Theoretical and Experimental Analysis of Hysteresis in Piezocomposite Airfoils Using Preisach Model

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In this paper, the classical (scalar) Preisach model is used to predict hysteresis observed in several Macro-Fiber Composite actuated piezocomposite bimorph devices: 1) two cantilevered beams, 2) a simply supported thin airfoil, and 3) a cascading bimorph thick airfoil. This paper contributes to the research field by examining the effectiveness of the scalar Preisach model for operational and wind-tunnel tested piezocomposite airfoils in the presence of aerodynamic and inertial loading and subjected to two different electrical boundary conditions. A low-speed open-circuit wind tunnel is used for experimental analysis. The flow speed is used as a parameter to understand the effect of nonuniform aerodynamic loading on the accuracy of the model. In addition, the excitation frequency is used as a parameter to understand the effect of inertial (mass) loading. It was observed that the aerodynamic loading, up to 19 m/s, has a negligible effect on the aerodynamic output of the airfoils considered in this research. The classical Preisach model is capable of predicting the hysteresis observed in all of the samples tested with an average prediction error less than 6.8\% for a 0.5 Hz input signal and less than 9.3\% for a 1 Hz input signal. The increasing–decreasing and decreasing–increasing first-order transition curves are both found to be equally successful in developing the Preisach model for all samples.

Introduction

The recent interest in smaller and lighter aircraft has driven a need to employ smart materials for flight and flow control. For example, some field-deployable aircraft have flexible wings that can be folded during transportation, and they can be unfolded for operation. These compliant wings can be realized with the integration of smart materials such as piezoelectric materials. For smart-material actuated devices, a challenge is found in operating a relatively compliant, thin structure (desirable for piezoelectric actuators) in situations where there are relatively high external forces. Establishing a configuration that is stiff enough to prevent flutter and divergence but compliant enough to allow the range of available motion is the central challenge in developing a piezocomposite wing. Novel methods of supporting the wing can take advantage of aerodynamic loads to reduce control input moments and increase control effectiveness. Another challenge is the aerodynamic demand for relatively large changes in shape. To induce these shape changes (even without external forces), the smart-material requires a large displacement output resulting in significant hysteresis nonlinearity.

Hysteresis is a type of nonlinearity that contains memory; thus, there may be several outputs for a given input. In general, smart materials exhibit large hysteresis, so open-loop or feedback control methods may result in amplitude-dependent phase shifts and harmonic distortions. The hysteresis effects are large when the input signals go through peak-to-peak values. Hysteresis can be observed in static and dynamic events. Static events include systems with slowly changing inputs. In dynamic events, the inputs and the response are fast changing, so time effects (e.g., inertia) cannot be ignored. Hysteresis falls into two major categories: scalar and vector.

Scalar and static hysteresis can further be split into two categories: 1) hysteresis nonlinearities with local memories and 2) hysteresis nonlinearities with nonlocal memories. The first case, hysteresis with local memories, is the simplest form of hysteresis nonlinearity. The system can have only one of the two outputs for any input. The past events exert their influence on the future events through the current value of output for a transducer with this type of hysteresis. In the second case with nonlocal memory, there are an infinite number of outputs for any input. The future value of output depends on both the current value of output and the past extremum of the input. The second major category is vector hysteresis. Scalar hysteresis (discussed previously) is a special case of vector hysteresis. In the vector case, both inputs and outputs are vectors, with two-dimensional (2-D) and three-dimensional vectors being the most relevant to practical applications. In vector hysteresis nonlinearity, the past extremum values of input along all possible directions may affect future values of output. The background and modeling of hysteresis is discussed in detail by Visintin [1], Mayergoyz [2], and Smith [3].

Piezoelectric materials offer relatively high force output in a wide frequency range. Although the strain output is very small for low excitation levels, the response is relatively linear. In the linear regime, the fast response of lead-zirconate-titanate (PZT) materials caused initial interest in the field of aerodynamic vibration control. Many researchers focused on the application of piezoelectrics to the blades of rotary wing aircraft to improve their performance and effectiveness. Steadman et al. [4] showed an application of a piezoceramic actuator for camber control in helicopter blades. Giurgiutiu et al. [5] has researched performance improvement on rotor blades using strain-induced actuation methods (using PZTs). Wind-tunnel experiments proved the control authority of the PZT actuators. Giurgiutiu [6] presented a comprehensive review on the application of smart-material actuation to counteract aerelastic and vibration effects in helicopters and fixed-wing aircraft.

In addition to the research in vibration control applications, static control of aerodynamic surfaces using piezoelectric materials started in the early 1990s for large aircraft. In static aerelastic control, researchers employed both the low-amplitude (mostly linear) and high-amplitude (nonlinear) excitation ranges of piezoelectric materials. Lazarus et al. [7] examined the feasibility of using...
representative box wing adaptive structures for static aeroelastic control. Pinkerton and Moses [8] discussed the feasibility of controlling the wing geometry employing a piezoelectric actuator known as the thin-layer composite-unimorph ferroelectric driver and sensor (THUNDER). Hysteresis nonlinearity was observed in the voltage-to-displacement relationship. Barrett and Stutts [9] proposed an actuator that uses a pair of piezoceramic sheets arranged in a push–pull assembly to turn a spindle of an aerodynamic control surface. Wind-tunnel testing showed the feasibility of the concept. Geissler et al. [10] adopted piezoelectric materials as the actuation element for a morphing leading edge (LE); however, in this case, the LE is an independent element able to rotate around an internal hinge. Munday and Jacob [11] developed a wing with conformal curvature driven by a THUNDER actuator internally mounted in a position to alter the upper surface shape of the airfoil, resulting in a variation of the effective curvature. Wang et al. [12] presented results from the Defense Advanced Research Projects Agency/U.S. Air Force Research Laboratory/NASA Smart Wing Phase 2 Program, which aimed to demonstrate high-rate actuation of hingeless control surfaces using several smart-material-based actuators, including piezoelectric materials. Grottemann et al. [13,14] presented the active trailing edge (TE) and active twist concepts applied to helicopter rotor blades. The paper presents the optimization of sizing and placement of piezocomposite patches so that the desired wing weight and pitching moment output is obtained.

The rapid development and the reduced cost of small electronics in the last decade led to the interest of using piezoelectric materials in small unmanned (and/or remotely piloted) fixed-wing, rotary, and ducted-fan aircraft. Eggleston et al. [15] presented the use of piezoceramic materials (THUNDER), shape-memory alloys (SMAs), and conventional servomotors in a small unmanned aircraft. Wind-tunnel testing showed the feasibility of the smart-material systems. Barrett et al. [16] employed piezoelectric elements along with elastic elements to magnify control deflections and forces. The so-called postbuckled-precompression (PBP) concept was employed as guide vanes in a small rotary aircraft. The PBP concept, in its earliest incarnation, was primarily intended to increase the coupling coefficient exhibited by piezoelectric transducer elements (Lesieutre and Davis [17]). Vos et al. [18,19] conducted research to improve the PBP concept for aerodynamic applications. Roll control authority was increased on a 1.4 m span unmanned air vehicle. Kim and Han [20] and Kim et al. [21] designed and fabricated a smart flapping wing by using a graphite/epoxy composite material and a Macro-Fiber Composite (MFC) actuator. A 20% increase in lift was achieved by changing the camber of the wing at different stages of flapping motion. Bilgen et al. [22,23] presented a unique application for piezocomposite actuators on a 0.76 m wingspan morphing wing air vehicle. In this application, two MFC patches were bonded to the wings of a small demonstration vehicle, and the camber of the wing was changed with voltage. Adequate roll control authority was demonstrated in the wind tunnel as well as in flight. All electronics, including MFC power electronics, were powered by an 11.1 V lithium-polymer battery, a common choice for remotely controlled aircraft. Perdices and Ciresa [24] implemented MFCs as actuators into an active composite wing. A scaled prototype wing was manufactured, and models were validated with static and preliminary dynamic tests of the prototype wing. Wickramasinghe et al. [25] presented the design and verification of a smart wing for an unmanned aerial vehicle. The proposed smart wing structure consists of a composite spar and ailerons that have bimorph active ribs consisting of MFC actuators. Butt et al. [26,27] and Bilgen et al. [28] presented a completely servoless, wind-tunnel and flight-tested, remotely piloted aircraft. The MFC actuators were used to create variable-camber continuous, piezocomposite wings instead of the traditional servomotor controlled, discrete control surfaces. This vehicle became the first fully solid-state piezoelectric controlled, non-tethered flight-tested fixed-wing aircraft.

The examples given above provide concrete proof of feasibility of piezoelectric materials in small aircraft. The research (to date) shows the capability of piezocomposite devices in terms of force, strain, and frequency range for a wide range of aerodynamic applications. Although most of these research identify the hysteresis nonlinearity (due to large input excitations), the modeling and compensation of hysteresis nonlinearity is neglected. On the other hand, the use of the Preisach model for predicting hysteresis in simple structures such as beams is common in the literature; however, there is a lack of analysis of hysteresis in complex devices (such as piezocomposite airfoils) in response to nonuniform aerodynamic loading and inertial loading. In the authors’ previous work, the structural and aerodynamic response of the two different piezocomposite airfoils revealed significant hysteresis nonlinearity. In this paper, the classical (scalar) Preisach model is used to predict hysteresis observed in several MFC actuated piezocomposite bimorph devices: 1) two cantilevered beams introduced in Bilgen [29], 2) a simply supported thin airfoil proposed in Bilgen et al. [30], and 3) a cascading bimorph thick airfoil proposed in Bilgen et al. [31]. The simply supported thin airfoil has been employed in an open-loop fashion in the fully piezocomposite aircraft discussed earlier (see [26–28]). Similarly, the cascading bimorph thick airfoil has been used in the wind-tunnel tests of a ducted-fan aircraft, and in an open-loop fashion (see Ko et al. [32] and Ohanian et al. [33] for details of the aircraft). The previous research dealt solely with identifying the static and peak-to-peak structural and aerodynamic characteristics; therefore, the modeling of hysteresis was not addressed. During wind-tunnel tests, the past input history of the piezoceramic materials were intentionally wiped off at each actuation step to reduce/eliminate the hysteresis memory buildup. This method allowed the tests to be repeatable and focused the tests to the fundamental peak-to-peak characteristics. In the current paper, the Preisach model prediction accuracy is tested by building up as much memory as possible into the piezocomposite airfoils.

This paper contributes to the research field by examining the effectiveness of the classical Preisach model in the presence of fluid and mass loading for operational, wind-tunnel, and flight-tested variable-camber piezocomposite airfoils. The paper first discusses the hysteresis phenomenon that is typically observed in smart-material systems. Next, the classical Preisach model is introduced and the numerical implementation is shown. Next, the experimental setup is presented and the hysteresis response prediction of the cases listed above is shown. The paper concludes with a discussion of the results.

**Classical Preisach Model**

The Preisach model was introduced in a landmark publication by F. Z. Preisach [34] in the German academic journal *Zeitschrift für Physik*. Since then, it has become a widely accepted model of hysteresis. The Preisach model is a completely intuitive (or phenomenological) approach for describing physical mechanisms of magnetization. The model was originally developed for ferromagnetic materials; however, it was later realized that physics of piezoceramic and SMA hysteresis is similar to that of ferromagnetics. Somewhat in parallel with the developments in the field of magnetism, the Preisach model was independently invented and then extensively studied and tested for adsorption hysteresis by Everett and Whitton [35] and their collaborators. Krasnosel’ski and Pokrovskii [36], two Russian mathematicians, separated the physical definition and represented the model in a purely mathematical form, which is a spectral decomposition of operators. Some early investigations of the Preisach model for ferroelectric compounds are by Sreepram et al. [32] and Hughes and Wen [38]. These researchers discussed both the unifying framework provided by the Preisach model for PZT, magnetic compounds, and SMAs and constructed inverse filters to compensate for hysteresis. Other investigations (e.g., Ge and Jouaneh [39,40] and Robert et al. [41]) focused on extensions of the theory for characterization and control design in systems employing piezoceramic transducers. Webb et al. [42] discussed an adaptive hysteresis model for model reference control with actuator hysteresis. Song et al. [43] presented the tracking control of a piezoceramic unimorph beam using the inverse Preisach model. Choi et al. [44] presented a fast Preisach modeling method for SMA actuators using major hysteresis loops. Viswamurthy and
Ganguli [45] discussed the modeling and compensation of piezoceramic actuator hysteresis for helicopter vibration control. Mayergoyz's book [2] is one of the most cited sources for the Preisach model; therefore, this paper follows a similar description and representation of hysteresis nonlinearity by the Preisach model. The mathematical treatment of hysteresis is completely phenomenological with the Preisach model, meaning that it provides no insight to the physical cause of hysteresis. There are fundamental models of hysteresis that attempt to explain experimental facts from first principles; however, these models are not employed in this paper. The mathematical description is as follows. Consider an infinite set of simple hysteresis operators, as shown in Fig. 1. The input is \( u \); the output is \( \gamma_{\alpha \beta} u \), where \( \alpha \) is the switching value for the increasing inputs; and \( \beta \) is the switching value for decreasing inputs. The constraint \( \alpha \geq \beta \) is required.

The relay operator represents the hysteresis with local memory. When the input is monotonically increasing from \(-\infty\), the output starts at a value of \(-1\). The output switches at \( \alpha \) and stays at \(+1\) until \(+\infty\). A similar response results due to a monotonically decreasing input, where the output switches from \(+1\) to \(-1\) at the input value of \( \beta \). Mathematically, the following relationship can be written:

\[
\frac{du}{dt} \geq 0 \quad \text{and} \quad u \geq \alpha, \quad \text{then} \quad \gamma_{\alpha \beta} u(t) = +1 \\
\frac{du}{dt} < 0 \quad \text{and} \quad u \leq \beta, \quad \text{then} \quad \gamma_{\alpha \beta} u(t) = -1
\]  

Now, consider the arbitrary weight function \( \mu(\alpha, \beta) \), which is referred to as the Preisach function. The Preisach model is then defined as

\[
f(t) = \Gamma u(t) = \int_{\alpha \leq \beta} \mu(\alpha, \beta) \gamma_{\alpha \beta} u(t) \, d\alpha \, d\beta \tag{2}
\]

where \( \Gamma \) is the Preisach hysteresis operator. The model is analogous to a system of parallel connected two-position switches. The effect of each switch is first multiplied by a weight function, and then it is integrated over the domain. The Preisach hysteresis nonlinearity is constructed by superposition of the simple \( \gamma_{\alpha \beta} \) operator with local memories to represent the complex operator \( \Gamma \). Functional analysis and spectral decomposition are a few well-known examples of the superposition method. This complex operator usually has a nonlocal memory.

The Preisach model depends on a special (and well-established) diagram technique that constitutes the mathematical foundation (see Mayergoyz [2]). The graphical method allows the model to detect local input extrema, accumulate them, and choose the right branches of hysteresis nonlinearity. In summary, the geometric definition describes the mechanism of memory formation. The Preisach model, in general, describes hysteresis nonlinearities with nonlocal memories. To represent the hysteresis behavior of a system, the Preisach function \( \mu(\alpha, \beta) \) has to be identified from experimental measurements (which will be presented later). First, the properties of the Preisach model are examined.

There are three main properties of the Preisach model. Property 1 states that the output value of \( f^+ \) in the state of positive saturation is equal to the negative of \( f^- \) in the negative saturation. In positive saturation, where all switches are up, the input is more than \( a_0 \). In negative saturation, where all switches are down, the input is less than \( b_0 \). The saturation values remain constant for \( u(t) \geq a_0 \) and \( u(t) \leq b_0 \). Partly due to this reason, the Preisach model does not describe the reversible components of hysteresis nonlinearities.

Property 2, the wiping-out property, shows that not all past extremum values are accumulated into the memory. Some past extremum values can be wiped out by subsequent input variations. For example, consider the finite decreasing input sequence \( \{u_1, u_2, u_3, u_4, u_5\} \) of local input maxima and an increasing input sequence \( \{u_2, u_4, u_6, u_8\} \) of local input minima. Now, assume an increasing input to \( u(t_0) = u_0 \), which is above \( u_3 \). After time \( t_0 \), all past inputs with \( \alpha < u_0 \) are wiped out. The wiping-out property works in the same manner for a set of monotonically decreasing inputs. The analytical formulation of the wiping-out property is as follows. First, consider the input variation shown in Fig. 2 for the interval \( t_0 \leq t \leq t' \). The pairs of input maxima \( (M_k) \) and minima \( (m_k) \) are calculated by

\[
M_k = \max u(t) \quad \text{in} \quad [t_{k-1}, t'] \quad u(t_k) = M_k \tag{3}
\]

\[
m_k = \min u(t) \quad \text{in} \quad [t_{k-1}, t'] \quad u(t_k) = m_k \tag{4}
\]

where \( t_{k-1} \) is the time for previous input minima, and \( t_k \) is the time for input maxima. The index \( k \) is equal to \( k = 1, 2, \ldots, n \). Only the alternating series of dominant input maxima \( (M_k) \) and minima \( (m_k) \) are accumulated by the Preisach model. All intermediate input extrema are erased. Note that the alternating series of dominant extrema is modified with time.

Property 3, the congruency property, is another property of the Preisach model that is valid for periodic input variations. Let \( u_1(t) \) and \( u_2(t) \) be two inputs that may have different past histories. Starting from \( t_0 \), the two inputs are subjected to the same two consecutive extremum values, \( u_+ \) and \( u_- \). These inputs result in minor hysteresis loops. This input sequence results in the periodic variations of outputs \( f_1(t) \) and \( f_2(t) \). The positions of these two loops are different in the (vertical) output \( f \) axis. The difference is due to the past histories of the two inputs. These two curves are congruent; therefore, a translation of the two curves in the \( f \) axis will cause the two curves to coincide.

**Identification of the Model**

This section briefly presents the identification of the Preisach weight function \( \mu(\alpha, \beta) \) for the classical Preisach model. The set of experimental first-order transition curves (that are experimentally obtained) will be used for the identification. These curves can be defined in two ways. The increasing—decreasing first-order transition curve can be obtained by the following method. First, the input is decreased below \( \beta_0 \) which brings all \( \gamma \) operators to a state of negative saturation. Next, the input is monotonically increased until it reaches some value \( \alpha' \). Starting from the negative saturation state, the ascending curve of the major loop is followed. This is called the limiting ascending branch. The output is \( f_+ \), where the input is \( u(t) = \alpha' \). The first-order transition curve is formed by the
monotonic increase (from \( \beta_0 \)) followed by a monotonic decrease. The curve is formed by the first reversal of input. The notation \( f_{\alpha_{0}\beta_{0}} \) is used for the output of the transition curve that comes after the limiting ascending branch (that is terminated at \( f_{\alpha_{0}} \)). The input is equal to \( \beta' \) at \( f_{\alpha_{0}'} \). The corresponding \( \alpha' - \beta' \) diagram is illustrated Fig. 3a, and the \( f' - u \) plot is presented in Fig. 3c.

A similar procedure can be followed to produce the decreasing–increasing first-order transition curves. Each of these curves are formed by a decrease along the limiting decreasing branch (starting at \( \alpha_{0} \)) followed by an increase in input. The notation \( f_{\alpha'_{0}\beta'_{0}} \) will be used for the value for the descending curve along the limiting decreasing branch. This value is achieved when the input is decreased from some value above \( \alpha_{0} \) to \( u = \beta' \). The notation \( f_{\alpha'_{0}\beta'_{0}} \) indicates the output of the first-order increasing transition curve that starts from the limiting decreasing branch at point \( f_{\alpha'_{0}} \) for input \( u = \alpha'' \). Figure 3b presents the \( \alpha' - \beta' \) diagram, and Fig. 3c presents the \( f' - u \) plot for the decreasing–increasing first-order transition curve.

Figure 4 shows the two types of experimental first-order transition curves inside the corresponding major (limiting) loop. These two types are described as 1) increasing–decreasing (\( \beta_0 - \alpha' - \beta' \)) and 2) decreasing–increasing (\( \alpha_0 - \beta'' - \alpha'' \)) first-order transition curves. Theoretically, the first-order or the higher-order transition curves can be used to calculate the weight functions \( \mu(\alpha, \beta) \) because all of them will lead to the same result. From a practical point of view, the first-order transition curves are preferred because 1) it is easier to obtain these curves experimentally, and 2) the measurements of these curves start from the negative or positive saturation states, which are well defined.

Limitations and Assumptions

The classical Preisach model has some intrinsic limitations. The most important of them are as follows:

1) The model describes hysteresis nonlinearities that exhibit congruency of minor loops formed for the same reversal values of input.

2) The model is rate-independent in nature, and it does not account for dynamic properties of hysteresis nonlinearity.

3) The model describes hysteresis nonlinearities with wiping-out property. Only one dominant cycle is required to wipe out the previous cycle.

4) A scalar output exhibits hysteresis variation with respect to only one scalar input.

5) The classical Preisach model deals only with scalar hysteresis nonlinearities.

From the summary above, certain restrictions and assumptions have to be made in order to apply the classical Preisach model to the piezocomposite samples driven in the quasi-static frequency range. As stated above, the classical Preisach model deals with scalar outputs in response to one scalar input. However, it is known that the strain output of a piezoelectric material is a function of (at least) two variables: electric field and mechanical stress. To help the discussion, the linear electromechanical constitutive relationship is given here as an example. From the IEEE Standard [46], the linear constitutive relationship (\( S_{ij} = s_{ijkl}T_{kl} + d_{ij}E_{k} \)) demonstrates that the mechanical strain \( S \) is a function of the mechanical stress \( T \) and the applied electric field \( E \). In reality, the coefficients of stress and strain are nonlinear due to hysteresis and this nonlinearity cannot be neglected for high-amplitude excitation with varying extrema. However, the linear constitutive relationship is still useful to aid the discussion here. When applying the scalar Preisach model, it is assumed that the mechanical stress, for the experimental cases considered here, is either negligible or it is proportional to the electric field. From the linear constitutive analogy, the piezoelectric constant \( d \) assumes a new effective value; therefore, the strain is assumed to be a function of only one variable. In addition, the external aerodynamic loads for the airfoil samples are neglected in the scalar model. One of the main goals of this paper is to determine if the aerodynamic loading has a measurable effect on the prediction accuracy of the scalar Preisach model within the operation velocity range of 5–19 m/s. It should be noted that the aerodynamic loading is expected to be dominant at
high flow velocities (and high dynamic pressures) and result in the overprediction of the response by the scalar Preisach model. As the mechanical and aerodynamic loads increase, the prediction error can be reduced by the use of the vector Preisach model; however, this is beyond the scope of this paper.

**Numerical Implementation of the Preisach Model**

The numerical implementation is relatively simple because the 2-D integrals can be avoided. First, a monotonically decreasing input will be discussed. Figure 5 demonstrates the \( \alpha - \beta \) diagram for a monotonically decreasing input where the final link of the staircase interface \( L(t) \) is vertical. As noted earlier, \( M_l \) and \( m_l \) form the series of dominant input extrema. Using Eq. (2), the response to a monotonically decreasing input can be written as [2]

\[
f(t) = \int_{-\infty}^{t} f(M_m, m_m - t) \, dM_m + f(M_l, u(t) - t) \tag{5}
\]

which is in terms of the increasing–decreasing first-order transition curves. In the expression, \( f^- \) is the minimum saturation of the output. Equation (5) can also be written in terms of the decreasing–increasing first-order transition curves. For a monotonically decreasing input, the response becomes

\[
f(t) = \int_{-\infty}^{0} f(M_m, m_m - t) \, dM_m + f(M_l, u(t) - t) \tag{6}
\]

where \( f^+ \) is the maximum saturation output value.

If the input \( u(t) \) is monotonically increased, the final link of the staircase interface \( L(t) \) is horizontal, as shown in Fig. 6. This case is realized when \( m_l(t) = M_l(t) = u(t) \). Using Eq. (2), the response to a monotonically increasing input can be written as [2]

\[
f(t) = \int_{0}^{\infty} f(M_m, m_m + t) \, dM_m + f(M_l, u(t) - t) \tag{7}
\]

which is in terms of the increasing–decreasing first-order transition curves. Similarly, the response to monotonically increasing input (in terms of the decreasing–increasing first-order transition curves) has the following form:

\[
f(t) = f^+ + \sum_{k=1}^{a(t)-1} (f(m_k, m_k - t) + f(u(t), m_{k-1}) - f(u(t), m_{k-1}) \tag{8}
\]

Equations (5–8) are the explicit expressions of the output \( f(t) \) in terms of the experimentally measured data. These expressions are used in the following experimental predictions.

It is important to note that the representation theorem reveals the limits of the Preisach model. The theorem states: “The wiping-out property and the congruency property constitute the necessary and sufficient conditions for a hysteresis nonlinearity to be represented by the Preisach model on the set of piecewise monotonic inputs.” The theorem allows for the explanation of the statistical instability of the Preisach model. The weight function can always be determined if the integrals of \( u(\alpha, \beta) \) over the triangles \( T(\alpha, \beta) \) are experimentally found. These integrals can be found from the first-order transition curves and from the higher-order transition curves. As the theory requires, if these transition curves are congruent and the wiping-out property holds, the integrals will have the same value, regardless of the order of the transition curves used. The statistical instability arises when the actual hysteretic transducer does not exactly satisfy the wiping-out and congruency properties. In this case, the Preisach model becomes an approximation of the actual hysteresis nonlinearity.

**Application to Piezocomposite Bimorphs**

The classical Preisach model is applied to 1) two cantilevered bimorph beams, 2) a simply supported thin airfoil, and 3) a cascading bimorph thick airfoil. All of these samples are in the bimorph electromechanical configuration, and they employ MFCs as the active element. The MFC was developed at NASA Langley Research Center (see Wilkie et al. [47]). The MFC is a layered, planar actuation device that employs rectangular cross-sectional unidirectional piezoceramic fibers (PZT-5A). The in-plane poling and subsequent voltage actuation allow the MFC to use the 33 piezoelectric effect, which is much stronger than the 31 effect used by traditional PZT actuators with through-the-thickness poling (Hagood et al. [48]). Williams [49] provides a detailed nonlinear characterization of the mechanical and piezoelectric behavior of the MFC actuator. Moses et al. [50] used both active fiber composites (AFCs; an earlier version of MFC) and MFCs to actively reduce vibration levels in the tail fins of a wind-tunnel model of a fighter jet subjected to buffet loads.

A MATLAB [51] code is developed to predict the output of the bimorph samples listed above. First, the \( \omega_u \) are computed using the experimentally measured first-order transition curves. Next, Eqs. (5) and (7) are used (along with increasing–decreasing first-order transition curves) or Eqs. (6) and (8) are used (along with decreasing–increasing first-order transition curves) to predict the output to a piecewise monotonically increasing and decreasing input \( u(t) \).

**Experimental Setup**

The hysteresis experiments for the two airfoil specimens are conducted in a low-speed open-circuit wind tunnel with a 2-D test-section configuration (Fig. 2). The two bimorph beams are tested outside of the test-section in a cantilevered configuration. The acrylic test section is 13.6 cm tall in the spanwise direction and 35.6 cm wide in the out-of-plane direction. A converging nozzle is used to condition the flow before entering the 2-D test section. Two fiberglass screens and an aluminum honeycomb flow straightener with \( \frac{1}{2} \) in. hexagonal cells are employed at the entrance of the inlet nozzle. Preliminary tests are conducted to characterize the empty test-section flow profile. Flow velocity is observed using a pitot-static tube. The differential pressure from the pitot-static probe is measured with a Setra 267 pressure transducer. The velocity profile is measured along the horizontal and vertical central axes. The pitot-static probe is located 16.0 cm upstream of the airfoil.

An approximately 4.5% maximum flow speed variation from the mean velocity is measured; therefore, the flow is assumed spatially uniform. The streamwise turbulence of the flow in the empty test section is measured by standard hot-wire anemometry technique.
using a TSI IFA-100 anemometer. An average 0.14% turbulence intensity is measured from the 0.1 Hz high-pass filtered hot-wire signal for the average flow velocity of 5 m/s. In addition, an average 0.33% turbulence intensity is measured for the average flow velocity of 19 m/s. The turbulence intensity monotonically increases as the flow velocity is increased. Note that the two airfoils are tested in the wind tunnel at the following velocity settings: 0, 5, 10, 15, and 19 m/s. The flow speed is used as a parameter to understand the effect of nonuniform aerodynamic loading on the accuracy of the classical Preisach model.

In this paper, the independent variable is the excitation voltage to a voltage divider circuit (discussed later). A Trek 623B high-voltage amplifier is used to supply voltage to the circuit, which then gets divided and sent to the two opposing sides of the bimorph. The single-axis out-of-plane displacement of the samples is considered as the dependent variable. An MTI Instruments LTC-200-100-SA laser displacement sensor is employed to measure the displacement of the specimens. A National Instruments (NI) cDAQ data acquisition system with an NI 9215 input module and an NI 9263 output module is used to control the multiparameter experiment. A triangle wave with variable amplitude and constant frequency is used to actuate the samples for both first-order transition curve measurements and the evaluation measurements. All of the samples are evaluated at the following frequencies: 0.5, 1, and 5 Hz. The input frequency is used as a parameter to understand the effect of inertial (mass) loading on the accuracy of the classical Preisach model.

Two electrical configurations are considered: 1) asymmetric excitation and 2) symmetric excitation. In the asymmetric excitation, a novel voltage divider circuit is employed (see Bilgen et al. [52]). The circuit is constructed using a pair of 1 and 3 MΩ resistors and diodes, and it results in a 3:1 division of the input voltage from the Trek amplifier. For example, if 2 kV is applied to the circuit, the front MFC receives 1.5 kV, which induces a bending motion away from the laser; hence resulting in a positive displacement. At the same time, the rear MFC receives −500 V and induces a bending motion away from the laser, hence resulting in an asymmetric bimorph actuation. The same 3:1 division is also applied for a −2 kV excitation. In the asymmetric configuration, the maximum voltages applied are −2 and +2 kV, resulting in peak-to-peak actuation of both MFCs. On the other hand, the symmetric electrical configuration (1:1) does not use the voltage divider. The opposing MFCs are connected to the amplifier in opposite polarities, therefore resulting in an electrically symmetric bimorph actuation. In the symmetric configuration, the maximum voltage is limited to −400 V and +400 V, since −500 V is the minimum input voltage to the MFC recommended by the manufacturer.

There are several important observations on the samples and the methodology employed in this paper. First, the excitation of the bimorphs should be noted. In the symmetric, 1:1 electrical configuration, a same-amplitude and opposite-polarity electric field is applied to the opposing active material on the bimorph. In this configuration, the neutral axis is assumed to coincide with the geometric midline of the bimorph. In the asymmetric 3:1 electrical configuration, one MFC receives a maximum of +1.5 kV where the opposing MFC receives −500 V. In this configuration, the neutral axis of the bimorph is no longer aligned with the geometric midline. Second, the structure of the bimorphs should be noted. For the cantilevered bimorph beam sample B1, discussed later in detail, there is no substrate material and the two Kapton layers, which are part of the packaging of the MFC actuator, are assumed to have negligible bending stiffness. The cantilevered bimorph beam sample B2 and the two piezocomposite airfoils have substrate materials. More specifically, both of the airfoils experience constraints due to the boundary conditions and the internal configuration. Possible errors arising from these observations will be discussed in the following sections.

Identification Method

For the measurement of the increasing–decreasing first-order transition curves, the excitation voltage is started at −2 kV for the 3:1 configuration and −400 V for the 1:1 configuration. A triangle wave at 0.5 Hz is used to bring the voltage to the desired $\alpha'$ value back to $\beta_0$ for the corresponding minor loop. First, the major loop is obtained, which is limited by $-2$ and $+2$ kV for the 3:1 configuration, and $-400$ and $+400$ V for the 1:1 configuration. The $\alpha'$ value is swept with decreasing amplitude from $\alpha_0$ to $\beta_0$ in 40 increments, resulting in 100 and 20 V $\alpha'$ resolution for the 3:1 and 1:1 configurations, respectively. It is important to note that a saturation loop, between $\alpha_0$ and $\beta_0$, is applied before the acquisition of each $\alpha'$ minor loop. The measurement of the decreasing–increasing first-order transition curves is similar to the discussion above. The major loop acquisition is started at +2 kV (for 3:1) or +400 V (for 1:1), which are the maximum saturation input levels. The minor loops and the corresponding $\beta''$ values are swept from $\beta_0$ to $\alpha_0$ with decreasing amplitude, also in 40 increments. For all transition curves, the displacement is recorded for 10 complete cycles, and these cycles are averaged.

Cascading Bimorph Thick Airfoil

The hysteresis analysis methodology for the four samples is the same; therefore, only the analysis of the cascading bimorph thick airfoil is presented in detail. This airfoil has the highest electrical and structural complexity when compared with the other samples. The specific thick airfoil sample employed is a variable-camber airfoil with two active surfaces (cascading bimorphs made with four MFC M8557P1 actuators each) and a single four-bar (box) mechanism as the internal structure. The unique choice of boundary conditions allows variable and smooth deformation in both directions from a flat camber line. The airfoil has 127 mm chord, 133 mm span, and 15 mm thickness. The two bimorphs used in the airfoil are fabricated by sandwiching a 0.027-mm-thick stainless-steel material with the MFC actuators. Wind-tunnel experiments were conducted previously to reveal static aerodynamic and structural performance results at a flow rate of 15 m/s and Reynolds number of 127,000. In summary, a 10.7° angle-of-attack (AOA) change and 7.59% camber change is observed for the peak-to-peak actuation range. The standard geometric definition of AOA is used where the angle between the streamwise axis and the chord line is taken as AOA. This angle is measured during tests using a single-point laser displacement sensor and a digital image of the cross section of the airfoil. The 2-D lift coefficient is measured as −0.677 at −1500 V and 0.865 at +1500 V static actuation. A change of 1.54 in lift coefficient is calculated for the peak-to-peak voltage input. The variable-camber airfoil produces a maximum 2-D lift-to-drag ($L/D$) ratio of 13.4 at 1500 V (AOA = +5.78°) and an $L/D$ ratio of −11.2 at −1500 V (AOA = −5.20°). Refer to Bilgen et al. [31] for the fundamental static aerodynamic characterization of this piezocomposite airfoil.

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In this paper, the hysteresis between the voltage input and displacement output is measured in the wind tunnel. The laser displacement measurement is taken at a single point, which is 102 mm from the LE (in the chordwise direction) and on the midspan line. Figure 8 shows the experimental setup. In Fig. 8a, the 2-D wind-tunnel test configuration is illustrated. The figure shows the test-section walls, the cascading bimorph airfoil, and the mounting rod. The mounting rod of the airfoil is 25.4 mm from the LE. In Fig. 8b, a picture of the acrylic test section with the cascading bimorph airfoil installed is presented. The voltage divider circuit is also shown in the picture. First, the first-order transition curves are measured to construct the Preisach model. Figure 9 presents the increasing–decreasing (βα′ − βα″) and decreasing–increasing (αα″ − βα′) first-order transition curves of the airfoil at a flow velocity of 10 m/s. Theoretically, these two types of first-order curves are symmetric, meaning that the 180° rotation of one curve should result in a matching pair between the two types. In reality, the curves are not completely symmetric due to the complex nonsymmetrical structure of the airfoil.

The curves correspond to the 3:1 electrical configuration. As noted earlier, a total of 40 curves are measured; however, most of these curves are removed for the plots in order to aid presentation. After the measurement of these curves, the Preisach model can be constructed. First, the wiping-out property is examined by exciting the airfoil using two different input signals (with different time history). The displacement is measured and compared with the prediction of the Preisach model. These tests are simultaneously used to 1) check that the samples behave as the Preisach model requires and 2) check the proper implementation of the model by the MATLAB code. Figure 10 presents the response of the cascading bimorph airfoil to a wiping-out experiment in the 3:1 electrical configuration at the flow velocity of 10 m/s. The inputs are labeled Inp. 1 and Inp. 2, and the outputs are labeled Exp. 1 and Exp. 2 (for the experiment) and Mdl. 1 and Mdl. 2 (for the Preisach model). Note that the two input signals, shown in

![Fig. 8 Experimental setup for hysteresis tests of the cascading bimorph thick airfoil: a) illustration of the boundary conditions, and b) airfoil in the wind-tunnel test section.](image)

![Fig. 9 First-order transition curves for the cascading bimorph thick airfoil at 10 m/s flow velocity in 3:1 configuration: a) increasing–decreasing, and b) decreasing–increasing.](image)

![Fig. 10 Wiping-out evaluation of the cascading bimorph thick airfoil at 10 m/s flow velocity in 3:1 configuration: a) voltage input (Inp.), and b) displacement (Disp.) output time histories.](image)
Inp. 2 have identical time histories in the [0, 1] s time frame and in the [4, 5] s time frame. Figure 10b shows the matching experimental final descent in the [4, 5] s time frame, confirming that the previous input history is wiped out. The Preisach model also predicts the wiping out very well. The set of increasing–decreasing first-order transition curves are used to make the prediction shown in Fig. 10b. The same test is also conducted starting at the positive saturation point. In this case, the decreasing–increasing first-order transition curves are employed. Figure 11 shows the wiping-out evaluation of the cascading bimorph airfoil and the Preisach model prediction with the decreasing–increasing transition curves. Similar to the increasing–decreasing curves, the decreasing–increasing curves result in a very good prediction of the wiping-out property. The cascading bimorph airfoil successfully passed the wiping-out property evaluation for the 3:1 electrical configurations. The tests are repeated at 1 and 5 Hz in order to determine the inertial effects in addition to the tests presented for 0.5 Hz.

The second property of the Preisach model is the congruency property. Another set of input time histories are generated and applied to the samples to check for this property. First, the 3:1 electrical configuration is evaluated at the flow velocity of 10 m/s. The increasing–decreasing transition curves are used in the model prediction. Figure 12 shows the corresponding congruency test results for the cascading bimorph airfoil. In the congruency tests, the two input time histories are identical in the [0, 1] s time frame and in the [3, 7] s time frame. As a result, the congruency of the two outputs is expected in the [4, 7] s time frame. Both the experiment and the model demonstrate the congruency of the two outputs in the [4, 7] s time frame. The output has the same form; however, it is at a different mean value due to the previous time history in the [1, 4] s time frame. Figure 13 shows the response to a similar congruency test and the model prediction using the decreasing–increasing first-order transition curves. As observed before, the decreasing–increasing curves also predict the congruency property well. The analysis above demonstrates that the cascading bimorph thick airfoil complies with both the wiping-out and congruency properties of the Preisach model.

The evaluation of the accuracy of the Preisach model is conducted by a decaying input time history that oscillates between consecutive decaying extrema. This type of signal accumulates more and more memory as time passes; therefore, the capability of the model is properly examined. Figure 14 shows the response of the cascading bimorph airfoil and the model prediction for the 3:1 electrical configuration. Two types of input signals (and the corresponding response) are presented, which employ both the increasing–decreasing and the decreasing–increasing first-order transition curves. The voltage input and the displacement output prediction using the decreasing–increasing first-order transition curves are labeled as Inp. 1 and Mdl. 1, respectively (with the corresponding experimental response labeled as Exp. 1). Similarly, the voltage input and the displacement output predictions using the increasing–decreasing first-order transition curves are labeled as Inp. 2 and Mdl. 2, respectively (with the corresponding experimental response labeled as Exp. 2). The maximum error (in the presented time frame) between the Preisach model and the experiment is 9.6 and 6.0% for the Inp. 1 and Inp. 2 voltage time histories, respectively. The error is given in terms of the peak-to-peak displacement. In contrast, an experimentally calibrated linear model (neglecting hysteresis non-linearity) would result in a 19% maximum error for the presented voltage time histories. In Fig. 14b, the model shows a small and constant offset from the experimental predictions. This offset is caused by the initial displacement of the sample at the saturation input voltage ($\alpha_0$ or $\beta_0$). The saturation deflection measured with the first-order transition curves is usually different than the saturation deflections observed in the decaying triangle input tests. The main reason for this is that the first-order transition curves are averaged from approximately 10 loops for every minor loop and the major loop; therefore, they represent a time-averaged value where
time-dependent effects are removed (e.g., creep). The second, less dominant reason is the very small inertial effects due to the use of a 0.5 Hz triangle input signal. The first-order transition curves can be obtained at much lower rates to reduce the dynamic effects; however, the creep effects will be increased.

The results presented above for a 0.5 Hz input frequency and 10 m/s flow velocity are given as an example of the analysis methodology. The tests above are conducted for flow speeds of 0.5, 10, 15, and 19 m/s, and for input frequencies of 0.5, 1, and 5 Hz. The model prediction error as a function of flow velocity and input voltage frequency is presented in Fig. 15. The labels (1) and (2) indicate that the decreasing–increasing and increasing–decreasing first-order transition curves are used, respectively. We immediately recognize a dominant effect of frequency (hence the inertial loading) on the accuracy of the classical Preisach model. Since the cascading bimorph airfoil has a significant amount of mass, the effect of inertia is apparent even at 1 Hz. In contrast, the external aerodynamic loading does not have a significant effect on the model prediction. In summary, the scalar Preisach model is capable of predicting the hysteresis observed in the cascading bimorph thick airfoil sample with an average 6.8% error for excitation at 0.5 Hz and with an average 9.3% error for excitation at 1 Hz. In contrast, an experimentally calibrated linear model results in an average 19.1% error for excitation at 0.5 Hz and an average 20.2% error for excitation at 1 Hz. These results are important to note because the scalar Preisach model is capable of making a reasonable prediction for three coupled hysteresis phenomena arising from each cascading bimorphs and the compliant-box mechanism. The increasing–decreasing and decreasing–increasing first-order transition curves are found to be successful in developing the $f_{u_{\beta_0}}$ and $f_{p_{\alpha_0}}$ databases for the Preisach model, respectively.

**Simply Supported Thin Airfoil**

The hysteresis analysis is extended to the simply supported thin airfoil. This airfoil is a single variable-camber thin bimorph with four MFC M8557-P1 (see footnote †) actuators of 127 mm chord and 133 mm span. Four stainless-steel pins (two on each end) are bonded to the airfoil at 5 and 50% chord (6.35 and 63.5 mm, respectively) from the LE. A parametric study of fluid–structure interaction is employed previously to find the pin locations along the chordwise direction that result in high lift generation. The bimorph is fabricated by sandwiching a 0.027-mm-thick stainless-steel sheet and bonding the laminate under vacuum. The MFCs are aligned at the LE in the chordwise direction. Two layers of 0.027-mm-thick stainless-steel metal (passive material) are bonded to the ends on the MFC actuators to complete the total chord to 127 mm. Wind-tunnel experiments are previously conducted to reveal static aerodynamic and structural characteristics at a flow rate of 15 m/s and Reynolds number of 127,000. The 2-D lift coefficient is measured as 0.657 at +1400 V and −0.807 at the −1400 V static excitation. A lift coefficient change of 1.46 is observed purely due to voltage excitation. A maximum 2-D $L/D$ ratio of 17.8 is recorded through voltage

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![Fig. 13](image-url)  
**Fig. 13** Congruency evaluation of the cascading bimorph thick airfoil at 10 m/s flow velocity in 3:1 configuration: a) voltage input, and b) displacement output time histories.

![Fig. 14](image-url)  
**Fig. 14** Decaying triangle input signal response of the cascading bimorph thick airfoil at 10 m/s flow velocity in 3:1 configuration: a) voltage input, and b) displacement output time histories.

![Fig. 15](image-url)  
**Fig. 15** Model prediction error as a percentage of peak-to-peak displacement output of the cascading bimorph thick airfoil in 3:1 configuration.
excitation. The variable-camber airfoil produces an $L/D$ ratio of $-17.3$ at $-1225$ V (AOA $= -4.52^\circ$) and an $L/D$ ratio of $17.8$ at $+1400$ V (AOA $= +3.48^\circ$). Refer to Bilgen et al. [30] for more details on the airfoil design and static aerodynamic characterization.

In this paper, the hysteresis between the voltage input and displacement output is measured in the wind tunnel. The laser displacement measurement is taken at a single point that is $111$ mm from the LE. Figure 16 shows the experimental setup. In Fig. 16a, the 2-D wind-tunnel test configuration is illustrated. The figure shows the test-section walls, the thin simply supported airfoil, and the four pinned boundary conditions. In Fig. 16b, a picture of the acrylic test section with the airfoil installed is presented. The hysteresis analysis is repeated for the simply supported thin airfoil as shown in the previous section for the cascading bimorph airfoil. First, the first-order transition curves are measured. Next, the wiping-out and congruency properties are checked. Finally, a decaying triangle waveform is measured. Next, the wiping-out and congruency properties are checked. The model prediction error as a function of flow velocity and input voltage frequency is presented in Fig. 17. The labels (1) and (2) indicate that the decreasing–increasing and increasing–decreasing first-order transition curves are used, respectively.

As noted earlier, the dominant effect on the accuracy of the classical Preisach model is excitation frequency and, more precisely, the inertial loading. Compared with the cascading bimorph airfoil, the simply supported airfoil weighs less than half; therefore, the inertial loading is smaller and the error in model prediction is less. Similar to the cascading bimorph airfoil, the external aerodynamic loading does not have a conclusive effect on the model prediction for the simply supported thin airfoil. In summary, the Preisach model is capable of predicting the hysteresis observed in the simply supported thin airfoil sample with an average $6.3\%$ error for excitation at $0.5$ Hz and with an average $5.8\%$ error for excitation at $1$ Hz. In contrast, an airfoil sample with an average $17.5\%$ error for excitation at $1$ Hz. The increasing–decreasing and decreasing–increasing first-order transition curves are found to be successful in developing the $f_{\ud{\phi}_0}$ and $f_{\ud{\phi}}$ databases for the Preisach model, respectively.

Cantilevered Bimorph Beams

The experiments presented above for the two airfoils revealed the effects of aerodynamic and inertial loading on the accuracy of the scalar Preisach model. From the beginning, the model neglected the presence of these two effects in addition to the hysteresis caused by the internal and external boundary conditions (constraints) of the airfoils. Now, the question becomes: How accurate is the classical Preisach model if the internal and external loadings and constraints are minimized so that the scalar input–output assumption is mostly satisfied? To answer this question, two cantilevered bimorph beams are evaluated outside of the wind tunnel (hence, without aerodynamic loading). The clamped-free boundary condition is chosen to minimize additional external hysteresis. The bimorph beam samples are labeled B1 and B2. Sample B1 employs two MFC M8507-P1 actuators (see footnote ‡), however, it has no substrate. Sample B2 is bimorph actuator with two MFC M8507-P1-type actuators and with a $0.027$-mm-thick stainless-steel substrate. Refer to Bilgen [29] for further details of these two samples. Figure 18 shows the important components of the benchtop experimental setup. The free length of the bimorph B1 is set to $85.90$ mm, and the laser measurement location is $80.60$ mm from the clamped base. The free length of the bimorph B2 is $86.65$ mm, and the laser measurement location is $80.88$ mm from the clamped base.

Similar to the analysis of the airfoils in the previous sections, several wiping-out, congruency, and decaying triangle input experiments are conducted for the beams. In addition to the high-amplitude 3:1 electrical configuration, the low amplitude 1:1 electrical configuration is also evaluated. The wiping-out property is observed both in the experiment and in the model. For the 1:1 configuration, a constant displacement offset between the actual displacement and the model prediction is observed. This is expected in the response when a low amplitude input variation (e.g., $-400$ to $+400$ V peak-to-peak excitation in the 1:1 configuration) is measured after an unknown high-amplitude past input history (e.g., $-2$ to $+2$ kV peak-to-peak excitation in the 3:1 configuration). In the 1:1 tests, the Preisach model is unaware of the previous high-amplitude tests that resulted in dominant extrema in the memory of the piezoceramic material. In summary, both bimorph samples passed the wiping-out property examination. The congruency property is also checked. The bimorph samples show excellent congruency in the 3:1 electrical configuration. The property is also demonstrated very well for the 1:1 electrical configuration. In summary, both bimorph beam samples comply with the two properties of the Preisach model.

Finally, the evaluation of the model prediction for bimorph beam samples is conducted by a decaying input time history that oscillates between consecutive decaying extrema. The model prediction error as a function of electrical configuration and input voltage frequency.
is presented in Table 1. The model prediction from both increasing-decreasing and decreasing-increasing first-order transition curves are presented. First, the effect of electrical configuration is noted where the low-amplitude 1:1 configuration results in higher prediction error when compared with the high-amplitude 3:1 case. There are several reasons for this: an important reason being that the experimental measurement accuracy is reduced (due to the finite resolution of the laser sensor and the analog-to-digital and digital-to-analog converter in the NI data acquisition system). As noted earlier, the $f_{\alpha \beta_{h}}$ and $f_{\alpha \beta_{a}}$ databases for the Preisach model are constructed using the experimentally obtained first-order transition curves. Second, the effect of substrate material is observed between the two samples. Bimorph beam B1 has no substrate and, almost consistently, the model prediction results in a lower error when compared with the bimorph beam B2 (which has a stainless-steel substrate). This is partly because the scalar model neglects the additional stress-dependent mechanical behavior of the substrate material and the two adjacent glue layers. Compared with sample B1, the additional glue layer and the substrate create a significant separation of the MFC actuators from the geometric midplane, which increases the area moment of inertia of the beam cross section.

The analysis of the cantilevered beams reveals another nonlinear phenomenon. As observed before, the accuracy of the classical Preisach model is dependent on the inertial loading. When we compare the response of both beams (in high-amplitude 3:1 excitation) at 0.5 and 1 Hz, the effect of piezoelectric creep is noticeable because the inertial effects are low for the beam samples (when compared with the airfoils). The increase of creep as the frequency is reduced explains the higher prediction error for 0.5 Hz when compared with the 1 Hz response. Note that piezoelectric creep is also amplitude dependent; therefore, low-amplitude 1:1 electrical configuration does not effectively reveal its presence. In summary, the model is capable of predicting the hysteresis observed in beam sample B2 (with stainless-steel substrate) with an average 7.3% error for excitation at 0.5 Hz and with an average 6.3% error for excitation at 1.0 Hz for the 3:1 electrical configuration. For the 1:1 electrical configuration, an average 10.4% error for excitation at 0.5 Hz and an average 10.0% error for excitation at 1.0 Hz is calculated for sample B2.

### Table 1 Model prediction error as a percentage of peak-to-peak displacement output for cantilevered bimorph beams

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Frequency, Hz</th>
<th>B1</th>
<th>B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3:1 ($V_{\text{max}} = +1.5 \text{ kV}, V_{\text{min}} = -0.5 \text{ kV}$)</td>
<td>0.5</td>
<td>5.5</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>4.8</td>
<td>6.2</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>10.6</td>
<td>11.0</td>
</tr>
<tr>
<td>1:1 ($V_{\text{max}} = +0.4 \text{ kV}, V_{\text{min}} = -0.4 \text{ kV}$)</td>
<td>0.5</td>
<td>7.2</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>10.4</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>22.5</td>
<td>26.7</td>
</tr>
</tbody>
</table>

### Conclusions

This paper presented the application of the classical Preisach model to predict hysteresis observed in MFC actuated devices: 1) cantilevered bimorph beams, 2) a simply supported thin airfoil, and 3) a cascading bimorph thick airfoil. The Preisach model is capable of predicting the hysteresis observed in all of the samples tested with an average prediction error less than 6.8% for a 0.5 Hz input signal and less than 9.3% for a 1 Hz input signal. On average, an experimentally calibrated linear model (neglecting hysteresis nonlinearity) results in a two to four times higher error when compared with the scalar Preisach model. Two electrical configurations, 3:1 and 1:1, are evaluated. The increasing-decreasing and decreasing-increasing first-order transition curves are both found to be equally successful in developing the $f_{\alpha \beta_{h}}$ and $f_{\alpha \beta_{a}}$ databases for the Preisach model, respectively, for all samples.

On the other hand, fine position control of these systems is not addressed. To achieve fine position control (which may be required from a flight control perspective), further nonlinear modeling can be conducted to include 1) structural dynamics, 2) piezoelectric creep, and 3) aerodynamic loading effects into the Preisach hysteresis model. This requires the employment of more generalized versions of the Preisach model (e.g., vector Preisach model). In addition to the Preisach model, more advanced theories for modeling hysteresis could also be employed. If the nonlinear effects are compensated, linear feedback controllers can be employed to achieve a desired proportional response. In addition, adaptive linear and nonlinear control laws can achieve further improvement in the response of the piezocomposite airfoils and similar bimorph or unimorph devices.

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