of prosthetic feet. The discovery of ‘critical values’ for the length and mass ratios for the inter-leg angle $\delta$ (and hence step length) bifurcation diagrams, or a ‘critical value’ for forefoot gain in the step period bifurcation diagram, is of particular interest as it would mean a constant step length or step period for a range of acceptable values of forefoot and hindfoot gains. The dependence of the horizontal roll-over shape length and hindfoot/forefoot gains on the actual stance leg angular velocity $\dot{\theta}_s$ and angular displacement ($\theta_s$) values along with the corresponding existence of critical values has also been demonstrated.

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1. Introduction

Human like feet are complex shaped and capturing the roll-over phase of the stance leg requires understanding of the knee–ankle–foot kinematics along with the effect of the foot curvature. The foot for a biped robot has been modelled as either a point, a flat or a curved/circular foot, even though it is generally known that the foot shape has direct influence on the stability of the walking process along with step period, inter-leg angle or step length, average velocity and energy.

Parallel to the robotics research, a ‘roll-over shape’ concept has been evolved over the last decade within the prosthetics research community. A roll-over shape is a trajectory of the centre of pressure as the foot rolls over the surface and is defined in a local co-ordinate system with $y$ axis being aligned with the ankle–knee axis. This shape is representative of the knee–ankle–foot kinematics and the curvature of the foot shape. One of the advantages of the roll-over shape is that it can be determined experimentally and hence can be input into the computational model.

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http://dx.doi.org/10.1016/j.apm.2013.02.048
A schematic diagram of foot motion during the human walking process is shown in Fig. 1. The walking step starts when a foot makes a heel contact with the ground (heel strike). Subsequent to the heel strike, the foot (also referred to as the stance leg foot) rolls over while the other leg (referred to as the swing leg) takes the next step. The roll-over phase continues until the toe pushes off the stance leg (toe off). At the same time, the swing leg makes the heel contact and becomes a stance leg for the next foot step.

The understanding of walking dynamics and mechanics of the foot-ground contact is important, particularly in the design of prosthetic feet. One of the objectives of prosthetic feet design is that the prosthetic foot closely matches its biological counterpart. The key design criteria are stability, energy requirements and walking speed. Although a large number of prosthetic foot types are commercially available, there are significant gaps in the clinical knowledge regarding the effects of different prosthetic components on various aspects of human mobility e.g. making transfers, maintaining balance, changing walking speed, negotiating ramps and obstacles [1]. Clinical gait analysis focuses on an assessment of a patient with walking disorders. The analysis usually consists of five elements: videotape examination, measurement of general gait parameters, kinematic analysis, kinematic measurement and electromyography. Until the mid-nineties the gait parameters did not include ‘roll-over shapes’ as a clinically relevant parameter [2]. However, it appears that at the same time the effective shape of prosthetic feet with ankle motion was studied by Knox [3]. The effective shape was referred to as the ‘roll-over’ shape of the foot. In other words, it is the path followed by the centre of pressure described in a coordinate frame attached to the prosthetic shank. The initial study discovered that the roll-over shape was insensitive to the walking speed making it an important parameter in the design and alignment of the prosthetic foot [4].

Experimental determination of ‘heel to toe’ roll-over shapes is not easy and techniques to determine the approximate shape have only emerged in recent publications. With the experimental evidence reviewed in Section 2 of this paper, it has become clear that understanding the effect of roll-over shapes on the walking dynamics of less able bodied individuals is indeed an important research area and must not be overlooked. The variation among roll-over shapes for different feet is significant. It is observed that the overall shape can be characterised by three parts, namely the hindfoot, the midfoot and the forefoot, with different lengths and slopes [5]. The schematic foot shape is shown in Fig. 1.

In this paper, the roll-over shapes are characterised by joining three quadratic polynomials, with first order continuity (continuity in slopes), in order to represent roll-over shapes created by the hindfoot, midfoot and forefoot parts of the foot. There is no evidence in the literature that suggests the effects of complex roll-over shapes on the walking dynamics of biped robots have been modelled computationally. A novel discrete pivot point rolling model coupled with the continuous angular velocity assumption for the swing leg is proposed in this paper to accurately model the complexities of roll-over shapes. The literature review in Section 2 is divided into two parts. The first part illustrates the importance of roll-over shapes as perceived from experimental research. The existing state of the art on the modelling and simulation of curved and flat feet shapes is summarised the second part. A discrete pivot point rolling model to take into account the effect of variable curvature in the roll-over shapes is described in Section 3. The roll-over shape is input to the proposed model and is reproduced by fitting three quadratic polynomials. The rolling contact is discretised with discrete pivot points. The influence of the impact after each pivot contact is modelled by introducing a transition phase so that the corresponding conditions in the swing and stance leg dynamics are updated accurately. The procedure is described in Section 4. This method can be used for any roll-over shape. The existence of a stable trajectory of the biped motion is defined by the robot parameters. The accuracy of the discrete pivot point model is investigated in Section 4.5. Results are discussed in Section 5 and the paper is concluded in Section 6.

Fig. 1. Foot movement during human walking process and the resulting roll-over shape characterised by four control points $S_1$ to $S_4$. The local co-ordinate system $(x − y)$ rotates as the stance leg rolls-over.
2. Literature review

2.1. Experimental roll-over shapes to account for knee–ankle–foot kinematics

The complexity of prosthetic feet give rise to many important characteristics, such as energy storage and return, hysteresis, the effective foot length and general stiffness properties. Klodd et al. [6] argued that “many of these properties are not independent. In the absence of an effective and accurate computational model, it is extremely difficult to understand which properties lead to specific gait deviations when they occur. The existing solution is to undertake controlled studies that attempt to change only one feature at a time in order to build core knowledge of these features and their effects on the gait”.

A common observation is that able bodied individuals often wear shoes of different heel heights without experiencing much difference in the overall stability during the walking process [7–8]. The most probable reason is that able bodied individuals are able to adapt their ankle motion in conjunction with other subtle changes at the knee to compensate for the effects introduced by different shoe heights. A recent study by Hansen and Childress [9] examined the effect of shoe heel height on the roll-over shapes of prosthetic ankle foot systems. They used seven prosthetic feet and plotted the corresponding roll-over shapes. The length and the slope of the forefoot part of the roll-over shape were noticeably different in each case. It was observed that the orientation of the roll-over shape changed for a prosthetic foot for different shoe heights and the prosthetic foot was unable to adapt to the ankle motion on its own. However, it was possible to align the roll-over shape by manually adjusting the heel height of the prosthetic shoe. Hansen et al. [10] also studied the effect of the roll-over shape arc length on the gait of unilateral trans-tibial prosthetic users. It was found that participants experienced higher loads on their sound limbs at various walking speeds when the roll-over arc length of the prosthetic foot was shortened. There was also an indication of a reduction in step length.

The effect of prosthetic forefoot flexibility was further studied by Klodd et al. [6] in a double blind randomized cross over study to determine the effects of forefoot flexibility on the gait of 14 unilateral transtibial prosthetic users. It was discovered that when prosthetic feet had excessively flexible forefoot sections, they provided shorter effective foot lengths, reduced ankle moments on the prosthetic side and introduced a ‘drop off’ effect when the body weight was transitioned from the prosthesis to the sound limb. The ‘one effect at a time’ approach is an expensive way of building knowledge and it is vital that the computational community attempts to fill this void. It is reported in the literature that lower limb prosthesis users expend more oxygen per unit distance during walking than non disabled participants. Klodd et al. [11] found that the forefoot flexibility does not influence the oxygen intake of persons with unilateral trans-tibial amputations even though young able bodied persons showed significant differences in the oxygen consumption rate when they wore shoes of different radii [12].

There is evidence that clinical conditions such as diabetes, stroke, hemiplegia or even old age influence the gait profile. The gait speed and step length (step period) influences the stability and increases the risk of falls [13]. Sawacha et al. [14] proposed a method to capture forefoot, midfoot and hindfoot motion during different gait tasks and reported that diabetic patients showed statistically significant differences in forefoot kinematics parameters over the full gait cycle. Kim and Eng [15] studied the kinematic and kinetic gait profiles in individuals with chronic stroke and found the magnitude and pattern of these profiles related to gait performances. Fatone and Hansen [16] reported distinctly different roll-over shapes in patients with hemiplegia following a stroke. The influence of foot postures on the cost of transport during walking in humans has been studied by Cunningham et al. [17]. The improvement of balance and stability has been related to the development of a ‘heel to toe’ roll-over pattern from the onset of independent walking until after one year of walking [18].

In order to understand the effect roll-over shapes on the walking dynamics, it is essential that the curvature of the roll-over shape is modelled accurately. The effect of rolling foot curvature on the work performed on the centre of mass during human walking was studied by Adamczyk et al. [19]. In their experiments subjects wore seven wooden arc shaped boots of varying radius and they found that the centre of mass work decreases with arc radius. Kwan and Hubbard [20] concluded that point feet are the least efficient and round feet are the most efficient. The efficiency of flat feet are somewhere between point and round feet.

One of the major limitations of most of the papers reviewed in this section is that they assumed a constant radius of curvature for the roll-over shape. This is, of course, not realistic as demonstrated by Curtze et al. [21]. The authors designed an experiment to investigate the roll-over characteristics of a number of prosthetic feet in combination with different shoes. The ‘roll-over’ shapes of these foot-shoe combinations were determined. The roll-over shapes created by human feet do not have constant curvature. Their experiments have shown that the forward travel does not progress linearly as predicted by circular feet with a constant radius of curvature. Their research implied that the roll-over shape is characterised by three regions with a mid region being flat or with a large instantaneous radius of curvature. The flat mid region gives greater stability during the stance phase. It was observed that the forward travel and the associated roll-over shapes were strongly different for various prosthetic feet and this was an important measure of the prosthetic foot characteristics, because of its influence on the gait stability.

The five reference points used by Major et al. [22] to characterize roll-over points clearly align with the schematic of the roll-over shape defined in Fig. 1. This experimental evidence has made a strong case for the computational community to model the effect of roll-over shapes (forefoot and hindfoot gain values and forefoot, hindfoot and midfoot length values as well as the roll-over shape arc length values) on gait descriptors that relate to gait stability, such as step period, step velocity, mechanical energy, and inter-leg angle.
2.2. Computational modelling of foot shape for biped robots

The dynamics of the human walking process is a complex phenomenon. Analysing passive dynamic walking to provide insights into human walking behaviour is a reasonable analogy. Collins et al. [23-24] extended McGeer’s [25] passive walking robot to a human like biped robot and showed that its efficiency is comparable to that of a human. Hansen et al. [26] compared the overall leg rocker radius in their experiment with the radius predicted by McGeer [25] for human walking even though McGeer’s biped robot’s circular arc feet replaced the knee–ankle–foot roll-over shape derived in their experiment.

McGeer [25] defined the concept of passive walking for biped robots on a sloping surface. Exploring the usefulness of this technique, extending it to multi-link biped robots, and understanding the linearisation process of the dynamic equations of a robot about an equilibrium point, are areas of active research [20,27-33]. Remy et al. [34] extended the biped robot theory to quadrupeds. However, all of these publications have either used point, flat or circular arc shaped feet.

Recent publications have modelled a flat foot with a segmented foot allowing rotation at the ankle and toe joints [35-36]. Li et al. [37] proposed an arched foot (inverted circular arc) along with circular arc and flat feet. Narioka et al. [38] replaced a circular arc foot with a flat foot that would create a circular roll-over shape. Their analytical model was an over simplified model with a single body mass, a weightless leg, an ankle joint and a flat foot which contacts the ground at the heel and the toe. However, they also constructed a robot and verified that for the same roll-over shape the robot’s stability was insensitive to the variation in the body mass. Ren et al. [39] used an actual roll-over shape with variable curvature. The analytical foot roll-over model was designed to predict ankle kinematics i.e. to predict the positions of ankle joints and the centre of pressure positions. The model for passive walking dynamics was not developed.

Replacing circular feet with roll-over shapes with variable curvature and maintaining the continuity of angular velocity of the swing leg during the roll-over phase requires re-development of the equations from first principles, as described by McGeer [25] and Goswami et al. [27]. In order to model the influence of variable curvature of roll-over shapes and the continuous angular velocity of the swing leg during the roll-over phase, a simple two linked biped robot has been considered in this paper, with a point leg mass located at a user defined location on the leg, and a point hip/body mass being located at the intersection of legs. The roll-over shape is discretised with a discrete pivot point model and the concept of the stance leg transition phase has been introduced to model a situation when two pivot points are in contact with the sloping surface. Between two subsequent stance leg transition phases, the stance leg pivots around a single pivot point and the dynamics of the swing leg (swing phase) is modelled with the point feet double pendulum model described by Goswami et al. [27].

3. Description of roll-over shape as a piecewise polynomial function

As discussed earlier, a roll-over shape is a path followed by the centre of pressure described in a coordinate frame attached to the ankle joint (or prosthetic shank). The roll-over shape does not change during the walking process and hence can be described by a polynomial function with variable curvature. The objective of this work is to study the influence of roll-over shapes on gait parameters for a stable walking process. This requires an ability to input complex geometry of roll-over shapes. As shown in Fig. 1, the roll-over shape has three distinct regions, hindfoot, midfoot and forefoot. Hence it is necessary to be able to control the length and curvature of the roll-over shape in each region independently by maintaining a C1 continuity at points S2 and S3.

The simplistic assumptions of circular roll-over shapes are not realistic and the proposed discrete point model allows a roll-over shape to be modelled and potentially optimised using the proposed mathematical formulation. The main impact occurs at heel strike. However, as a result of discretising roll-over shapes, the effects of smaller subsequent impacts need to be modelled in the mathematical formulation. The sensitivity of the accuracy of the simulation to the number of pivot points is described in Section 4.5.

In order to satisfy these conditions, a set of polynomial functions \( f_{S1}(x) \) is proposed by combining three different polynomial functions \( f_{S1z}(x), f_{S1a}(x) \) and \( f_{S1a}(x) \) (Eq. (1)) representing the hindfoot (rear part of the foot, including heel, from \( S_1 \) to \( S_2 \)), the midfoot (from \( S_2 \) to \( S_3 \)) and the forefoot (front part of the foot, including toes, from \( S_3 \) to \( S_4 \)) respectively. The length of the roll-over shape is characterised by four control points \( S_1, S_2, S_3 \) and \( S_4 \). The curvature of the roll-over shape is characterised by three positive coefficients \( r_p, r_m \) and \( r_f \) referred to as the hindfoot gain, the midfoot gain and the forefoot gain, respectively.

These functions are derived from a family of parabolic functions whose origins are defined at a zero slope point, and given by

\[
  f_{S1z}(x) = \begin{cases} 
  f_{S1z}(x) = \frac{1}{r_p} (x - x_h)^2 + y_h & \text{if } S_1 \leq x \leq S_2 \\
  f_{S1a}(x) = \frac{1}{r_m} (x - x_m)^2 + y_m & \text{if } S_2 \leq x \leq S_3 \\
  f_{S1a}(x) = \frac{1}{r_f} (x - x_f)^2 + y_f & \text{if } S_3 \leq x \leq S_4 
  \end{cases}
\]  

In order to construct a roll-over shape with continuity in slope at control points \( S_2 \) and \( S_3 \), the co-ordinates of the zero slope points of the three parabolic functions are translated to \( (x_8, y_h), (x_8, y_m) \) and \( (x_f, y_f) \) respectively. The roll-over shape requires that a local co-ordinate system is defined with origin at the ankle joint and y axis along the stance leg. The mid foot function \( f_{S1z}(x) \) passes through the origin and the ankle joint, and is a polynomial function with a large radius of curvature. If \( h \) is defined as the horizontal distance between the origin and the zero slope point of \( f_{S1z}(x) \) then
The values \((x_h, y_h)\) and \((x_f, y_f)\) are determined by ensuring the continuity of slope at \(S_2\) and \(S_3\).

\[
x_h = s_2 - \frac{r_h}{r_m} (s_2 - x_m),
\]

\[
y_h = f_{S_2}(s_2) - \frac{r_h}{r_m} (f_{S_2}(s_2) - x_m)^2,
\]

\[
x_f = s_3 - \frac{r_f}{r_m} (s_3 - x_m),
\]

\[
y_f = f_{S_3}(s_3) - \frac{r_f}{r_m} (f_{S_3}(s_3) - x_m)^2.
\]

4. Dynamics of passive walking with a discrete pivot point roll-over model

As illustrated in Fig. 1, the roll-over shape is discretised by \(e\)-pivot points during the analysis. As \(e\) tends to infinity the shape of the roll-over curve would approach the original smooth curve. This will correspond to \((e - 1)\) stance leg transition phases, a double support transition phase and \(e\) swing phases. The overall assumptions of the biped motion are described below:

(A1) The scuffing problem of the swing leg, which is inevitable for straight-legged walkers, is neglected during the swing phase [25,40].

(A2) The impacts are inelastic and there is no sliding at the pivot points.

(A3) There is no actuator or controller.

(A4) The robot configuration remains unchanged for \(i \geq 1\), as well as the angular velocity of the swing leg for \(i > 1\) during instantaneous transitions.

(A5) Angular momentum is conserved through the pivot-point impact for the whole robot about the impacting pivot points \((i \geq 1)\), and for the former stance leg about the hip \((i = 1)\).

As a result of using a discrete pivot point model, small impacts occur during the rolling contact. For a sufficiently large number of pivot points, as shown in Section 4.5, the results converge to a unique solution. However, the resulting impact when the next pivot point makes a contact with the ground needs to be modelled. The assumption A4 ensures that the leg angular velocity is same before and after the smaller impacts resulting from contacts during the single support phase.

A schematic diagram with nine pivot points is shown in Fig. 2. As the stance leg pivots around pivot point 1 during the rolling contact, the swing phase dynamics occurs as per the double pendulum model until pivot point 2 comes into contact with the sloping surface. This defines the stance leg transition phase. The consequence of the swing phase and the stance leg transition phase during the single support phase will continue until the swing leg contacts the sloping surface which is described by the double support transition phase. The initial conditions for the double pendulum motion about each pivot point consist of virtual leg lengths \((l_{v1}\text{ and } a_{v1}\text{ in Fig. 2})\), initial angular velocities and initial angular displacements. The calculation procedure for virtual leg lengths and initial angular displacements about a given pivot point is described in Section 4.1. Initial angular velocities during the single support phase and the double support phase are presented in Sections

![Fig. 2. Discretising roll-overs shape to develop a multi-pivot point rolling contact model.](image-url)
4.1 The contact point of the roll-over shape with the surface during swing phase

In Fig. 2, the hip mass and leg mass are shown as point masses on the stance leg. The ankle joint is at the intersection of the stance leg and the roll-over shape. The local co-ordinate axis is defined at the ankle joint. In Fig. 2 the ankle joint is shown at pivot point 7 (it should be noted that the ankle joint may not coincide with a pivot point). When the stance leg is rolling at pivot point 1, a virtual stance leg connecting pivot point 1 and the hip mass is introduced. Similarly, a virtual stance lower leg is defined that connects pivot point 1 with the leg mass. The angular positions and lengths of the actual stance leg, virtual stance leg and the virtual stance lower leg are represented by \((\theta_i, L), (\theta_{vs}, f_{vs})\) and \((\theta_{pol}, a_{pol})\) respectively (Fig. 3). The virtual stance leg and virtual stance lower leg are used instead of the actual stance leg to analyse the biped motion. During the swing phase, angular velocities of virtual stance leg, the stance leg and the virtual stance lower leg are kept equal. This means that the grey coloured triangle made by joining hip mass, pivot point and ankle joint \((C_i, \Omega_{s1}, \Omega_s)\) remains constant (Fig. 3) during the swing phase that occurs between two consecutive stance leg transition phases. The angular position and velocity of the swing leg is denoted by \(\theta_{ns}\) and \(\theta_{ms}\) respectively (Fig. 3) and the superscript \(i\) denotes the \(i\)th time step. In the subsequent swing phase, the foot will roll-over around pivot point 2. The shape and the angular position of the triangle will change due to the position of \(\Omega_{s2}^i\) and the angular position of the stance leg \(\theta_i\) as shown in Fig. 2. The virtual stance leg and actual stance leg become identical as the pivot point approaches the ankle joint (see point 7 in Fig. 2). Note that the local co-ordinate system defined at the ankle joint also changes during the rolling contact.

The distance between two consecutive pivot points is a function of the roll-over shape, the time-step and the initial conditions for each time-step. The length, angular position and angular velocity of the virtual stance leg and the virtual stance lower leg change in every time-step and depend on the position of the pivot point.

The global coordinate system \((X, Y)\) and the local coordinate system \((x, y)\) (as shown in Fig. 3) are used to track the position of the pivot point on the roll-over shape of the stance leg. The gradient of the inclined surface about the local coordinate system is \(-\tan(\phi + \theta_i - \pi)\). Just after the double support transition phase, the first pivot point for the foot of the stance leg foot in contact with the ground during the swing phase, \(\Omega_{vs}^1 = (x_p^1, y_p^1)\), can be derived from

\[
f_{S_{11}}(x_p^1) + \tan(\phi + \theta^i - \pi) = 0 \quad \text{and} \quad y_p^1 = f_{S_{12}}(x_p^1).
\]

(7)

The point \(\Omega_{vs}^1\) is not necessarily located on the control point \(S_1\) (as shown in Fig. 1) and it can be somewhere inside the roll-over curve. If the \(\Omega_{vs}^1\) is outside the roll-over curve then its co-ordinates are given by:

\[
x_p^1 = x_i \quad \text{and} \quad y_p^1 = f_{S_{12}}(x_p^1).
\]

(8)

The following condition is checked at each time step as the stance leg begins to rotate about the control point \(S_i\)

\[
f_{S_{12}}(x_p^1) + \tan(\phi + \theta^i - \pi) \geq 0.
\]

(9)

![Fig. 3](image-url) The angular positions and lengths of the actual stance leg (solid line), the virtual stance leg (dashed line connected to the hip mass) and the virtual stance lower leg (dashed line connected to the leg mass). Figure on the right shows the roll-over of the stance leg between two pivot points. Rotation of the local co-ordinate system \(x_i - y_i\) to \(x_{i+1} - y_{i+1}\) is shown.
At a particular time step, the Eq. (9) will not be satisfied. This leads to two intersection points between the roll-over function and the inclined surface. The first point is $S_1$ and the second intersection point becomes the next pivot point. The pivot point of $ith$ time step in the local coordinate system $(x_i, y_i)$ is given by:

$$f_{fsa}(x'_p) - f_{inc}(x'_p) = 0 \quad \text{and} \quad y'_p = f_{fsa}(x'_p),$$  \hspace{1cm} (10)

where $f_{inc}(x'_p) = -(x'_p - x_{p}^{-1}) \tan(\phi + \theta'_{1p} - \pi) + y_{p}^{-1}$ is the equation of the inclined surface in the local coordinate system of the $ith$ time step.

At every pivot point the initial conditions, i.e. angular positions $\theta'_{rs}$, $\theta'_{rel}$, and virtual lengths $\ell'_{rs}$, $a'_{rel}$ and $\lambda'_{rel}$, $\lambda'_{mcl}$ (as shown in Fig. 2) required for the subsequent double pendulum differential equation are updated. Using the constant triangle assumption and trigonometry, the initial conditions of the biped motion for the pivot point of the $ith$ time step are given by:

$$\theta'_i = \theta'_1 - \pi,$$  \hspace{1cm} (11)

$$\lambda'_i = \tan^{-1} \left( \frac{b + a - y'_p}{x'_p} \right),$$  \hspace{1cm} (12)

$$\theta'_i = \begin{cases} \lambda'_i + \frac{\pi}{2} & \text{if} \quad x'_p < 0 \\ \lambda'_i + \frac{\pi}{2} & \text{if} \quad x'_p > 0 \end{cases};$$  \hspace{1cm} (13)

$$\lambda'_i = \tan^{-1} \left( \frac{a - y'_p}{x'_p} \right),$$  \hspace{1cm} (14)

$$\theta'_i = \begin{cases} \lambda'_i + \frac{\pi}{2} & \text{if} \quad x'_p < 0 \\ \lambda'_i + \frac{\pi}{2} & \text{if} \quad x'_p > 0 \end{cases};$$  \hspace{1cm} (15)

$$a'_{rel} = \sqrt{(a + b - y'_p)^2 + x'_p^2},$$  \hspace{1cm} (16)

$$\ell'_{rs} = \sqrt{(a + b - y'_p)^2 + x'_p^2}.$$  \hspace{1cm} (17)

4.2. The stance leg transition phase

As the stance leg rotates about a pivot point a small impact occurs when the next pivot point makes contact with the sloping surface. During this stance transition phase, $\theta'_{rs}$ and $\theta'_{rel}$ for the $ith$ time step are derived by using Eqs. (13) and (15). It is assumed that both the swing leg and the virtual stance leg angles remain constant during the impact giving the conditions

$$\theta'_m = \theta'_m \quad \text{and} \quad \theta'_i = \theta'_rs.$$

The conservation of angular momentum (Assumption A5) and the assumption of constant angular velocity of the swing leg (Assumption A4) during the stance leg transition phase leads to the condition

$$\dot{\theta'} = H^i(\theta')\dot{\theta'},$$

where

$$H^i_{11} = \frac{ma_{rel}a^{-1} \cos(\theta'_{rel} - \theta'_{mcl}) + (m_H + m)\ell'_{rs} \cos(\theta'_{rs} - \theta'_{mcl}) - mcl'_{rs} \cos(\theta'_{rel} - \theta'_{mcl})}{m(1 + cl'_{rs})^2 + ma_{rel}^2 + mcl'_{rs}^2 - mcl'_{rs} \cos(\theta'_{rs} - \theta'_{mcl})}.$$  \hspace{1cm} (20)

$$H^i_{12} = H^i_{21} = 0 \quad \text{and} \quad H^i_{22} = 1.$$  \hspace{1cm} (21)

The – and + superscripts denote the pre and post-pivot point impact variables for the $ith$ time step respectively.

4.3. The double support transition phase

The impact resulting from the swing leg’s contact with the sloping surface (heel-strike) is modelled via the double support transition phase. During this phase the angular position of virtual stance leg and the swing leg are related by

$$\theta'_{rs} + \theta'_{rel} = -2\phi + 2\pi.$$  \hspace{1cm} (22)
where $\phi$ is the slope of the walking surface and $2\pi$ is the correction added because of the definition of angular positions (Fig. 3).

It is assumed that both the swing leg and the virtual stance leg angles remain constant during the impact giving the condition

$$\theta^{1+} = J\theta^e,$$  
(23)

where

$$J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. $$  
(24)

The matrix $J$ exchanges the support and the swing leg angles for the upcoming swing phase.

The conservation of angular momentum (Assumption A5) leads to the condition

$$\dot{\theta}^{1+} = \frac{H^1(\theta)}{H^d_2} J \dot{\theta}^e,$$  
(25)

where the elements of $H^1(\theta)$ are

$$H^1_{11} = m_i m b^2 l^4 l^4_{ls} \cos (\theta_{ls} - \theta_{ls}^e) + m^2 b^2 l^4 l^4_{ls} \cos m (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1) + m^2 b^2 a^2_{ls} l^2_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_5^1 - \lambda_4^1)$$

$$+ m^2 b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_5^1 - \lambda_4^1)$$

$$H^1_{12} = -m b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1)$$

$$H^1_{21} = m m_{ls} b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1) + m^2 b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1)$$

$$+ m^2 b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1) + m m_{ls} b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1)$$

$$+ m^2 b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1)$$

$$H^1_{22} = -m b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1)$$

$$H^1_{3} = m b^2 l^4 l^4_{ls} \sin (\lambda_6^2) \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1) + m^2 b^2 l^4 l^4_{ls} \sin (\lambda_6^2) + m b^2 l^4 l^4_{ls} \sin (\lambda_6^2)$$

$$+ m b^2 l^4 l^4_{ls} \sin (\lambda_6^2) \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1)$$

$$+ m^2 b^2 l^4 l^4_{ls} \sin (\lambda_6^2) \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1)$$

and

$$H^1_{4} = m b^2 \left[ m b^2 l^4 l^4_{ls} \cos (\theta_{ls}^e - \theta_{ls}^{1+} + \lambda_6^2 - \lambda_5^1) \right].$$

Again, the $-$ and $+$ superscripts denote the pre and post-pivot point impact variables for the $ith$ time step respectively. The $e$ and $1$ superscripts denote the biped parameters during the swing phase before and after impact.

4.4. Dynamics of the swing phase

The dynamic equations of the swing phase model are the standard double pendulum equations given by Goswami et al. [27]. In their work, the swing phase covered the entire step period and the equations were solved only once for each step period. In this implementation, the influence of transition phase is accounted by solving the following equations. The equations are initialised when the stance leg begins to rotate about every pivot point.

$$M(\theta) \ddot{\theta} + C(\theta, \dot{\theta}) \dot{\theta} + K(\theta) = 0,$$  
(31)

where $\theta = [\theta_{ls} \theta_{m}]^T$ and $\theta_{ls}$ and $\theta_{m}$ are measured from the negative $y$-axis, as shown in Fig. 3.

The inertia matrix, $M(\theta)$, centrifugal terms, $C(\theta, \dot{\theta})$, gravitational stiffness matrix, $K(\theta)$, and initial conditions for the pivot point of the $ith$ time step, are

$$M(\theta) = \begin{bmatrix} m a_{ls}^2 + m b_{ls}^2 + m l_{ls}^2 - m l_{ls}^2 \cos (\theta_{ls} - \theta_{m}) \\ -m l_{ls}^2 \cos (\theta_{ls} - \theta_{m}) \end{bmatrix},$$  
(32)

$$C(\theta) = \begin{bmatrix} 0 \\ m l_{ls}^2 \cos (\theta_{ls} - \theta_{m}) \end{bmatrix},$$  
(33)

$$K(\theta) = \begin{bmatrix} m g a_{ls} l_{ls} \sin (\theta_{ls} + \lambda_6^2 - \lambda_5^1) + (m b_{ls} l_{ls} + m g l_{ls}^2) \sin (\theta_{ls}) \\ -m g \sin (\theta_{m}) \end{bmatrix},$$  
(34)
\[
\begin{align*}
\theta^{\text{initial}}_{\text{ps}} &= \theta^{\text{initial}}_{\text{ss}}, \\
\dot{\theta}^{\text{initial}}_{\text{ps}} &= \dot{\theta}^{\text{initial}}_{\text{ss}}, \\
\ddot{\theta}^{\text{initial}}_{\text{ps}} &= \ddot{\theta}^{\text{initial}}_{\text{ss}}, \quad \text{and} \quad \dddot{\theta}^{\text{initial}}_{\text{ps}} = \dddot{\theta}^{\text{initial}}_{\text{ss}}.
\end{align*}
\] (35)

For a given time step, the angular position of the actual stance leg \(\theta^{\text{ss}}_{i+1}\), the angular position of the swing leg \(\theta^{\text{ss}}_{i+1}\) and the local coordinate system \((x_{i+1}, y_{i+1})\) for the next time-step is determined using numerical integration in MATLAB using ode45.

4.5. Sensitivity study of modelling roll-over shape using discrete pivot points

In order to investigate the accuracy of the proposed discrete pivot point model two examples are presented. The first example (Section 4.5.1) demonstrates the ability of the discrete pivot point model to simulate the dynamics of the rolling motion of a disk on an inclined surface with a point mass located at its centre. The convergence to the analytical solution is demonstrated in Fig. 4. The second example illustrates that as the number of pivot points that define the roll-over shape are increased the oscillation in step period and inter-leg angle in subsequent gait cycles is reduced and converge to a unique value. This provides an indication of number of pivot points required for a given analysis (e.g. 3000 pivot points).

4.5.1. Rolling motion of a disk on an inclined surface

The displacement time profile of the centre of a rolling disk on an inclined surface is calculated using an exact analytical solution. The acceleration of the stationary disk as it rolls down the slope with angle \(\phi\) is given by \(a = g \sin(\phi)\). For this example the angle \(\phi\) is taken as 4°. The first four seconds of the motion are simulated in order to study the accuracy of the discrete pivot point model. In the discrete model the contact point of the disk and the inclined surface is modelled as a pivot point of the inverted pendulum with the mass located at the centre of the disk. The mass rotates about the pivot point with a constant time step and the post pivot point is given by the second intersection point of the disk and the inclined surface (similar to the discussion in Section 4.1). The virtual link which connects the post pivot point and the mass is the post inverted pendulum with initial conditions obtained by conserving angular momentum about the contact point. Fig. 4 illustrates that by increasing the number of pivot points (NP) the trajectory of the discrete model motion converges to the analytical solution and for sufficiently large number of pivot points the discrete pivot point model is able to accurately model the rolling motion of a disk on an inclined surface.

4.5.2. Rolling motion of the biped robot for a fixed roll-over shape

The roll-over shape determines the location of the contact point with reference to the stance leg position as the stance leg rolls over during the swing phase. As described in Section 4 this causes a change in the initial conditions of the double pendulum governing equations. It is difficult to determine the optimal number of pivot points required to accurately model a complex roll-over shape since an equivalent analytical solution is not available. As the numbers of pivot points to model a biped robot walking on an inclined surface were increased, the resulting step period and inter-leg angles for stable and periodic walking converged. The corresponding values for mass ratio, length ratio, slope angle, hindfoot length, forefoot length and roll-over gain are 2°, 1°, 2°, 16 (cm), 16 (cm) and 0.8. Fig. 5 illustrates that the step periods and inter-leg angles for stable and periodic walking converge to a unique value. The minimum number of pivot points used need to be more than the corresponding value, i.e. 3000 in this example.

5. Results and discussion

5.1. Modelling roll-over shapes with polynomials

The robot parameters of interest in this paper are the forefoot and the hindfoot curvature values, defined by the corresponding gain values \((r_f, r_h)\), along with the corresponding length values \((L_f, L_h)\). The midfoot gain and the midfoot

![Fig. 4.](image-url)
length are described by variables $r_m$ and $L_m$ respectively. As shown in Fig. 1, the length of the roll-over shape is characterised by four control points $S_1$, $S_2$, $S_3$ and $S_4$. The distance between $S_1$ and $S_2$ is the hindfoot length, $L_h$, whereas the forefoot length, $L_f$, is the distance between $S_3$ and $S_4$.

Figs. 6 and 7 illustrate the sample roll-over shapes corresponding to values of the gains and lengths given in Tables 1 and 2 respectively. Note that a gain value equal to zero corresponds to a point foot and the roll-over shape tends to become flat as the gain value increases. In both Figures the single solid point in the centre represents the ankle joint. It can be seen that the roll-over shape arc length increases as the $r_f$ and $r_h$ values increase. The projection of the roll-over shape on the horizontal axis is referred to as horizontal roll-over shape length. The dashed lines in Figs. 6 and 7 represent the extended roll-over shapes and the solid lines are the actual roll-over shapes used by the passive biped robot and are referred to as effective roll-over shapes. This is discussed further in Section 5.2.7.

### 5.2. Bifurcation diagrams

Bifurcation diagrams are commonly used to describe the evolution of gait descriptors (e.g. average velocity, step period, mechanical energy or inter-leg angle) as a function of robot parameters (e.g. ground slope angle, mass ratio, length ratio). In order to compare the bifurcation diagrams with the ones published by Goswami et al. [41], identical values for the mass

![Fig. 5. Sensitivity study to determine the minimum number of pivot points required. Under stable and periodic walking conditions and with a sufficient number of pivot points, the step period and the inter-leg angle value for consecutive walking steps converges to a unique value.](image)

![Fig. 6. Roll-over shapes for different forefoot gain values (refer to Table 1 for all input values).](image)
ratio, length ratio and slope values are chosen, i.e. 2, 1 and 2° respectively. The values used for various bifurcation diagrams are given in Table 3. The first column denotes the robot parameters used on the x-axis of the bifurcation diagram and the corresponding values used for \( r_f \) and \( r_h \) are shown in the subsequent columns. The midfoot length and midfoot gain values used are shown in the last column.

The evolution of the following six gait descriptors due to changes in the five robot parameters (mass ratio, length ratio, slope angle, forefoot gain value and hindfoot gain value) are analysed, followed by the sixth robot parameter, namely the horizontal roll-over shape length.

1. Average velocity.
2. Mechanical energy.
3. Step period.
4. Inter-leg angle (as defined in Fig. 11).
5. Forefoot and hindfoot length.
6. Roll-over shape arc length and inter-leg length.

The bifurcation diagrams (Figs. 8–10) are similar to the corresponding diagrams given by Goswami et al. [41] for point feet, but in addition show the influence of various roll-over shapes defined by different forefoot, midfoot, hindfoot gain and length values. The bifurcation diagrams are plotted either until the passive walking has a period two response or becomes unstable.

Prosthetic foot designers have the ability to influence various gain and length values by changing the stiffness at various points in the prosthetic foot. There are other adjustments that are also undertaken to make the prosthetic foot specific to individual needs. The knowledge of inter-dependence of step period, inter-leg angle, average velocity and mechanical energy with various gain and length values and length and mass ratios directly relate to the human comfort, and hence are of interest to prosthetic foot designers. This inter-dependence is described below (Sections 5.2.1–5.2.7).

### Table 1

| Various length and gain values used to describe roll-over shapes. Note that the hindfoot gain value is constant and all length values are in measured incm. The corresponding roll-over shapes are shown in Fig. 4. |
|---|---|---|---|---|---|---|
| \( r_h \) | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| \( r_f \) | 0.33 | 0.41 | 0.49 | 0.57 | 0.65 | 0.73 |
| \( L_f \) | 5.13 | 6.67 | 8.37 | 10.22 | 12.3 | 14.7 |
| \( L_m \) | 8.27 | 8.41 | 8.57 | 8.7 | 8.9 | 9.07 |
| \( L_m \) | 2 | 2 | 2 | 2 | 2 | 2 |

### Table 2

| Various length and gain values used to describe roll-over shapes. Note that the forefoot gain value is constant and all length values are in measured incm. The corresponding roll-over shapes are shown in Fig. 5. |
|---|---|---|---|---|---|---|
| \( r_h \) | 0.01 | 0.09 | 0.17 | 0.25 | 0.33 | 0.41 |
| \( r_f \) | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| \( L_f \) | 7.86 | 7.98 | 8.08 | 8.18 | 8.33 | 8.39 |
| \( L_m \) | 0.13 | 1.25 | 2.45 | 3.75 | 5.15 | 6.69 |
| \( L_m \) | 2 | 2 | 2 | 2 | 2 | 2 |

### Table 3

| Legend and the corresponding values used for the bifurcation diagrams. |
|---|---|---|---|---|
| Robot Parameter (x-axis) | Solid line | Dashed line | Dotted line | Midfoot length \( L_m \) (cm) and midfoot gain value \( r_m \) |
| Mass ratio | \( r_f \) and \( r_h = 0.8 \) | \( r_f \) and \( r_h = 0.5 \) | Point feet | 0 and 0 |
| Length ratio | \( r_f \) and \( r_h = 0.8 \) | \( r_f \) and \( r_h = 0.5 \) | Point feet | 0 and 0 |
| Slope angle | \( r_f \) and \( r_h = 0.8 \) | \( r_f \) and \( r_h = 0.5 \) | Point feet | 0 and 0 |
| Forefoot gain | \( r_f = 0.5 \) | \( r_f = 0.2 \) | \( r_f = 0.01 \) | 0.02 and 1 |
| Hindfoot gain | \( r_f = 1 \) | \( r_f = 0.8 \) | \( r_f = 0.5 \) | 0.02 and 1 |
| Horizontal roll-over shape length | \( r_f = 0.5 \) | \( r_f = 0.5 \) | 0 and 0 | 0 and 0 |
5.2.1. Average velocity (Fig. 8)

The average velocity increases significantly with increase in the foot gain value for a given mass ratio. However, for a given foot gain value, the increase in average velocity with respect to mass ratio is moderate.

The range of single periodic stability (i.e. the value of length ratio at which the gait motion switches over to double periodic) increases as the foot gain value increases. Fig. 8 also illustrates that there is an exponential drop in the average velocity value as the length ratio increases. However, it is seen that the difference between the average velocity for different roll-over shapes, for a given mass or length ratio for a range of gain values and point feet, remains constant.

Fig. 8. Bifurcation diagrams for average velocity. Refer to Table 3 for the legend.
As the slope angle increases, the increase in the average velocity becomes significantly higher when the gain values are increased. It is also observed that the average velocity increases as the forefoot and hindfoot gain values increase. The influence of forefoot and hindfoot gain values on the stability of the passive walking can be observed in the last two figures. The forefoot gain bifurcation diagram illustrates that the passive walking becomes unstable for lower forefoot gains and higher hindfoot gains (solid line). The hindfoot gain bifurcation diagram also confirms this pattern (dotted line).

5.2.2. Mechanical energy (Fig. 9)

The mechanical energy is the summation of the total kinetic and potential energies of the three point masses (hip mass and mass for each leg). An increase in the average velocity will result in an increase in the kinetic energy and hence the trends for various robot parameters are similar to the average velocity bifurcation diagrams.

Fig. 9. Bifurcation diagrams for mechanical energy. Refer to Table 3 for the legend.
5.2.3. Step period (Fig. 10)

For a given mass ratio, the effect of foot gain on the step period is moderate. However, there is an initial steep increase in step period followed by a moderate increase as the mass ratio increases. For a given length ratio, the effect of foot gain on the step period is also moderate, with a similar pattern to the mass ratio. The stability range of single period motion also increases with increased foot gain values (refer to the length ratio bifurcation diagram). The stability range initially increases with the slope angle (dotted and dashed lines). However, as the foot gain values increase, passive walking becomes unstable even at lower slope values as the gain value is further increased (solid line).

The rate of increase in the step period value with respect to forefoot gain becomes higher as the hindfoot gain value increases. In the last two bifurcation diagrams, it can be seen that the behaviour of the step period changes at $r_f$ and $r_h$ values around 0.8. For a forefoot gain value of around 0.8, the three bifurcation graphs cross over. This value is referred to as the 'critical forefoot gain value' since the step period would remain constant for every acceptable value of hindfoot gain. This is an interesting observation with potential benefit in the design of control systems. However, it also illustrates the need
to further understand the non-linear dependence of various robot parameters on gait descriptors and also highlights the limitations of existing experimental approaches where one robot parameter is changed at a time.

5.2.4. Inter-leg angle (Fig. 12)

The inter-leg angle is defined in Fig. 11. The corresponding distance between the contact points of two legs is defined as the inter-leg length. The analysis of the inter-leg angle is important as it effectively controls the step length. In the case of a rolling contact the calculation of the step length is complex. The step length is equal to the distance of the contact points at the double support phase plus the arc length of the roll-over shape.

For different forefoot and hindfoot gains (described by solid, dashed and dotted lines) and point feet (dotted line) the variation in $\alpha$ is much smaller than the variation in $\delta$. However, the existence of a critical mass ratio (at about 1.8) is seen, where the inter-leg angle $\delta$ remains constant for any acceptable foot gain. As the mass ratio increases beyond the critical mass ratio, any increase in the foot gain decreases the inter-leg angle $\delta$. The behaviour is opposite for mass ratio values lower than the critical mass ratio value.

The length ratio variation is similar to the mass ratio variation and demonstrates an initial increase in the inter-leg angle for smaller length ratios. A critical length ratio (~1.1) occurs when all of the lines intersect and the inter-leg angle $\delta$ is constant for all acceptable values of forefoot and hindfoot gain. The figure also shows that the range of single period stability (i.e. the value of length ratio at which the gait motion switches over to period two motion) increases as the foot gain increases. In the last two bifurcation diagrams, where the variation in $r_f$ and $r_h$ are considered, the mass ratio and length ratio is 2 and 1. These values are very close to the critical values of ~1.8 and ~1.1 respectively. Hence, the value of $\delta$ remains approximately constant for a range of forefoot and hindfoot gains.

5.2.5. Forefoot and hindfoot lengths ($L_f$ and $L_h$) (Fig. 13)

The robot uses more horizontal hind foot length $L_h$ to remain stable as the foot gain increases for a given mass or length ratio or slope angle. It also tends to use higher hind foot length as the mass or length ratio or slope angle increases for a given value of foot gain. Similar observations are seen for horizontal fore foot length $L_f$. The observations in the last two bifurcation diagrams are trivial. As the forefoot gain increases, i.e. as the roll-over shape becomes wider or tends to become flat, the horizontal forefoot length, $L_f$, increases. In contrast the horizontal hindfoot length, $L_h$, remains relatively constant. Similar observations are seen for the hindfoot gain variation.

5.2.6. Roll-over shape arc length and inter-leg length ($L_e$ and $L_i$) (Fig. 14)

The inter-leg length is directly influenced by the inter-leg angle $\delta$. A similar pattern of variation with the existence of critical mass and length ratio values is observed. The bifurcation diagrams for variation in $r_f$ and $r_h$ also show that the inter-leg length remains almost constant for various values of forefoot and hindfoot gains. The pattern of variation for the effective roll-over shape arc length, $L_e$, is similar to that for the hind foot and fore foot lengths.

5.2.7. Effect of the horizontal roll-over shape length on gait descriptors (Fig. 15)

The roll-over shape arc length is expected to be determined experimentally and input into the computational model proposed in this paper. If the arc length is too long the biped robot will choose a shorter roll-over shape arc length, referred to as the effective roll-over arc length, based on the governing equations. Hence the resulting horizontal roll-over shape length will be reduced. It can be seen that the average velocity increases linearly with the horizontal roll-over shape length until it is less than or equal to the maximum roll-over shape arc length for which the robot kinematics allows stable walking. The excess arc length is effectively ignored and hence the velocity value remains same. For large gain values, the passive walking becomes period two (solid line), however, the forefoot and hindfoot lengths remain single period ($L_f$ and $L_h$ bifurcation diagrams). This is because the input roll-over shape is smaller or equal to the effective roll-over shape chosen by the robot.
5.3. Phase plane limit cycles

The dependence of the actual stance leg angular velocity ($\dot{\theta}_s$) and angular displacement ($\theta_s$) on the horizontal roll-over shape lengths, and the hindfoot and forefoot gains, is plotted using phase plane limit cycle diagrams (Fig. 16). Table 4 gives the legend used for Fig. 16. The mass/length ratio and slope angle values used the analysis are 2, 1 and $2^\circ$ respectively.

The phase plane limit cycle diagrams show that the maximum value of the actual stance leg angular velocity and displacement increases with larger values of horizontal roll-over shape lengths and forefoot gains. The jump in the actual stance
leg angular velocity during the double support phase when the swing leg becomes the stance leg (the bottom right hand corner of the phase plane limit cycle diagram) is different as the horizontal roll-over shape lengths and forefoot gains are changed. The reversal of this trend is observed at the critical values of horizontal roll-over shape length (10 cm) and hindfoot gain (0.7). It is also seen that for a forefoot gain of 0.4 the angular velocity before and after the double support contact remains the same.

Fig. 13. Bifurcation diagrams for the forefoot and hindfoot lengths. Refer to Table 3 for the legend.
6. Conclusions

Modelling the rolling contact of a robotic foot is a challenging task, and previous research has mostly relied on point, flat or curved/circular feet. For a human foot, the centre of pressure moves forward as it rolls from the heel strike position to the toe off position. Over the last decade, experimental research on prosthetic foot design has introduced a new concept referred to as 'roll-over shape'. The roll-over shape is a locus of centre of pressure defined in a co-ordinate frame such that

Fig. 14. Bifurcation diagrams for the roll-over shape arc length and the inter-leg length. Refer to Table 3 for the legend.
always aligned with the straight line joining the ankle and knee or the line joining the ankle and hip [42]. Roll-over shapes, although independent of walking speed, depend on various conditions such as age, medical conditions, shoe type, shoe heel height, shoe flexibility, slope of the walking surface, and can be determined experimentally.

A discrete pivot point roll-over model has been developed that takes the roll-over shape as an input parameter and computes its effect on the stability and kinematics of passive biped robot. The fundamental double pendulum equations proposed by Goswami et al. [27] have been revisited to model the influence of variable curvature of roll-over shapes and the

**Fig. 15.** Bifurcation diagrams to illustrate the effect of horizontal roll-over shape length. Refer to Table 3.
Fig. 16. Phase plane limit cycles for different horizontal roll-over shape lengths, hindfoot gains and forefoot gains.

Table 4
Legend and the corresponding values used for the phase plane limit cycle diagrams.

<table>
<thead>
<tr>
<th>Phase plane limit cycle</th>
<th>Thin solid line</th>
<th>Dashed line</th>
<th>Thick solid line</th>
<th>Gain values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal roll-over shape length</td>
<td>Point feet</td>
<td>10 (cm)</td>
<td>20 (cm)</td>
<td>$r_h = 0.8 \ r_m$ and $L_m = 0$</td>
</tr>
<tr>
<td>Hindfoot gain ($r_h$)</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>$r_h = 0.8 \ r_m$ and $L_m = 0$</td>
</tr>
<tr>
<td>Forefoot gain ($r_f$)</td>
<td>0.4</td>
<td>0.8</td>
<td>1</td>
<td>$r_h = 0.5 \ r_m$ and $L_m = 0$</td>
</tr>
</tbody>
</table>
continuous angular velocity of the swing leg during the roll-over phase. The traditional swing phase and stance phase is replaced with three phases: the stance leg transition phase, the swing phase and the double support transition phase (when the swing leg contacts the ground and the stance leg becomes the new swing leg and vice versa). The corresponding governing equations have been re-developed. The initial conditions for the double pendulum model are updated at each stance leg transition phase and the procedure has been described.

A parametric study was undertaken for various gains ($r_f$ and $n_b$), lengths ($L_f$ and $L_b$), and mass and length ratios, on various slopes, to study the effect on the average velocity, step period, inter-leg angle and mechanical energy of the system. This is important as the experimental research reported in Section 2.2 has shown evidence that the analysis of roll-over shapes for the optimal design of prosthetic feet has become an increasingly important area of research. The stiffness and other adjustments used in an optimal prosthesis design influence the shape and size of the corresponding roll-over shape. The results of the parametric study provide a first step in discovering trends in order to gain further insights in the complex inter-dependency among various gait parameters. This information is expected to influence the design process of future prosthetic feet.

Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version, at http://dx.doi.org/10.1016/j.apm.2013.02.048.

References


