A practical technique for quantifying the performance of acoustic emission systems on plate-like structures

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ABSTRACT

A model for quantifying the performance of acoustic emission (AE) systems on plate-like structures is presented. Employing a linear transfer function approach the model is applicable to both isotropic and anisotropic materials.

The model requires several inputs including source waveforms, phase velocity and attenuation. It is recognised that these variables may not be readily available, thus efficient measurement techniques are presented for obtaining phase velocity and attenuation in a form that can be exploited directly in the model. Inspired by previously documented methods, the application of these techniques is examined and some important implications for propagation characterisation in plates are discussed. Example measurements are made on isotropic and anisotropic plates and, where possible, comparisons with numerical solutions are made.

By inputting experimentally obtained data into the model, quantitative system metrics are examined for different threshold values and sensor locations. By producing plots describing areas of hit success and source location error, the ability to measure the performance of different AE system configurations is demonstrated. This quantitative approach will help to place AE testing on a more solid foundation, underpinning its use in industrial AE applications.

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1. Introduction

Acoustic emission is a sensitive non-destructive testing (NDT) technique capable of detecting many types of defects in various materials. Relying on damage mechanisms to generate transient elastic waves, it is an attractive option for localised defect detection. Coupling the sensitivity of AE with the need for relatively few sensors, it is a candidate technology for structural health monitoring (SHM) applications. The SHM approach is different from conventional inspection schemes as damage detection is conducted in situ reducing system downtime and maintenance costs.

Five general goals for successful damage detection were described by Kessler et al. [1]:

- determination of presence of damage,
- estimation of severity of damage,
- damage classification,
- damage location, and
- estimation of damage size.

The difficulty in meeting these goals increases as the list progresses but considering the large numbers of papers on the subject, it would appear AE has the potential to meet all of them. However, significant limitations exist in AE testing which must be addressed before this is the case. In a review of AE testing in composite materials, Hamstad [2] discussed some of the repeated fundamental errors that occur in AE testing, including a lack of understanding of how wave propagation influences AE signals. It was recognised that even small propagation distances yield significant changes in the properties of AE signals, most evidently amplitude, and have serious implications for source characterisation and location.

The best examples of AE studies incorporating the effects of propagation are given by moment tensor studies [3–5]. Moment tensor studies are generally conducted on isotropic materials where the propagation effects are well-known. Propagation in anisotropic materials is more challenging due to the effects of steering [6] and attenuation. To overcome this problem, Kinjo et al. [7] conducted a series of independent tests to characterise the propagation of elastic waves in fibre-reinforced composite materials which were then applied in their AE measurements. However, although the ability to include the effects of propagation has been demonstrated, the vast majority of recent literature, including modal AE studies [8,9], fail to incorporate the effects.
Specimen geometry also influences elastic wave propagation and can make interpretation of propagation characteristics and, consequently, AE signals more difficult. Gorman [10] examined the transferability of results from small laboratory specimens (most commonly used in AE research) to larger field tests. Whilst it was concluded that transfer is possible, a sound understanding of wave propagation and sensor response was required. Since few recent AE works incorporate these effects, AE results are generally case specific and the ability to transfer information from AE research into industrial applications of the technology is difficult leading to a gap between the two. Such a gap was recognised by Drouillard [11], who described a need to bridge the gap between research and the technical application of AE, in his detailed review of the field. No clear framework exists for linking AE source characterisation measurements with measurements of AE system performance in practical monitoring situations. A recent exception is the work of Wilcox et al. [12, 13] who proposed a transfer function approach (similar to that proposed by Schmerr for modelling ultrasonic NDE [14]) to describe the entire AE system.

The work described in this paper aims to use the transfer function framework for quantifying the performance of AE systems, with the intention of placing the use of AE on a solid foundation. Work commences with a detailed description of the framework and its constituent elements. It is recognised that all required inputs may not be available using available mathematical techniques, so efficient experimental techniques that fit within the framework of the model are described. In particular, a simple measurement method that allows the complex propagation of Lamb waves in anisotropic composite plates to be included in the model is demonstrated. After validating the model transfer function, inputs are placed into the model and example quantitative outputs for different threshold amplitudes and sensor configurations are shown.

2. Examining the performance of AE systems

The technique for evaluating AE system performance is conducted in two stages. In the first stage, source waveforms, representative of the acoustic event being monitored, are obtained. In the second stage, the source waveforms are coupled with the propagation characteristics of the specimen to determine the received waveforms at arbitrary positions relative to the source. The performance of the AE system is evaluated by simulating the waveforms from sources located at many different positions relative to the AE system sensors.

To model the entire radiation field of waves from a specified AE source and their subsequent interactions with geometrical features is a difficult process. An efficient alternative is ray-tracing which works by considering only the wave energy which interacts with a chosen point of interest. In addition to greatly reducing the amount of propagation information required, ray-tracing allows the propagating wave to be described as the product of several linear transfer functions in the frequency domain [12]. The frequency spectrum of a signal received at a sensor, $H(\omega)$, may be written as:

$$H(\omega) = \sum \left[ S(\omega)P(\omega)A(\omega)B \prod_{\text{reflections}} R_{\text{r}}(\omega) \prod_{\text{transmissions}} T_{\text{t}}(\omega) \right]$$

(1)

where $S(\omega)$ is a mode and material dependent source frequency spectrum, $P(\omega)$ is the phase delay due to propagation transfer function, $A(\omega)$ is the attenuation transfer function, $B$ is the beam spreading transfer function, $R_{\text{r}}(\omega)$ are the reflection coefficients of all features at which the ray is reflected and $T_{\text{t}}(\omega)$ are the transmission coefficients of all features which the ray has traversed. The summation is performed over all possible mode and ray-path combinations that pass through the point of interest. System elements, such as sensors and pre-amplifiers, can be included in the model by incorporating additional transfer function elements [12, 13].

2.1. Source frequency spectra

The first stage of the technique involves obtaining source frequency spectra which are representative of the acoustic event being studied. The source frequency spectra can be obtained from theoretical sources, generated using moment tensors [3–5] or finite element analysis [15] or from experimental measurements of a real source. Whichever technique is used to derive the source frequency spectra, it is important to note that the spectra must meet certain criteria. The source frequency spectra must be free from the effects of geometrical features and, where the source or specimen demonstrate angular variations, the spectra must be obtained at a range of angular locations. Furthermore, the separate frequency spectrum of each propagating mode in each direction must be obtained. To examine the performance of an AE system monitoring different types of defect, the source frequency spectra for each type of defect must be obtained and subsequently substituted into the model. It is important to note that different types of source can yield different performance metrics and, as a result, the performance of an AE system monitoring an industrial structure should be evaluated for each type of expected source.

The AE system model described in Eq. (1) assumes a point-like source which may not be accurate for certain real damage mechanisms where the source has significant spatial extent. For cases where the propagation distance between a source and sensor is small relative to the spatial extent of the source, the point-like source assumption does not hold. To reduce this effect in the performance model, the source frequency spectra should be obtained in the far-field, where the propagation distance is large relative to the size of the source. In experimental measurements, the source frequency spectra are obtained from time domain waveforms making separation of the propagating modes more difficult. Time domain separation of propagating modes can generally be achieved if experimental measurements are made far enough from the source.

2.2. Elastic wave propagation

The second stage of the technique is concerned with elastic wave propagation. The general equation describing a signal received at a sensor is applicable to wave propagation in any material or structure. This work is concerned with plate-like structures, as these are frequently used in industrial structures. Propagation in plates differs from bulk materials as the plate surfaces act as waveguides creating Lamb waves [16, 17]. Dispersive in nature, that is the phase velocity is frequency dependent, Lamb waves have two types of propagating modes: symmetric (denoted by the letter S) and anti-symmetric (denoted by the letter A). In the low frequency–thickness regime only fundamental Lamb wave modes exist and these are identified by zero subscripts: $S_0$ and $A_0$. A third mode exists in the form of a non-dispersive horizontal shear mode (denoted by the letters SH) but this is not a Lamb mode.

The general equation for a signal received at a sensor, Eq. (1), is simplified by removing the effects of geometrical features. The removal of geometrical features for the acquisition of source frequency spectra has already been discussed. Further simplification is achieved by examining the propagation of a single mode on a single ray-path. Given the mode and material dependent source frequency spectrum measured at a far-field nominated distance from the source, $S(\omega)$, the signal received at a sensor location, $H(\omega)$, is given by:
The phase delay component implicitly includes dispersion effects which result in a "spreading out" of the wave over time due to the frequency dependent phase velocity. No energy is lost through dispersion, although a decrease in signal amplitude is seen to the frequency dependent phase velocity. No energy is lost effects which result in a "spreading out" of the wave over time due

\[ P(\omega) = \exp\left(-\frac{iod}{v_{ph}(\omega)}\right) \]  

(3)

where \( \omega \) is the angular frequency, \( d \) is the propagation distance and \( v_{ph}(\omega) \) is the frequency dependent phase velocity.

2.4. Beam spreading, \( B \)

The expansion of the wave-front as it radiates outwards is described by the beam spreading transfer function and results in a further reduction in amplitude. Beam spreading is a function of propagation distance from the source alone. Similar to dispersion, beam spreading conserves energy. However, since the energy is spread over an increasing length of wave-front the amplitude on a chosen ray-path decreases. The beam spreading transfer function is modelled for both isotropic and anisotropic media [18] as:

\[ B = \frac{1}{\sqrt{d}} \]  

(4)

2.5. Attenuation, \( A(\omega) \)

The final signal decrement mechanism is attenuation. This represents wave energy lost through absorption and backscatter and is described by the attenuation transfer function:

\[ A(\omega) = e^{-\alpha(\omega)d} \]  

(5)

where \( \alpha(\omega) \) is the frequency dependent attenuation coefficient.

2.6. Angular variations in the source waveforms and propagation characteristics

In the work which follows, the acoustic source is chosen to be a piezo-ceramic element with a diameter of 3 mm. This is chosen purely for illustrative purposes, to show how source characterisation measurements are linked with wave propagation measurements and ultimately used to predict AE system performance. Whilst the source has no angular amplitude variations, the cross-ply plate has an angular excitability pattern and, as a result, the source must be characterised by measurements of radiated modal amplitudes in several directions. The equivalent characterisation of a real AE source also requires simultaneous measurement in multiple directions. The angular variations in the cross-ply plate also require that the propagation be characterised in different directions. Once obtained, the inclusion of angular variations in the source and propagation characteristics into the rest of the model is straightforward.

The nominated propagation distance between the acoustic source and sensors during the acquisition of the source frequency spectra is 150 mm. The propagation distance is of sufficient extent to allow separation of the \( S_0 \) and \( A_0 \) propagating modes. The source frequency spectra of each mode are obtained from measurements of out-of-plane surface displacement. Alternatively the frequency spectra based on in-plane displacements could be used, but this does not provide additional information as at each frequency the in-plane and out-of-plane displacements for each mode are in a fixed proportion to one another according to the relevant mode shape.

2.7. Inclusion of AE sensor response

The electrical output from a conventional piezoelectric sensor is a function of the displacement field under the active area of the sensor. Complete characterisation of sensors is challenging as they may be sensitive to both in-plane and out-of-plane displacement components, and have independent frequency and wavelength sensitivity (the latter due to the finite size of the transducer aperture). As the model implicitly predicts guided wave modal amplitudes, all these effects can in theory be included if the data is available. However, without suitable data, an appropriate first approximation is to assume that AE sensors are sensitive to out-of-plane displacement only. As the source characterisation in this paper is also performed using out-of-plane displacement measurements no further conversion is required. However, if for example, the AE sensors were sensitive only to in-plane displacements, then it would be straightforward to use knowledge of the guided wave mode shapes to predict the measured in-plane displacement instead for a given source.

3. Experimental methods for obtaining phase velocity and attenuation

In many plates, elastic material properties are known and, as a result, propagation can be predicted using numerical solutions of Lamb's homogeneous equation [19]. In this section situations where material properties are unknown and propagation characterisation relies on experimental measurements will be considered.

In the framework of the model, phase velocity and attenuation are the only variables required to characterise propagation. Experimental techniques, inspired by previously reported work, are employed to make example measurements on two large plates: one aluminium and one uni-directional (UD) glass fibre composite (GFC). Known material properties for these plates are reported in Appendix A and are used to predict dispersion curves using a numerical approach in the software package DISPERSE [19].

Large plate dimensions allow time domain separation of the \( S_0 \) and \( A_0 \) modes as they propagate from the source and help avoid unwanted reflections from the specimen edge. To facilitate the isolation of single modes, an acoustic source was required which had a relatively flat frequency response to minimize resonance. Resonance of the acoustic source extends the length of a waveform in the time domain, making subsequent isolation of modes using time domain windowing more difficult. After examination of several transducers the most suitable candidate was found to be an in-house produced circular pz-27 piezo-ceramic transducer, with 3 mm diameter and 3 mm thickness. A schematic of the phase velocity measurement in the aluminium plate is shown in Fig. 1. A Hanning windowed tone burst was used to pulse this transducer which was permanently mounted on the plates using commercial superglue. The centre frequency and bandwidth of the tone burst were varied according to the measurement being taken and are reported in Appendix B.

To remove unwanted transducer and angular excitability variations, the source was mounted on the specimen surface and all
measurements were made at different distances along the same ray-path. Measurements of absolute plate displacement were made using a Polytec OFV-505 laser vibrometer. This allowed consistent measurements of displacement at various points on the ray-path. The laser suffered from a low signal to noise ratio and, as a result, the signal was averaged over at least 1000 measurements. The signals were recorded and averaged using a LeCroy Waverunner 6030 digital oscilloscope, with a sampling frequency of 10 MHz.

3.1. Phase velocity

Phase velocity was measured using a phase delay technique [20] where frequency/phase relationships at two points along a ray-path are compared and the phase velocity, \( v_{ph}(\omega) \), calculated using:

\[
v_{ph}(\omega) = \frac{\omega(d_2 - d_1)}{2\pi n + |\psi_1(\omega) - \psi_2(\omega)|}
\]  

where \( \omega \) is the angular frequency, \( n \) is the number of cycles undertaken by the progressing wave between the two measurement points and \( \psi(\omega) \) is the phase function defined as:

\[
\psi(\omega) = \tan^{-1}\left[\frac{\text{Im}(M(\omega))}{\text{Re}(M(\omega))}\right]
\]  

where \( M \) is the frequency spectrum of the measured waveform.

The use of the frequency domain for calculations allowed a large frequency range (equivalent to the bandwidth of the acoustic source) to be measured in a single measurement. Since beam spreading and attenuation have little or no effect on the phase content of the propagating wave-front, energy lost due to these mechanisms has no effect on the phase velocity measurement.

At most frequencies, the ratio of propagation distance to wavelength is relatively large, and thus the number of cycles the wave has undertaken is unknown. This leads to an immediate problem, since a value of \( 2\pi n \) is required to estimate phase velocity from the measurements. Waveform measurements were made at distances of 200 and 300 mm from the source. The effect of substituting different values of \( n \) into the phase delay equation Eq. (6) is shown in Fig. 2a. It can be seen that this causes the value of phase velocity to vary, with the correct value of phase velocity achieved when \( n = 12 \).

Group velocity describes the velocity at which energy is conveyed along the wave and is that which is ultimately witnessed in simple time-based measurements. Group velocity, \( v_g(\omega) \), is defined as:

\[
v_g(\omega) = \frac{\partial \omega}{\partial k}
\]  

where \( k \) is the angular wavenumber.

The group velocity is calculated from the experimentally measured phase velocity curve by first computing \( k \) and then numerically differentiating according to Eq. (8). In practice, the measured phase velocity is first fitted with a curve which is subsequently used in this calculation. In the differentiation, a fitted phase velocity curve provides a much smoother group velocity curve.

The value of \( n \) has no effect on group velocity. The importance of this result will be demonstrated later in this paper and for now is just noted. A comparison between the group velocity, calculated with any value of \( n \), and the numerical solution obtained using DISPERSE is shown in Fig. 2b. Similar results can be obtained for waves propagating in off-principal axis directions in anisotropic media, where elastic wave propagation is complicated by the differing directions of phase and group velocities. The differing propagation directions of phase and group velocity is known as steering and is described in detail by Neau et al. [6] and Neau [21]. A geometrical representation of the steering phenomenon is shown in Fig. 3.

The magnitude of the phase velocity is observed in the phase velocity direction, \( \theta_p \). The group velocity direction, \( \theta_g \), which corresponds to this phase velocity direction is perpendicular to the tangent of the slowness, \( S \) curve. The slowness is the reciprocal of the phase velocity and is shown in Fig. 3. For a phase velocity direction, the corresponding group velocity, \( V_g \), is obtained using Eq. (8) which differs from the group velocity direction by the steering angle, \( \phi \). The phase velocity of the waveform measured on the plate...
at two points on the same radial line from a point source is the component of phase velocity propagating in the group velocity direction. Thus to obtain the true phase velocity a geometrical correction must be applied:

\[ v_p = v_0 \cos \phi \]  

(9)

where \( v_p \) is the measured component of phase velocity propagating in the group velocity direction.

The experimental set-up for measurements on a UD GFC plate is shown in Fig. 4. Waveforms were measured at 50 and 150 mm from a PZT source (mounted on the edge of the specimen), 15° from the fibre direction. The phase velocity component in the group velocity direction, \( v_g \), was calculated using the phase delay technique Eq. (6).

Fig. 5 shows that clear discrepancies exist between the experimentally measured phase velocity and that predicted by the numerical solution when no compensation for steering is applied. As the propagation velocity and measurement distances were different from those in the aluminium example, different values of \( n \) were required. However, by varying the value of \( n \), no improvements were seen.

To correct the phase velocity measurement for the effects of steering, the slowness curve over a large angular range must be known. By using this curve, the steering angles can be used to determine the phase velocity angle, \( \theta_p \), that leads to the waveform propagating at the group velocity angle, \( \theta_g \), of 15°. The steering angle curve, shown in Fig. 6, indicates that at 15° the steering angle is 39°. The summation of the group velocity angle and steering angle gives a phase velocity angle of 54°. Thus the phase component of waves propagating at 54° generates the measured waveforms and the phase velocity predicted by DISPERSE at 54° is comparable to the steering corrected experimental phase velocity measurement at 15°.

A comparison between the experimental and numerical solutions is shown in Fig. 7. It can be seen that the correct experimental phase velocity is achieved when \( n = 8 \). It should be noted that for simulating propagation using the transfer function approach, the measured phase velocity with no correction for steering should be used. Correction for steering is only required for comparison with numerically generated curves. Thus, no knowledge of steering angles is required a priori for application of experimentally measured phase velocity within the model.
3.2. Attenuation

The second measurement required to describe Lamb wave propagation is attenuation. Previous work has often grouped the three signal decrement mechanisms together, but this is an inaccurate representation since the behaviour of each mechanism is dependent in a different way on the distance of the propagating wave from the source. In adopting the definition of attenuation described earlier by Eq. (5), accurate measurements require that these other signal decrement mechanisms be accounted for. The effects of dispersion can be removed by studying entire wave packets along a single ray-path and the effects of beam spreading can be removed with knowledge of the distance between the source and measuring stations. This approach is similar to those reported by Kinjo et al. [7] and Neau [21] but however includes a term to account for the beam spreading element of the decrement. Similarly to the phase velocity measurement, study of attenuation in the frequency domain allows a large bandwidth to be considered in a single measurement. The attenuation coefficient, \( \alpha(\omega) \), is then estimated by:

\[
\alpha(\omega) = \frac{1}{(d_1 - d_2)} \ln \left[ \frac{M_2(\omega)}{M_1(\omega)} \right] \sqrt{\frac{d_2}{d_1}}
\]

(10)

where subscripts denote the measuring stations.

Attenuation measurements were made on both the aluminium and the UD GFC plate. Due to the low material damping of aluminium and the large difference in acoustic impedance between the material and surrounding air, the attenuation was expected to be small in relation to the measurement sensitivity. The attenuation is approximately 0 Np/m across the bandwidth of the measurement, shown in Fig. 8. Large errors, indicating regions of apparently negative attenuation, occur at the extremes of the measurement bandwidth where the amount of energy in the input signal is small.

The S\(_0\) attenuation in UD GFC, shown in Fig. 9, is much greater than in aluminium. Higher attenuation in GFC is an expected result and is thought to be due to the viscoelastic nature of the matrix material. The attenuation shows an increasing trend with frequency. Although similar attenuation magnitudes are reported elsewhere [21], there is no direct comparison available, either experimental or numerical. Since the amplitude of measured signals has a large influence in the output of AE systems, an accurate value for attenuation is required. The inability to obtain analytic attenuation values is a good example of why experimental attenuation measurements need to be taken.

4. Predicting propagation along a ray-path

The simplified general equation, Eq. (2), may be used to predict the waveform at any point along a ray-path, given the AE source waveform at a specified propagation distance. In addition to being the basis of the AE system model operation, it also provides a useful confirmation of the validity of the proposed experimental method. By using the propagation transfer functions in Eq. (2), waveforms may be calculated for different points along a single ray-path. The required inputs are a source waveform, measured at a prescribed distance, the phase velocity and attenuation.

In the first example, a signal measured in the aluminium plate 400 mm from the source, shown in Fig. 10a, was back-propagated to a point 100 mm from the source. A comparison between the calculated waveform and that measured at 100 mm is shown in Fig. 10b with excellent agreement seen. It should be noted the agreement seen is over a relatively large propagation distance (300 mm).

In the second example, the forward propagation of an S\(_0\) mode in the GFC plate described previously at 15° was attempted. In or-
order to excite the $S_0$ mode, a transducer of the type described previously was mounted on the edge of the plate. The source waveform measured at a distance of 75 mm is shown in Fig. 11a. A comparison between the calculated appearance of the propagated waveform at 125 mm and that measured experimentally is shown in Fig. 11b.

Again, excellent agreement is seen between the calculated waveform and measured waveform both in terms of phase and amplitude. A reflected $S_0$ wave is observed in the measured signal which is not accounted for. It is accepted that the propagation distance here is small (50 mm) and errors in the attenuation and phase velocity measurement will become more apparent for large propagation distances. However the result here shows that the adopted approach for measuring and subsequently being able to predict propagation is valid.

5. Quantitative model outputs

By extending the propagation prediction approach described in the earlier section to consider ray-paths in all directions, full-field AE system response can be examined. The next section describes a series of quantitative parametric studies for a large ($400 \text{ mm} \times 400 \text{ mm}$) cross-ply (CP) carbon fibre composite (CFC) plate. For simplicity, the displayed outputs are limited to the $A_0$ mode.

### 5.1. Source angular dependency

The model requires a waveform input from which the waveforms at different points in the plate can be calculated. Any waveform, whether derived through a mathematical or experimental approach, can be entered into the model and should include all excitability effects whether they are due to the source or material. In the simplified examples which follow, the input waveform is derived from a PZT transducer permanently mounted at the centre of the plate (Appendix B). From this source, angular excitability dependence is expected from both the transducer coupling and material. Consequently, the source must be characterised to account for these excitability effects in the model.

The presence of angular variations in excitability requires that several waveforms be recorded at different angular locations which are subsequently entered into the model. Variations in amplitude are examined over an angular range from $0^\circ$ to $360^\circ$ in $15^\circ$ intervals at a set distance around the transducer of 150 mm. Although it is impossible to distinguish between variations in amplitude due to the transducer coupling and those due to the material, Fig. 12 shows clear increases in amplitude in the $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$ directions. Since these coincide with the fibre directions, the angular amplitude behaviour is thought to be

![Fig. 10. Backward propagation of the $A_0$ mode on the aluminium plate (a) measured waveform at $d = 400 \text{ mm}$ and (b) calculated and measured waveforms at $d = 100 \text{ mm}$. Waveforms are offset by $\pm 250 \text{ mV}$.](image)

![Fig. 11. Forward propagation of the $S_0$ mode on the UD CFC plate at 15° (a) measured waveform at $d = 75 \text{ mm}$ and (b) calculated waveform and measured waveform at $d = 125 \text{ mm}$. Waveforms are offset by $\pm 75 \text{ mV}$ to allow a clear comparison.](image)
dominated by the excitability of the CFC plate [18]. The excitability variation, irrespective of origin, is accounted for in the model by matching the angular location of the input waveform measurement to the modelled direction of the ray-path propagation from the source.

5.2. Propagation characteristics

By using the experimental phase velocity and attenuation methods described earlier, $A_0$ propagation was characterised in the CFC plate. Due to the symmetry of the specimen, an angular range between $0^\circ$ and $90^\circ$, in $30^\circ$ intervals, was sufficient. Two waveforms were measured at different distances along those directions at 100 and 200 mm, respectively. Whilst the angular resolution is quite sparse, the amount of variation is quite small and thus not many angular points are required. If modes were studied that have higher angular variations, for example the $S_0$ mode, it would be better to increase the angular resolution.

The $A_0$ propagation measurements made in the CP CFC plate at $0^\circ$ are shown in Fig. 13. The experimental phase velocity measurement and that predicted by DISPERSE are in reasonable agreement. The $A_0$ attenuation measured in the CP CFC plate is much greater than the $S_0$ mode attenuation measured in the UD GFC plate. Again the attenuation coefficient has a frequency dependency.

5.3. Hit success and source location

Threshold amplitude is an important parameter in AE studies as it describes the point at which the system identifies a hit. Its value determines whether the transducer triggers on a passing waveform and, if so, at what point in time the hit is recorded. Consequently, it determines whether the system is successful in registering hits and provides the input to source location algorithms based on the arrival time of waveforms.

The specimen geometry, including plate dimensions and receiver locations, was entered into the model. The model starts by dividing the plate into a series of discrete source location points. Each source location point is considered in turn and the propagation distance and angle to each of the three specified receiver locations is calculated. By using the supplied propagation characteristics and input waveforms, the shape of the waveform arriving at each receiver location is then calculated for every source location in turn. Finally, using the specified threshold amplitude, quantitative information is taken from each waveform, in this case hit success and arrival time. Since no receiver characteristics are entered into the model, the output is given as units of absolute out-of-plane surface displacement.

Fig. 14 shows the hit success plot for a triangular arrangement of sensors with the threshold set at 300 pm. A central region exists in which all three receivers trigger on the $A_0$ mode, indicated as three $A_0$ hits. Outside this central region, the amplitude of the

![Fig. 12. Angular amplitude variations on the CP CFC plate.](image)

![Fig. 13. $A_0$ propagation measurements on the CP CFC plate at $0^\circ$ (a) phase velocity and (b) attenuation.](image)

![Fig. 14. $A_0$ hit success on the CP CFC plate for a threshold amplitude of 300 pm. Dots denote sensor locations.](image)
waves is too low to enable triggering on all three receivers. The inability to trigger on a mode, or indeed triggering on an incorrect mode, is detrimental to the source location as it leads to large errors in the arrival time.

The location error is evaluated using a simple source location algorithm. By measuring the difference in arrival time (delta-t time) between two or more sensors, an array of sensors provides a unique combination of delta-t values for a given source point. By matching this delta-t combination to a set already created from a highly refined mesh with non-dispersive propagation, source location error can be estimated.

Fig. 15 shows that the region of good source location is smaller than the central region in which all three sensors register a hit, intuitively the region of good source location. It can be seen that the errors within this region are as large as 150 mm, thus the quality of source location is heavily compromised by the propagation of the $A_0$ mode, with dispersion, attenuation and beam spreading all affecting the accuracy of the measurement.

By reducing the threshold to 200 pm, the region where the $A_0$ mode is received on all three channels expands. Fig. 16 shows that although the sensors mark the three corners of an isometric triangle, the region of three hits clearly does not resemble an isometric triangle. This is due to anisotropic propagation, notably the subtle variations in angular attenuation coefficients, and the angular excitability of the source.

The location error calculated for the reduced threshold amplitude is shown in Fig. 17. Again the region of good source location is much smaller than the area in which each sensor registers an $A_0$ hit. The final example demonstrates an alternative system configuration, including different sensor geometry and threshold amplitude. Sensors are now located in three random positions on the plate and the threshold lowered to 100 pm.

The hit success for this new arrangement is shown in Fig. 18. The region in which each sensor registers a hit is non-intuitive and would be difficult to estimate, showing how the approach can be used to represent complicated systems.

The location error of this configuration is shown in Fig. 19. It can be seen that very large errors, up to 510 mm, are expected at several points within the region where each sensor registers a hit. Regions of exceptionally high location error are generally “behind” the sensors, where small changes in delta-t value yield large changes in source location. Thus, the region of good source location is limited to the area between the three receiver locations.

6. Conclusions

This paper has described a model for quantifying the performance of AE systems. Coupling a transfer function approach with characterisation of the system elements, the model provides a powerful tool for evaluating the performance of different AE system configurations.

For the model presented, knowledge of frequency dependent phase velocity and attenuation is required. With a complete knowledge of material properties, phase velocity and attenuation can be determined using mathematical approaches. However,
complete and accurate knowledge of material properties is not always available, especially in the case of composites. For this reason, efficient approaches have been presented for determining phase velocity and attenuation as a function of frequency in a form which can be directly incorporated into the model with no knowledge of material properties required a priori. Excellent agreement has been observed with available numerical approaches, although it has been noted that the success of such measurements relies heavily on the ability to isolate single propagating modes.

By incorporating the propagation results into the model, the ability to calculate the waveform at any point on a ray-path has been demonstrated. This is a powerful tool in AE as, with knowledge of the propagation distance, source characterisation can be conducted with compensation for propagation effects.

By expanding this approach into all propagation directions, the full-field performance of several simplified AE systems has been modelled. Lowering the threshold was found to increase the area in which all three sensors register a hit. A simple anisotropic source location technique has been demonstrated for the calculation of location error. It was found that the area in which all three sensors register a hit from the same mode does not correspond with the area in which good source location is achieved. This is caused by changes in waveform shape due to propagation resulting in inconsistent estimation of arrival times. Regions outside that defined by the receivers have steep gradients in expected delta-t, where a small change in arrival time yields a large change in source location.

The model can be expanded to represent more complex AE systems through the inclusion of further geometrical features, receiver characteristics and the waveforms from real source mechanisms. Continued work on quantitatively characterising these elements shows great promise for allowing the AE user to make informed decisions on AE system settings and configurations.

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Appendix A. Material properties

T6082 Aluminium plate
Geometry
Length = 1000 mm
Width = 1000 mm
Thickness = 2.5 mm
Density = 2.7 g/cm³

Engineering constants
\[ E_{xx} = 70.0 \text{ GPa}, \quad G_{xy} = 26.3 \text{ GPa}, \quad v_{xy} = 0.33 \]
\[ E_{yy} = 70.0 \text{ GPa}, \quad G_{xz} = 26.3 \text{ GPa}, \quad v_{xz} = 0.33 \]
\[ E_{zz} = 70.0 \text{ GPa}, \quad G_{yz} = 26.3 \text{ GPa}, \quad v_{yz} = 0.33 \]

Stiffness matrix
\[
\begin{bmatrix}
103.72 & 51.08 & 51.08 & 0.00 & 0.00 & 0.00 \\
51.08 & 103.72 & 51.08 & 0.00 & 0.00 & 0.00 \\
103.72 & 51.08 & 103.72 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 26.30 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 26.30 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 26.30 \\
\end{bmatrix}
\]

E-Glass/913 uni-directional glass fibre-reinforced plate
Geometry
Length = 300 mm
Width = 300 mm
Thickness = 2.0 mm
Density = 2.0 g/cm³

Engineering constants
\[ E_{xx} = 43.9 \text{ GPa}, \quad G_{xy} = 4.34 \text{ GPa}, \quad v_{xy} = 0.30 \]
\[ E_{yy} = 15.4 \text{ GPa}, \quad G_{xz} = 4.34 \text{ GPa}, \quad v_{xz} = 0.30 \]
\[ E_{zz} = 15.4 \text{ GPa}, \quad G_{yz} = 5.31 \text{ GPa}, \quad v_{yz} = 0.45 \]

Stiffness constants
\[
\begin{bmatrix}
49.59 & 9.49 & 9.49 & 0.00 & 0.00 & 0.00 \\
9.49 & 21.13 & 10.51 & 0.00 & 0.00 & 0.00 \\
9.49 & 10.51 & 21.13 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 5.31 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 4.34 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 4.34 \\
\end{bmatrix}
\]

Appendix B. Input waveforms

Aluminium plate: two cycle Hanning windowed tone-burst, centre frequency 250 kHz.
GFC plate: two cycle Hanning windowed tone-burst, centre frequency 300 kHz.
CFC plate: two cycle Hanning windowed tone-burst, centre frequency 150 kHz.

References