Equivalent models of composite corrugated cores with elastomeric coatings for morphing structures

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\textbf{Article Info}

Article history:
Available online 13 May 2013

Keywords:
Coated corrugated core
Homogenization
Composites
Morphing skin

\textbf{Abstract}

Coated composite corrugated panels have wide applications in engineering, especially in morphing skin applications. The optimal design of these structures requires simple models of the panels that may be incorporated into multi-disciplinary system models. Therefore, equivalent structural models are required that retain the dependence on the geometric parameters of the coated corrugated panels. Taking into account the geometric and mechanical properties of the coated corrugated panel, an analytical homogenization model is investigated in this paper. The importance of this work is that it provides a simple equivalent analytical model which uses the geometric and mechanical properties of panel as variables that can be applied for further optimization studies. In this regard, two analytical solutions to calculate the equivalent tensile and bending flexural properties of a coated composite corrugated core in the longitudinal and transverse directions are presented. Then different experimental and numerical models are investigated to verify the accuracy and efficiency of the presented equivalent model. The comparison studies demonstrate the suitability of the proposed method for application in further complex design investigations.

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1. Introduction

Sandwich structures have been used for a long time in applications where low weight is important, such as packaging, civil, naval, automotive and aerospace industries due to their low mass to stiffness ratio and high impact absorption capacity [1–5]. Some instances of their applications in daily life are cardboard sandwich cores used for packaging, metal corrugated roofs, ship hulls, automotive chassis and bumpers, fuselages and morphing wings. In nature, where mechanical performance has to be optimized, sandwich structures are used, for example in the human skull which is made up of two layers of dense compact bone separated by a “core” of lower density material.

Composite corrugated panels, as a subdivision of sandwich structures, have exceedingly anisotropic behavior. They are stiff and flexible along and transverse to the corrugation direction, respectively. Composite corrugated panels have been proposed as a candidate for application in morphing wings. This is due to the fact that wing structures must be stiff so as to withstand bending due to aerodynamic forces, and flexible so they can deform efficiently in flight. Another advantage of using sandwich structures (made of metals or composites) with corrugated cores is that they have high fatigue resistance. Moreover, composite corrugated panels decrease the number of parts used in a wing structure which increases the speed of assembly and reduces the manufacturing costs [6].

Numerous investigations have been carried out on the mechanical behavior of corrugated boards for general applications. Luo et al. [7] investigated analytically the effect of different shapes of the corrugated medium on the bending stiffness of corrugated board. Gilchrist et al. [8] considered geometric and material nonlinearities and different loading configurations in their finite element analysis and they obtained results that correlated reasonably well with the experimental measurements. In terms of optimization, Daxner et al. [9] conducted a study on a specific kind of corrugated board with the aim of weight-reduction and increased buckling strength. Kazemahvazi et al. [10,11] modeled and tested hierarchical composite corrugated cores with better strength properties. They investigated failure mechanism maps for the different failure modes and showed a good agreement between the analytical predictions and the experimental observations. Leekitwattana et al. [12] proposed the concept of a bi-directional steel corrugated core and derived the transverse shear stiffness of a sandwich beam structure using analytical methods.
The development of smart materials motivated designers towards the concept of a morphing wing which may provide superior aircraft performance. Therefore, morphing wings are an important application of composite corrugated panels and in many cases the design of the skins has been identified as a major issue. The highly anisotropic behavior of composite corrugated panels is very effective in morphing wing applications; the panels are stiff along the corrugations to withstand the aerodynamic loads and flexible transverse to the corrugations to allow deformation. Yokozeki et al. [13] experimentally evaluated the in-plane stiffness of corrugated composite laminates as a proposed skin for a flapping flexible wing and analytically developed a simple model for the initial stiffness of the laminates. As an extension to this study, tensile and flexural characteristics of trapezoidal corrugated laminates made of aramid and glass fiber were experimentally, numerically and analytically investigated by Thill et al. [14] and Dayyani et al. [15], respectively. They investigated the local failure mechanisms, energy dissipation due to delamination in the structure and the three-stage mechanical behavior of the composite corrugated core in tensile and bending.

On the other hand, the numerical modeling of composite corrugated sheets can be very expensive and time-consuming if many corrugation periods are spanned or an actuation system and internal structure are involved. Therefore, a need exists to represent the structural properties of the corrugated panels with equivalent structures accounting for mesh densities much lower than needed for the corrugation geometry. The investigations in this field can be classified in two main subcategories: equivalent modeling of uncoated and coated corrugated cores.

Briassoulis [16] and Yokozeki et al. [13] calculated the equivalent extensional and flexural rigidities of corrugated sheets made of isotropic and composite material using Castigliano's second theorem. Using thin shell theory, Kress et al. [17,18] derived accurate analytical expressions of equivalent orthotropic plates made of multidirectional laminates for circular corrugations. They extended their study by developing a two-dimensional finite element model for the analysis of corrugated laminates. Xia et al. [19] presented a homogenization-based analytical model considering the coupling stiffness effect. They calculated the average equivalent internal forces and moments over a unit cell corrugation and validated their model with finite element simulation.

Recently, the homogenization of coated corrugated cores has been investigated and more elaborate theories have been used to study these structures due to their complex geometry. Aboura et al. [20] analyzed experimentally and analytically the elastic behavior of corrugated cardboard with skins. They used the classical laminate theory of plates in order to homogenize the coated corrugated core and developed a finite element model showing that the simplified homogenized procedure is adequate. Furthermore, Talbi et al. [21] studied both laminated and sandwich plate theories and presented an analytical homogenization model for a coated composite corrugated core. They considered shear forces and torsion moments and developed a 3-node shell element for the linear and buckling analyses. Abbès and Guo [22] presented an analytical homogenization model for the torsion of corrugated cardboard. They decomposed the plate torsion into two orthogonal beam torsion rates and then developed a shell element for the 2D modeling of corrugated cardboard.

As demonstrated in the literature, a critical component of morphing structures is a skin that is stiff to withstand the aerodynamic loads and flexible to enable to morphing deformations. However, as a proposed candidate for the skin of a morphing wing, the behavior of the corrugated panels must be investigated comprehensively and optimized in terms of aero-elastic effects and the boundary conditions arising from the internal wing structure. The optimal design of these structures requires a simple equivalent analytical model to incorporate into multi-disciplinary system models. However, due to the complex geometry of the coated corrugated core, so far homogenization has been mainly numerical and therefore, still costly and expensive, especially with regard to morphing wing applications.

The importance of this work is that it provides a simple equivalent analytical model which uses the geometric and mechanical properties of the panel as variables that can be applied for further optimization studies. In this paper, two analytical solutions are presented to calculate the equivalent tensile and bending flexural properties of a coated composite corrugated core in the longitudinal and transverse directions. Then to verify the accuracy and efficiency of the presented equivalent model, different experimental and numerical models are investigated. The results obtained by the present model are compared to those given by numerical simulations and experiments. The comparison demonstrates the suitability of the proposed method for application in further design investigations.

2. Analytical homogenization methods

In this section, two analytical solutions to calculate the equivalent tensile and bending flexural properties of a coated composite corrugated core in the longitudinal and transverse directions are presented. Fig. 1a shows a schematic of the corrugated core coated with elastomeric skins in tension. The panel is assumed to have periodic corrugations in the longitudinal direction only. The corrugation pattern consists of trapezoidal segments. The objective is to approximate the response of the coated corrugated panel using two longitudinal and transverse beam models whose properties are selected to be equivalent to those of the original panel. Since the ratio of the elastomer Young’s modulus to the glass fiber Young’s modulus is very small, a good assumption is to neglect the elastomer coating in the areas overlapped with the composite corrugated core. This assumption is reasonable because these two materials are well adhered together and have the same displacement. Thus the strain energy terms of the elastomer in contact with the glass fiber may be neglected. Furthermore, since the out of plane and compression stiffnesses of the elastomer coatings are very low, they may be modelled as springs that undergo only tension. In other words, their action resists the gap opening between two adjacent corners of each unit cell of the corrugated core. On the other hand, since the structure is indeterminate in terms of loading in the longitudinal direction, the equilibrium and compatibility equations are used to find the force distribution in the elastomeric members. Then the whole structural stiffness of the panel is obtained using Castiglione’s second theorem. Finally, the analytical solutions are compared to experimental and numerical results.

2.1. Longitudinal in-plane stiffness

2.1.1. Problem definition

Fig. 1a shows a schematic of the constructed corrugated core coated with elastomeric skins in tension. The required parameters to define the geometry of the coated corrugated core are shown in Fig. 1b.

2.1.2. Solution

To calculate the in-plane displacement of the corrugated core, a unit cell of the corrugated core is considered as shown in Fig. 1b. Since the unit cell is symmetric along the axis passing through the center of the unit cell, only one half of the cell is studied in the calculation. Considering the concept of periodicity and Saint-Venant’s principle [23] in the theory of elasticity, the global applied force would pass through the neutral axis located at the center of...
the panel. In other words the eccentric local effect of the load becomes very small at sufficiently large distances from the load segment, where the external load is applied.

As illustrated in Fig. 2, in order to avoid dealing with the indeterminate loading configuration of the structure, half of the corrugated core unit cell is subdivided into two corrugated parts such that each has half of the original thickness of the structure. The idea is to calculate the stiffness of each subdivision and then add them together. Considering the nonlinearity in the formulation of the second moment of area with respect to the thickness, it is important to state that this parameter is calculated about the separating centre line of the original configuration, so that the total stiffness of the structure would be the same as the original configuration after the summation of these two substructures. Thus, from the parallel axis theory, 

\[
I_c = \frac{b t^3}{36}
\]

Where \( b \) and \( t \) denote the width and thickness of the corrugated core unit cell respectively. Moreover, because of the periodicity and symmetry, it is assumed in Fig. 2 that the nodes at the end of spring \( S_1 \) and the corrugated unit cell have equal displacement.

Fig. 3 illustrates a schematic of the second substructure of corrugated core in tension. In this figure \( f_4 \), \( R \) and \( M_R \) are the force from spring \( S_2 \), reaction force and reaction moment, respectively. According to Castigliano’s second theorem in order to calculate the displacement at node \( P \), the virtual force \( g \) must be applied to the structure at this node. Therefore the strain energy of each member due to the bending moment and axial forces may be calculated as:

\[
U_i = U_{iA} + U_{iB}, \quad i = 1, 2, 3
\]

where

\[
U_{iA} = \frac{F^2h_1}{2E_cA_c}, \quad U_{iB} = \frac{F^2h_1^3}{8E_cI_c}
\]

Here \( h, E_c, A_c \) and \( I_c \) represent the height, Young’s modulus, cross section and second moment of area of the subdivided corrugated core respectively. Moreover \( c \) and \( s \) denote \( \cos(\theta) \) and \( \sin(\theta) \), respectively. Differentiating the strain energy of each member with respect to this virtual force \( g \), the displacement of each member would be obtained from following relations.

\[
\delta_{iA} = \left. \frac{\partial U_{iA}}{\partial g} \right|_{g=0}, \quad \delta_{iB} = \left. \frac{\partial U_{iB}}{\partial g} \right|_{g=0}, \quad i = 1, 2, 3
\]
By following a straightforward formulation and adding these terms, the total displacement would be expressed as:

$$
\delta = \frac{(F - f_0)(c^2l_2 + l_1) + \frac{1}{2Ec_x} \left[ -Fs_1^2 + 2(F - f_0)s^2l_1^2 \right]}{E_c A_c} + \frac{h^2l_3((F/2) - f_0)}{E_c l_2}
$$

(6)

Considering that the force in the elastomeric members are proportional to the applied structural load, i.e. $f_0 = \alpha_0 F$, the force displacement relation may be rewritten in the following form:

$$
\delta = \frac{\left( (1 - \alpha_0)(c^2l_2 + l_1) - 3hs_1^2 + 4(1 - \alpha_0)s^2l_1^2 + 6h^2l_3(1 - 2\alpha_0) \right)}{12E_c l_2}
$$

(7)

Compatibility implies that $\delta = \frac{\delta_1}{Ec_x} = \frac{\delta_2}{Ec_y} = F$ for the elastomeric member, where $E_c$ and $A_c$ represent the Young's modulus and cross section of the elastomeric coating respectively. Therefore $\alpha_0$ is obtained as:

$$
\alpha_0 = \frac{(c^2l_2 + l_1) + 3hs_1^2 + 4(1 - \alpha_0)s^2l_1^2 + 6h^2l_3(1 - 2\alpha_0)}{12E_c l_2}
$$

(8)

Considering Fig. 2, and by repeating the same procedure for the first substructure of the half of periodic unit cell, $\alpha_0$ is obtained as:

$$
\alpha_0 = \frac{(c^2l_2 + l_1) + 3hs_1^2 + 4(1 - \alpha_0)s^2l_1^2 + 6h^2l_3(1 - 2\alpha_0)}{12E_c l_2}
$$

(9)

After finding the distribution of forces in the elastomeric members i.e. $\alpha_0$ and $\alpha_0$, the total mechanical behaviour of the second substructure of half of the unit cell would be calculated by adding the displacement of the first member as:

$$
\delta_{14} = F_{14} + \frac{h^2l_3((F/2) - f_0)}{E_c l_2}
$$

Therefore the equivalent longitudinal tensile modulus can be presented as:

$$
\frac{AE}{E_0} = (l_1 + l_2 + l_3)(K_1 + K_2)
$$

(13)

2.2. Longitudinal out of plane stiffness

As mentioned before, since the out of plane and compression stiffness of the elastomer coatings are very low, it is assumed that they act like springs that undergo only tension. In other words, the role of the upper elastomeric coating which is subjected to tensile loading is modelled as springs resisting the gap opening between two adjacent corners of each unit cell of the corrugated core. The minor effect of the coating subjected to compression forces during the bending is neglected.

2.2.1. Problem definition

As illustrated in Fig. 4a, the coated corrugated core is symmetric along the axis passing through the center of the panel. Hence only one half is considered in the analytical solution. Fig. 4a depicts the schematic of generic coated corrugated core which includes $N + 1$ unit cells in half of the span of the simulated three-point bending experiment. It is expected that the maximum gap opening in the structure, and consequently the maximum stretching of the elastomeric coating, happens at node $N + 1$ which is in the center of the panel. The more distance the marked nodes in Fig. 4a have from the symmetry line of the panel, the less gap opening and consequently the less resisting force they would have. The idea here is to find a relation for the distribution of forces in the elastomeric members.

2.2.2. Solution

In order to find the distribution of the forces in the elastomeric members and to avoid solving complex coupled multi equations of compatibility for these members, the Euler–Bernoulli beam in a three point bending test is considered as illustrated in Fig. 4b. In this figure, for any two arbitrary points restin on a line parallel to the neutral axis, the ratio of their off axis strain in the x direction, are equal to the ratio of their coordinates, i.e.:

$$
\varepsilon_x \frac{X_2}{X_1} = \varepsilon_x \frac{X_2}{X_1}
$$

(14)

By assuming a displacement model for the corrugated structure based on an Euler–Bernoulli beam, and by analogy to Eq. (14), the force of the elastomer coating in each unit cell due to the gap opening is proportional to forces in the other cells. Thus it is assumed that the upper coating of the sandwich panel acts like a beam in three-point bending. That is, Eq. (14) is used here as an assumption to make a proportional relation between the forces of the elastomeric members and the one located in the center of panel. This relation can be stated as:

![Fig. 4. Schematic representation of a beam in three-point bending (b) versus the coated corrugated core (a).](image-url)
Table 1
Ratio of strains and stresses corresponding to nodes for the elastomeric members marked in Fig. 4a.

<table>
<thead>
<tr>
<th>$(X_1/X_{N+1})$</th>
<th>Coordinate ratios</th>
<th>FE strain ratios</th>
<th>Error%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(X_1/X_{10})$</td>
<td>0.0341</td>
<td>0.0346</td>
<td>1.47</td>
</tr>
<tr>
<td>$(X_2/X_{10})$</td>
<td>0.1414</td>
<td>0.1430</td>
<td>1.13</td>
</tr>
<tr>
<td>$(X_3/X_{10})$</td>
<td>0.2488</td>
<td>0.2520</td>
<td>1.29</td>
</tr>
<tr>
<td>$(X_4/X_{10})$</td>
<td>0.3561</td>
<td>0.3610</td>
<td>1.38</td>
</tr>
<tr>
<td>$(X_5/X_{10})$</td>
<td>0.4634</td>
<td>0.4700</td>
<td>1.42</td>
</tr>
<tr>
<td>$(X_6/X_{10})$</td>
<td>0.5707</td>
<td>0.5790</td>
<td>1.45</td>
</tr>
<tr>
<td>$(X_7/X_{10})$</td>
<td>0.6780</td>
<td>0.6880</td>
<td>1.47</td>
</tr>
<tr>
<td>$(X_8/X_{10})$</td>
<td>0.7854</td>
<td>0.7960</td>
<td>1.35</td>
</tr>
<tr>
<td>$(X_9/X_{10})$</td>
<td>0.8927</td>
<td>0.9050</td>
<td>1.38</td>
</tr>
</tbody>
</table>

To see how valid this assumption is, the ratio of strains and in plane axial forces for the nodes marked in Fig. 4a for 9 periodic unit cells, i.e. $N = 9$, are calculated, tabulated and compared with FE results in Table 1. (More details of the FE simulation are presented in Section 3.2.)

The matrix form of the forces in the elastomeric members can be written as following symmetric matrix relation, in which $f_{el}(ij)$ for $i, j = 1: (N + 1)$ is the elastomeric force that the corners of the unit cells exert on each other, as shown in Fig. 5b. Thus

$$f_{el} = f_{el}(ij) = \begin{cases} \frac{I_{el}}{L_{np} + L_{uc}}, & i + 1 = j \\ 0, & i + 1 \neq j \end{cases}$$  \quad (15)

where $L_{np}$ and $L_{uc}$ are the length of non-periodic part and periodic unit cell respectively. Next, in order to determine the distribution of force in the elastomeric member located at the center of panel, i.e., $f$ in Eq. (15) and Fig. 5a, the displacement at node $q$ is calculated first. In this regard, again according to Castigliano’s second theorem, the virtual force $g$ is applied to the structure at this node. Differentiating the strain energy of each member respect to this force, the displacement in each member can be obtained. As mentioned before the strain energy of each member is due to the axial force and bending moment:

$$U_{ij} = U_{ij}^a + U_{ij}^b, \quad i = 1 : N + 1; \quad j = 1, 2, 3, 4, 5$$  \quad (16)

where the indices $i$ and $j$ denote the cells and members of each cell respectively, as shown in Fig. 5b.

Considering equilibrium equations, the axial forces and bending moments of the illustrated members in Fig. 5a, i.e., $i = N + 1$ and $j = 4, 5$ are

$$T_{(N+1)4} = (g - f)$$  \quad (17)

$$M_{(N+1)5} = F(d - x) + (g - f)h$$

where $d = L_{np} + N_{uc}$ and represents half of the bending span, as shown in Fig. 5a. By differentiating the obtained strain energy due to the axial forces and bending moments with respect to the virtual force, the relative displacement in each member is obtained. In Eqs. (18)–(22) the factor $w$ has been introduced so as to keep the format of equations simple and clear. Thus

$$\delta_{(N+1)4} = f_{el} \left( -\frac{I_{el}}{E_A} \right) = fw_1$$  \quad (18)

$$\delta_{(N+1)5} = F \left( \frac{h_d (d - l_4)}{2E_I} \right) + f \left( -\frac{l_4 h_d^2}{E_I} \right) = Fw_2 +fw_3$$

Likewise for the next member, i.e. $i = N + 1$ and $j = 5$,

$$T_{(N+1)5} = (g - f) c - Fs$$

$$M_{(N+1)4} = (F(d - l_4) + b(g - f)) - x((g - f)s + Fc)$$

and for the tensile axial and flexural displacement:

$$\delta_{(N+1)5} = F \left( \frac{l_{el}}{E_A} \right) + f \left( \frac{h_d^2}{E_A} \right) = Fw_4 +fw_5$$

$$\delta_{(N+1)4} = F \left( \frac{-h_d (b - l_4) - c (h_d^2 + \frac{1}{2})}{2k} \right) + f \left( \frac{-h_d^2 (b - l_4) - c (h_d^2 + \frac{1}{2})}{2k} \right) = Fw_6 +fw_7$$  \quad (20)

Considering the compatibility of the elastomeric member, the displacement of node $q$ which is the summation of these relative displacements is

$$\delta = \sum_{j=4}^{5} (\delta_{(N+1)4j} + \delta_{(N+1)5j}) = F(w_2 + w_4 + w_5) + f(w_1 + w_3 + w_5 + w_7) = \frac{f(l_c/2)}{A_E}$$  \quad (21)

where $l_c$ is the length of the elastomeric member which is located between two adjacent corners of two consecutive unit cells of corrugation. Then the relation between the applied global force and the resisting force due to the gap opening in the elastomeric member is

$$f = F \left( \frac{w_2 + w_4 + w_5}{2} - \frac{w_1 + w_3 + w_5 + w_7}{2} \right) = 2F$$  \quad (22)

Obtaining Eq. (22) and considering Eqs. (15) and (16) the force distribution in the elastomeric members are identified and hence the structure is determinate. In other words the force in each elastomeric member is proportional to the global applied force $F$, i.e., $f_{el-i+1} = x_{el-i+1}$. Similar to Eq. (15) $x_{el-i+1}$ is the symmetric matrix given by $x_{el-i+1} = \frac{L_{np} + L_{uc}}{2}$. By differentiating the whole strain energy of the structure with respect to the global applied load $F$ in Fig. 5b, the vertical displacement for the center of the panel would be obtained. Considering Fig. 5b, Eq. (16) can be divided into periodic and non-periodic parts as:

$$U = U_{ap} + \sum_{i=2}^{i=N+1} \sum_{j=1}^{i} U_{ij}$$

Considering Eqs. (15) and (22) and following the same procedure, the axial forces and bending moment of the members of the non-periodic part, i.e. $i = 1$, can be calculated. The axial and bending deformation of these members are calculated as

Fig. 5. Schematic of half of the coated corrugated core in bending.
\[ \delta_{i4a} = F \left( \frac{s + \alpha_{i2} C_l^2 l_i}{E A_t} \right) = FC_{14a} \]
\[ \delta_{i4a} = F \left( \frac{(s + \alpha_{i2} S - C l_i)}{3E I_t} \right) = FC_{14a} \]
\[ \delta_{i5a} = F \left( \frac{(l_i + l_C - x)}{3E I_t} \right) = FC_{15a} \]
\[ \delta_{i5a} = F \left( \frac{(l_i + l_C - x)^3}{3E I_t} \right) = FC_{15a} \]

In Eqs. (23)–(35), \( C_{ij} \) is the flexibility factor that has been introduced so as to keep the format of equations simple and clear. According to Fig. 5b, for the periodic unit cells of the structure, the length parameter \( L_i \) is introduced as:

\[ L_i = \left[ \frac{n_p}{2} + (i - 2)L_{uc} \right] + \sum_{j=1}^{N} L_j \quad i = 2 : N + 1 \text{ and } j = 1, 2, 3, 4, 5 \]

where \( L_i \) is the distance from the support to the member of each periodic unit cell. Considering Fig. 5b, for the first member of each periodic unit cell, i.e. \( i = 2 \) and \( j = 1 \), the axial and bending moment is stated as:

\[ T_{i1} = f_{i1} C = F(s - \alpha_{i1} C) \]
\[ M_{i1} = f_{i1} l_i h - F(x + L_i) = F(\alpha_{i1} h - L_i - x) \]

Similarly, for the third member of each periodic unit cell, i.e. \( j = 3 \), the axial and bending moment is:

\[ T_{i3} = 0 \]
\[ M_{i3} = F(L_i + x) \]

Therefore the deformations corresponding to these forces are:

\[ \delta_{i3a} = F \left( \frac{(s + \alpha_{i1} C l_i)^2 l_i}{E A_t} \right) = FC_{i3a} \]
\[ \delta_{i3a} = F \left( \frac{(s + \alpha_{i1} S - C l_i)^2 l_i}{3E I_t} \right) = FC_{i3a} \]

Likewise, for the fourth member of each periodic unit cell, i.e. \( j = 4 \), the axial force and bending moment are:

\[ T_{i4} = F(s + f_{i4} c) = F(s + \alpha_{i4} C) \]
\[ M_{i4} = f_{i4} l_i h + F(x + L_i) = F(\alpha_{i4} h - L_i - x) \]

Similarly, the deformations corresponding to these forces are:

\[ \delta_{i4a} = F \left( \frac{(s + \alpha_{i1} C l_i)^2 l_i}{E A_t} \right) = FC_{i4a} \]
\[ \delta_{i4a} = F \left( \frac{(s + \alpha_{i1} S - C l_i)^2 l_i}{3E I_t} \right) = FC_{i4a} \]

Finally, for the fifth member of each periodic unit cell, i.e. \( j = 5 \), the axial and bending moment is:

\[ T_{i5} = f_{i5} C = F(\alpha_{i5} C) \]
\[ M_{i5} = f_{i5} l_i h - F(x + L_i) = F(\alpha_{i5} C) \]

and the deformations are:

\[ \delta_{i5a} = F \left( \frac{(s + \alpha_{i1} C l_i)^2 l_i}{E A_t} \right) = FC_{i5a} \]
\[ \delta_{i5a} = F \left( \frac{(s + \alpha_{i1} S - C l_i)^2 l_i}{3E I_t} \right) = FC_{i5a} \]

The vertical displacement for the half of the panel then is obtained as:

\[ \delta = \left( \sum_{j=4}^{i-5} (\delta_{ij4} + \delta_{ij3}) + \sum_{i=2}^{i-3} \sum_{j=1}^{i-5} (\delta_{ij3} + \delta_{ij2}) \right) \]
\[ = F \sum_{j=1}^{i-5} \sum_{j=1}^{i-5} (C_{ij4} + C_{ij3}) \]

Comparing this relation, with the cantilever beam with length equal to half of the original three point bending span, i.e. \( d \), the equivalent flexural stiffness would be obtained as:

\[ (EI)_{\text{eq}} \left( \frac{L_{uc}}{N_{\text{uc}}} \right)^3 \]
It is worthwhile to notice that $C_{\text{total}}$ is a function of the number of periodic unit cells and hence the equivalent flexural stiffness is dependent to the number of periodic unit cells.

### 2.3. Transverse stiffness

In order to predict the transverse stiffness modulus of the coated corrugated core, a straight Euler–Bernoulli beam is considered. As illustrated in Fig. 6a and b, calculation of the in plane and out of plane stiffness of the beam in the transverse direction leads to formulation of the equivalent transverse modulus of stiffness.

#### 2.3.1. Transverse in plane stiffness

Composite corrugated core and elastomeric members act like parallel springs in the transverse direction. Therefore, the total stiffness of a unit cell of coated corrugated core in the transverse direction is given by:

$$K_T = K_{eT} + K_{cT} = \frac{A_e E_e + A_c E_c}{W_T}$$  \hspace{1cm} (37)

where $A_e$, $A_c$, and $W_T$ denote the cross section area of the corrugated core and elastomeric coating and the width of the panel in the transverse direction, respectively. The axial stiffness for the equivalent bar is:

$$K_{eq} = \frac{(AE)_{eqT}}{W_T}$$  \hspace{1cm} (38)

Hence by comparing Eqs. (37) and (38) the equivalent stiffness in the transverse direction is:

$$(AE)_{eqT} = A_e E_e + A_c E_c \approx A_e E_c$$  \hspace{1cm} (39)

#### 2.3.2. Transverse out of plane stiffness

As mentioned in the previous section, since the composite corrugated core and elastomeric members act like parallel springs in the transverse direction, the total out of plane stiffness of a unit cell of coated corrugated core is given by:

$$EI_{eq} = EI_{eT} + EI_{cT} \approx EI_{eT}$$  \hspace{1cm} (40)

It is evident that in Eqs. (39) and (40) that the role of the elastomeric members in the transverse direction may be neglected since the elastomer Young’s modulus is small compared to the composite material.

### 3. Validation

Different experimental and numerical models are considered in this section to verify the accuracy and efficiency of the presented equivalent model. In the experimental part, both coated and uncoated corrugated cores are studied in tensile and three point bending tests and their mechanical behavior are compared to those predicted by the analytical solution. More details of the experiments for the uncoated corrugated cores are presented in [15].

#### 3.1.1. Problem definition

##### 3.1.1.1. Coated corrugated core fabrication

To manufacture the composite corrugated cores, prepreg laminates of glass fiber plain woven cloth were hand-laid on a trapezoidal machined aluminium mould. More details of the manufacturing process as well as the schematic of the trapezoidal mould and the prepreg laminates of glass fiber are presented in the author’s previous work [15]. Afterwards, both the upper and lower faces of the corrugated core were covered by elastomer which is widely used in applications where low stiffness and a high elastic strain are required.

##### 3.1.1.2. Coated corrugated core geometry

The length of the coated composite corrugated core investigated in this section is 300 mm. The widths of tensile and bending test specimens are 25 mm and 100 mm, respectively. Fig. 7 illustrates the constructed coated composite corrugated core which included 10 unit cells. The thickness of the corrugated core and elastomer skin is 1.00 mm and 0.80 mm, respectively. The values of the dimensions given in Fig. 1b are tabulated in Table 2.

#### 3.1.1.3. Coated corrugated core material characterization

The corrugated core is made of three-plies of woven glass fibers with epoxy resin where the orientation of each ply was 0° or 90° with respect to the corrugation direction. Moreover, to evaluate the mechanical properties of this material, uniaxial tensile tests were carried out on five flat samples with unidirectional embedded fibers. The average Young’s modulus of fibers, $E_{\text{fiber}}$, is 9 GPa, while the effect of the matrix is neglected. The elastomer coatings used in this paper are made of synthetic rubber Polyurethane (PU) which is knitted by a circular interlock weft method. To evaluate the mechanical properties of the elastomer, a total of six strips of elastomer sheet were cut and subjected to simple tensile tests. Table 3 describes the average Young’s modulus of the elastomer coating in three phases of stretching.

#### Table 2

Dimensions of the corrugated core unit cell.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Values (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>3.75</td>
</tr>
<tr>
<td>$a_2$</td>
<td>5.5</td>
</tr>
<tr>
<td>$a_3$</td>
<td>5.3</td>
</tr>
<tr>
<td>$h$</td>
<td>9.5</td>
</tr>
</tbody>
</table>

#### Table 3

Young’s modulus of the elastomer in different phases of stretching.

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Initial stretch</th>
<th>Medium stretch</th>
<th>Final stretch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>13.5 MPa</td>
<td>10.25 MPa</td>
<td>108.00 MPa</td>
</tr>
</tbody>
</table>
3.1.2. Experiments on coated composite corrugated core

3.1.2.1. Tensile test of coated composite corrugated core. Using the ASTM D3039 standard [24] as a basis for the tensile standard testing, six specimens of the coated composite corrugated core were tested experimentally in tension, transverse to their corrugations. Fig. 8a illustrates the investigated composite corrugated core with elastomeric coatings for the tensile test. A comparison of the average data from the experiment for the corrugated core with and without elastomeric coatings in the tensile tests is shown in Fig. 9a.

3.1.2.2. Three-point bending test of a coated composite corrugated core. Likewise, according to ASTM C393 [25], six specimens of the coated composite corrugated core were tested experimentally by a three point bending test. The span and diameter of the support rollers were 193 mm and 5 mm, respectively. Fig. 8b illustrates testing machine and the coated composite corrugated core during the three point bending test. A comparison of the average data from the experiment for the corrugated core with and without elastomeric coatings in the three-point bending test is shown in Fig. 9b.

3.1.3. Analytical modeling of experiments

The analytical modelling consists of two steps: first, the equivalent isotropic material properties for the plain woven fabrics are estimated and second, the equivalent tensile and bending flexural property of the coated composite corrugated core for 10 unit cells are evaluated. In the first step, since the plain woven fabrics are a type of heterogeneous orthotropic material, the equivalent Young’s modulus is calculated based on the finite element model described in detail in [15,26]. However the equivalent Young’s modulus can also be calculated from the rule of mixtures. Since the analytical solution is proposed for linear small deformations of the structure, the initial stretching behaviour of the elastomer is considered. Therefore, the equivalent Young’s modulus of the composite, $E_c$ and the elastomer $E_e$ are estimated as 4.5 GPa and 13.5 MPa respectively.

3.1.4.1. Longitudinal out of plane stiffness. The span in the three-point bending test was 193 mm. Hence, considering the values listed in Table 2 and Fig. 5b, one half of the bending span included one non-periodic part and three periodic unit cells. In this regard Eq. (15) is rewritten as:

\[
\mathbf{f}_{el} = \begin{bmatrix}
0 & f_{12} \\
f_{21} & 0 & f_{23} \\
f_{32} & 0 & f_{34} \\
f_{43} & 0 & f_{45} \\
f_{54} & 0 & f_{55}
\end{bmatrix}
= \begin{bmatrix}
0 & 0.1 \\
0.4 & 0 & 0.7 \\
0.1 & 0 & 0.4 \\
0.7 & 0 & 1 \\
1 & 0 & 0
\end{bmatrix}
\]

The force displacement curves for both coated and uncoated structures are plotted in Fig. 9b. By setting the Young’s modulus of the elastomer to zero the proposed analytical solution would be the same as the analytical method proposed in [15] and matches with the first phase of mechanical behaviour of uncoated corrugated core in the experiment. However by increasing the Young’s modulus of elastomer to 13.5 MPa, the proposed analytical solution correlates with the first phase of the mechanical behaviour of elastomeric coated corrugated core in the bending experiment.

3.2. Finite element validation

In this section the mechanical behavior of coated corrugated cores predicted by the analytical model is compared to the finite element results obtained by ABAQUS simulation. Two different effects on the mechanical behavior of the structure are investigated and verified by numerical simulations. In both cases the geometric
dimensions and material properties are selected as those presented in Section 3.1.1. In both cases a fine mesh and linear displacement theory were considered.

3.2.1. The effect of the number of unit cells

As mentioned in Section 2.1.2 the coated corrugated core acts as a series of springs in tension. However, for the bending case the story is slightly different. It is evident in Eqs. (23)–(36) that the flexibility factor, i.e. $C_{ij}$, itself is a quadratic function of the number of unit cells. On the other hand, considering Eq. (36) the bending stiffness is a cubic function of the length of the bending span which is dependent to the number of unit cells. As a result, in both tensile and bending cases the stiffness has an inverse relation with the number of unit cells. Fig. 10a compares the tensile stiffness of the panel versus $N$, the number of periodic unit cells, obtained by the equivalent model and the finite element simulation. Fig. 10b

Table 4

<table>
<thead>
<tr>
<th>Method/stiffness</th>
<th>Ratio of horizontal force to horizontal displacement (N/mm)</th>
<th>Ratio of vertical force to vertical displacement (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite element model</td>
<td>3.3820</td>
<td>0.0388</td>
</tr>
<tr>
<td>Analytical equivalent model</td>
<td>3.5500</td>
<td>0.0362</td>
</tr>
<tr>
<td>Error (%)</td>
<td>4.73</td>
<td>6.70</td>
</tr>
</tbody>
</table>

Fig. 9. Force-displacement behaviour of coated and uncoated corrugated core from tensile experiments and analytical solution.

Fig. 10. The tensile and bending stiffness of the panel versus the number of unit cells.
Fig. 11. The relation between Alpha and the Young's modulus of the elastomer in longitudinal tensile and bending tests.
shows a comparison of the bending stiffness of the panel versus the number of periodic unit cells in half of the bending span, obtained by the equivalent model and the finite element simulation.

3.2.2. The effect of combined loading

This section shows the suitability of the proposed method to replace the coated corrugated core with an equivalent structure under a complex combination of load and boundary conditions, by analyzing two finite element models corresponding to these structures.

Firstly a coated corrugated core with equivalent material properties given in Section 3.1.1.3 with the same dimensions of the tensile specimens given in Section 3.1.1.2, were modeled as beam elements. The boundary conditions were selected such that one end of the panel was fixed and the other was subjected to a horizontal displacement of 10 mm in the longitudinal direction. Furthermore, in terms of loading, one side of the coated corrugated core was subjected to a vertical uniform pressure of 1000 kPa. Secondly an equivalent beam model was simulated with an in-plane stiffness, i.e. $(EA)_{eq}$, of 1035.8 N and an out of plane stiffness, $(EI)_{eq}$, of 12931 N/mm²; the same boundary conditions and loading were modeled independently in ABAQUS.

In both cases fine meshes and linear small displacement theory were considered. The results obtained corresponded to the reaction forces at the fixed end and the nodal displacements of the moving end are tabulated and compared in Table 4. The error in both the horizontal and vertical cases is less than 6.7%, which demonstrates the efficiency of the proposed method under complex combined load and boundary conditions.

4. Discussion

Considering the geometric dimensions and material properties of the panel presented in Section 3.1.1, and regarding Eqs. 8, 9, and 22, the relation between the coefficient of the force distribution in the elastomeric members, i.e. $\alpha$, and the Young's modulus of the elastomer is shown in Fig. 11 for the tension and bending cases in the longitudinal direction respectively. The curve shows a plateau when the Young's modulus of the elastomer tends to infinity, which is expected because of the geometric constraints. The physical description of the behaviour is that by increasing the Young's modulus of the elastomer sufficiently there would be a rigid geometric constraint between two adjacent corners of the unit cells. In other words the rigid link would prohibit any change in the distance between two adjacent corners of subsequent unit cells. Therefore the rigid link would have a constant axial force. However it must be mentioned that this behaviour is true only in terms of theory. In reality, by increasing the Young's modulus of the elastomer up to a certain level the mechanical behaviour of the elastomer would change from membrane into shell behaviour. Moreover the deformation mechanism would also change since the skins of the sandwich panel would have a greater Young's modulus than the composite corrugated core. Fig. 11a and b illustrate the relation between $\alpha$ and the Young's modulus of the elastomer in the longitudinal tensile and bending tests respectively. In addition, the effect of the length of the elastomeric members on the distribution of force in each elastomeric member (i.e. different $z_4$ and $z_5$), which is due to the different geometric parameters, are shown in fig. 11a.

5. Conclusion

Two analytical solutions to calculate the equivalent tensile and bending flexural property of the coated corrugated core in the longitudinal and transverse directions are presented based on Castigliano's second theorem. The results obtained by the analytical model were compared to those given by numerical simulations and experiments. In the experimental part, both coated and uncoated corrugated cores were studied in tensile and three point bending tests. The ratio of the in-plane and out of plane stiffness of coated corrugated structure to uncoated corrugated structure was 2.28 and 2.14, respectively. This provides a better insight into the mechanical behavior of coated composite corrugated panels as candidates for morphing wing applications. Furthermore, the effect of combined loading and the number of unit cells on the mechanical behavior of the coated corrugated core are investigated and verified with numerical simulations. The physical description of the behavior that the relation between the coefficient of the distribution of forces in the elastomeric members and the Young's modulus of the elastomer converges when the Young's modulus of elastomer tends to infinity was also discussed. The comparison studies demonstrate the suitability of the proposed method for further design investigations.

Acknowledgement

The authors acknowledge funding from the European Research Council through Grant No. 247045 entitled “Optimization of Multi-scale Structures with Applications to Morphing Aircraft”.

Reference


