Initial sizing optimisation of anisotropic composite panels with T-shaped stiffeners

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Abstract

This paper provides an approach to perform initial sizing optimisation of anisotropic composite panels with T-shaped stiffeners. The method divides the optimisation problem into two steps. At the first step, composite optimisation is performed using mathematical programming, where the skin and the stiffeners are modelled using lamination parameters accounting for their anisotropy. Skin and stiffener laminates are assumed to be symmetric, or mid-plane symmetric laminates with 0°, 90°, 45°, or −45° ply angles. The stiffened panel is subjected to a combined loading under strength, buckling and practical design constraints. Buckling constraints are computed using closed form solutions and an energy method (Rayleigh-Ritz). Conservatism is partially removed in the buckling analysis by considering the skin-stiffener flange interaction and decreasing the effective width of the skin. Furthermore, the manufacture of the stiffener is embedded within the design variables. At the second step, the actual skin and stiffener lay-ups are obtained using genetic algorithms, accounting for manufacturability and design practices. This two step approach permits the separation of the structural analysis (strength, buckling, etc.), which is performed at the first step, from the laminate stacking sequence combinatorial problem, which is dealt efficiently with genetic algorithms at the second step.

Keywords: Optimisation; Anisotropic; Lamination parameters; Composite stiffened panels; Closed form solutions; Buckling; Rayleigh-Ritz; Genetic algorithm

1. Introduction

Aerospace manufacturers are increasingly employing laminated composites to replace metallic materials in primary structures in order to reduce aircraft weight. Composite stiffened panels, especially with T-shaped stiffeners, are commonly used to design flight primary structures such as wings or fuselages. In general terms, composite materials present high specific strength and stiffness ratios [1] and offer the advantage over their metallic counterparts of being stiffness tailored. This latter feature is closely associated with their design and manufacture. Laminated composite materials have usually been restricted to symmetric, or mid-plane symmetric laminates with 0°, 90°, 45°, and −45° ply angles, due to practical manufacturing requirements. In addition, the design of composite stiffened panels becomes more complex when considering the manufacture of the stiffener.

Composite optimisation is a non-linear problem. A number of optimisation techniques have been developed over the years to design composite structures [2–29]. In the 1970s, early attempts on optimisation of laminated fibre composites were performed by Schmit and Farshi [2,3]. They optimised symmetric laminated fibre composites with homogeneous and orthotropic properties, considering ply thicknesses as continuous design variables. Stroud and Agranoff [4] followed the same trend and optimised composite hat-stiffened and corrugated panels using a simplified set of buckling equations as constraints. Laminares were assumed to be orthotropic and the design variables were the dimensions of the panel’s cross-sections. However, composites might exhibit a certain degree of flexural anisotropy. Ashton and Waddoups [5] reported the effect of the flexural anisotropy on the stability of composite plates. Chamis [6] concluded that neglecting
flexural anisotropy to assess buckling behaviour could lead to non-conservative results. Nemeth [7] provided the bounds within which flexural anisotropy has a significant effect. Weaver [8,9] recently developed simple closed form (CF) solutions to include the effect of flexural anisotropy on compression and shear loading.

Tsai et al. [10] introduced an alternative representation of the stiffness properties of a laminated composite by the use of lamination parameters. Miki and Sugiyama [11] proposed the use of lamination parameters to deal with the discrete laminate stacking sequence problem. They considered symmetric and orthotropic laminates. Optimum designs for constraints such as in-plane stiffness or buckling, were obtained, from geometric relations between the lamination parameters feasible region and the objective function. Fukunaga and Vanderplaats [12] used lamination parameters and mathematical programming (MP) techniques to carry out stiffness optimisation of cylindrical shells with orthotropic properties. Haftka and Walsh [13] used integer-programming techniques to carry out lay-up optimisation under buckling constraints on symmetric and balanced laminated plates. They used zero-one integers as design variables that were related to stiffness properties via lamination parameters and showed that the problem was linear. Nagendra et al. [14] extended that work and optimised the stacking sequence of symmetric and balanced composite laminates with stability and strain constraints. The drawback of integer-programming techniques is that they require large computational resources especially when structure complexity increases. Fukunaga et al. [15] used MP techniques and lamination parameters to maximise buckling loads under combined loading of symmetrically laminated plates including the bending-twisting couplings (flexural anisotropy). They showed that under shear and shear-normal loading flexural anisotropy could increase or decrease the critical buckling load.

Le Riche and Haftka [16] and Nagendra et al. [17,18] adopted a different approach. They employed genetic algorithms (GAs) to solve the discrete lay-up optimisation problem. GAs are search algorithms based on the mechanics of natural selection and natural genetics [19], which do not require gradient information to perform the search and have the ability to tackle search spaces with many local optima [20]. Furthermore, Nagendra et al. [17] investigated the application of a GA to the design of blade stiffened composite panels. VIPASA [21] was used as the analysis tool and results were compared with PASCO [22], which uses VIPASA as the analysis tool and CONMIN [23] as optimiser. It was concluded that the designs obtained by the GA offered higher performance than the continuous designs. However, it was recognised that great computational cost was associated with the GA. More recently, Liu et al. [24] employed VICONOPT [25] to perform an optimisation of composite stiffened panels under strength, buckling and practical design constraints. A bi-level approach was adopted. VICONOPT was employed at the first level to minimise the panel weight, employing equivalent orthotropic properties for the laminates with continuous thickness, whereas at the second level laminate thicknesses were rounded up and associated with pre-determined design lay-ups.

A two-level optimisation strategy employing lamination parameters, MP and GAs, was initially proposed by Yamazaki [26]. The optimisation was split into two stages. Firstly, a gradient-based optimisation was performed using the lamination parameters as design variables. Secondly, the lamination parameters from the first level were targeted using a GA. In this paper, volume, buckling load, deflection and natural frequencies of a composite panel were optimised without accounting for either membrane or flexural anisotropy. Autio [27], following a similar approach to Yamazaki’s, investigated actual lay-ups, where certain lay-up design rules were introduced as penalties in the fitness function of the GA.

However, with the exception of Fukunaga and Vanderplaats [12], none of the previous authors considered the feasible region in the lamination parameter space that relates in-plane, coupling and out-of-plane lamination parameters. Liu et al. [28] used lamination parameters and defined the feasible region between two of the four membrane and bending lamination parameters to maximise the buckling load of unstiffened composite panels with ply angles restricted to 0°, 90°, 45°, and −45°. They compared their approach against one using a GA and concluded that the use of lamination parameters in a continuous optimisation produced similar results to those obtained by the GA except in cases where laminates were thin or had low aspect ratios. Diaconu and Sekine [29] performed lay-up optimisation of laminated composite shells for maximisation of the buckling load, using the lamination parameters as design variables and including their feasible region. They fully defined, for the first time, the relations between the membrane, coupling and bending lamination parameters for ply angles restricted to 0°, 90°, 45°, and −45°.

The authors’ previous work [30], based upon a two-step optimisation approach, which coupled MP with GAs, showed that composite anisotropy could be used to improve structural performance. Design constraints such as strength, local and global buckling as well as practical design rules were considered. Buckling was addressed by finite elements (FE) and CF solutions. It was shown that CF solutions introduced a high degree of conservatism in the buckling analysis and hence heavily penalised the optimum solutions. However, CF solutions significantly increased the computational efficiency.

The purpose of this paper is to provide an approach to perform initial sizing optimisation of anisotropic composite panels with T-shaped stiffeners. The method divides the optimisation problem into two steps. At the first step, composite optimisation is performed using MP, where the skin and the stiffeners are modelled using lamination parameters accounting for their anisotropy. The skin and stiffener laminates are assumed to be symmetric, or
mid-plane symmetric laminates with 0°, 90°, 45°, or −45° ply angles. The stiffened panel is subjected to a combined loading under strength, buckling and practical design constraints. Buckling constraints are computed using CF solutions and an energy method (Rayleigh-Ritz). Conservatism is partially removed in the buckling analysis by considering the skin–stiffener flange interaction and decreasing the effective width of the skin. Furthermore, the manufacture of the stiffener is embedded within the design variables. At the second step, the actual skin and stiffener lay-ups are obtained using GAs, accounting for manufacturability and design practices. This two step approach permits the separation of the structural analysis (strength, buckling, etc.), which is performed at the first step, from the laminate stacking sequence combinatorial problem, which is dealt efficiently with GAs at the second step.

2. Panel geometry and loading

As in Ref. [30] the composite stiffened panel is assumed to be long, wide and composed of several skin–stiffener elements or superstiffeners under combined loading. Each superstiffener element consists of three flat plates (skin, stiffener flange and stiffener web) that are assumed to be rigidly connected along their longitudinal edges. The superstiffener element is assumed to model the panel’s behaviour. Fig. 1 defines the superstiffener element geometry, the material axis as well as the positive sign convention for the loading.

Due to the stiffener’s manufacture, four different stiffener configurations are considered. The stiffener is

![Figure 1](image1)

![Figure 2](image2)
manufactured as a back-to-back angle (Fig. 2a), adding capping plies in the stiffener flange (Fig. 2b), or extra plies in the stiffener web (Fig. 2c), and finally the combination of the previous configurations (Fig. 2d).

3. Laminate constitutive equations

The classical laminate theory (CLT) [1] is applied to the skin, stiffener flange and stiffener web, respectively, assuming laminates are symmetric or mid-plane symmetric. Thus,

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} = \begin{bmatrix}
A & 0 \\
0 & D
\end{bmatrix} \begin{bmatrix}
\varepsilon^0 \\
\kappa
\end{bmatrix},
\]

(1)

where \( [A] \) is the membrane stiffness matrix, \([D]\) is the bending stiffness matrix, \([N]\) is the vector of the in-plane running loads, \([M]\) is a vector of the running moments, \([\varepsilon^0]\) is the vector of in-plane strains and \([\kappa]\) is the vector of the middle surface curvatures.

Note that this paper does not consider transverse shear deformation. However, it is recognised that transverse shear deformation could significantly affect the response of composite laminates (e.g. Ref. [31]) especially if they have low width to thickness ratios.

The membrane and bending stiffness matrices can be expressed in terms of material stiffness invariants \((U)\) and eight lamination parameters \((\varepsilon)\) [10]. Plies are considered orthotropic and with fibre angles restricted to 0°, 90°, 45°, and -45°. As a result, the lamination parameters are further reduced to six. Hence,

\[
\begin{bmatrix}
A_{11} \\
A_{12} \\
A_{22} \\
A_{66} \\
A_{16} \\
A_{26}
\end{bmatrix} = \begin{bmatrix}
1 & \varepsilon_A^1 & \varepsilon_A^2 & 0 & 0 \\
0 & 0 & -\varepsilon_A^2 & 1 & 0 \\
1 & -\varepsilon_A^1 & \varepsilon_A^2 & 0 & 0 \\
0 & 0 & -\varepsilon_A^2 & 0 & 1 \\
0 & \varepsilon_A^2 & 0 & 0 & 0 \\
0 & 0 & \varepsilon_A^2 & 0 & 0
\end{bmatrix} \begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4 \\
U_5
\end{bmatrix},
\]

(2)

\[
\begin{bmatrix}
D_{11} \\
D_{12} \\
D_{22} \\
D_{66} \\
D_{16} \\
D_{26}
\end{bmatrix} = \begin{bmatrix}
1 & \xi_A^1 & \xi_A^2 & 0 & 0 \\
0 & 0 & -\xi_A^2 & 1 & 0 \\
1 & -\xi_A^1 & \xi_A^2 & 0 & 0 \\
0 & 0 & -\xi_A^2 & 0 & 1 \\
0 & \xi_A^2 & 0 & 0 & 0 \\
0 & 0 & \xi_A^2 & 0 & 0
\end{bmatrix} \begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4 \\
U_5
\end{bmatrix}.
\]

(3)

The material stiffness invariants \((U)\) are given as follows:

\[
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4 \\
U_5
\end{bmatrix} = \frac{1}{8} \begin{bmatrix}
3 & 2 & 3 & 4 \\
4 & 0 & -4 & 0 \\
1 & -2 & 1 & -4 \\
1 & -6 & 1 & -4 \\
1 & -2 & 1 & 4
\end{bmatrix} \begin{bmatrix}
Q_{11} \\
Q_{12} \\
Q_{22} \\
Q_{66}
\end{bmatrix}.
\]

(4)

The ply stiffness properties \((Q)\) are related to the ply Young’s moduli and Poisson’s ratios by the following equations:

\[
Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}},
\]

(5)

\[
Q_{12} = \frac{v_{12}E_{22}}{1 - \nu_{12}\nu_{21}},
\]

(6)

\[
Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}},
\]

(7)

\[
Q_{21} = Q_{12},
\]

(8)

\[
Q_{66} = G_{12},
\]

(9)

\[
v_{21} = \frac{E_{22}}{E_{11}}.
\]

(10)

The membrane and bending laminate parameters are given by the following integrals:

\[
\bar{\xi}_1^{d12} = \frac{1}{h^2} \int_{-h/2}^{h/2} \left[ \cos 2\varphi \cos 4\varphi \sin 2\varphi \right] dz,
\]

(11)

\[
\bar{\xi}_1^{d12} = \frac{12}{h^3} \int_{-h/2}^{h/2} \left[ \cos 2\varphi \cos 4\varphi \sin 2\varphi \right] z^2 dz,
\]

(12)

where \( \varphi \) represents the fibre orientation angle at position \( z \) and \( h \) is the laminate thickness.

4. Optimisation strategy

The optimisation strategy follows Ref. [30] and is presented in Fig. 3. This strategy divides the optimisation into two steps. At the first step, the optimum dimensions and lamination parameters of the superstiffener are found by employing gradient-based techniques (MP) under strength, buckling and practical design constraints. At the second step, a GA code is used to target the optimum laminate parameters to obtain the actual lay-ups for both the skin and the stiffener. Note that after the second step the actual superstiffener (optimum discrete design) is checked against the first step constraints to verify its feasibility.

4.1. First step—gradient-based optimisation

MATLAB [32] is employed to conduct the gradient-based optimisation. The non-linear mathematical optimisation problem can be stated as follows:

Minimise \[ M(x), \]

subject to \[ G_i(x) \leq 0, \quad i = 1, \ldots, n_c, \]

\[ x_j^l \leq x_j \leq x_j^u, \quad j = 1, \ldots, n_c, \]

where \( M \) is the objective function and represents the mass of the superstiffener element, \( G \) are the inequality constraints (such as strength, local and global buckling or
practical design rules), \( n_c \) is the number of design constraints, \( \vec{x} \) is the vector of the design variables and \( n_c \) is the number of design variables.

4.1.1. Objective function

The objective function is the mass of the superstiffener element. The mass as a function of the design variables, materials properties and geometry is given by

\[
M(\vec{x}) = a(\rho_{\text{skin}} A_{\text{skin}}(\vec{x}) + \rho_{\text{stg}} A_{\text{stg}}(\vec{x}))
\]

with

\[
A_{\text{skin}} = tb,
\]

\[
A_{\text{stg}} = A_{sf} + A_{sw} = t_{sf} b_{sf} + t_{sw} h_{sw},
\]

where \( A_{sf} \) is the area of the stiffener flange, \( A_{stg} \) is the area of the stiffener web, \( t \) is the thickness of the skin, \( t_{sf} \) is the thickness of the stiffener flange, \( t_{sw} \) is the thickness of the stiffener web, \( b \) is the stiffener pitch, \( b_{sf} \) is the width of the stiffener flange and \( h_{sw} \) is the height of the stiffener web.

4.1.2. Design variables

The manufacturing requirements of the stiffener are embedded in the design variables. The design variables for the superstiffener element, depending on the stiffener type, are listed in Table 1 [30], noting that \( \zeta \) are the lamination parameters (e.g. Ref. [10]).

### Table 1

<table>
<thead>
<tr>
<th>Stiffener type</th>
<th>Design variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skin</td>
<td>Stiffener flange</td>
</tr>
<tr>
<td></td>
<td>( h )</td>
</tr>
<tr>
<td>( a )</td>
<td>( \zeta_{1}^{A,D} )</td>
</tr>
<tr>
<td></td>
<td>( \zeta_{1}^{A,D} )</td>
</tr>
<tr>
<td>( t = h )</td>
<td>( t_{sw} = 2t_{a} )</td>
</tr>
<tr>
<td>( b )</td>
<td>As stiffener type a, knowing that</td>
</tr>
<tr>
<td></td>
<td>( t_{sf} = t_{a} )</td>
</tr>
<tr>
<td>( c )</td>
<td>( h )</td>
</tr>
<tr>
<td></td>
<td>( \zeta_{1}^{A,D} )</td>
</tr>
<tr>
<td></td>
<td>( \zeta_{1}^{A,D} )</td>
</tr>
<tr>
<td>( d )</td>
<td>As stiffener type e, knowing that</td>
</tr>
<tr>
<td></td>
<td>( t_{sf} = 2t_{a} )</td>
</tr>
</tbody>
</table>

Note that the stiffener web laminate is not the same as the web laminate. The stiffener web laminate is made of two stiffener flange laminates for stiffener type \( a \), is equivalent to the stiffener flange laminate for stiffener type \( b \), and is composed of three sub-laminates (two outer stiffener flange laminates and one inner web laminate) for stiffener types \( c \) and \( d \).

4.1.3. Design constraints

The design constraints considered are: lamination parameters feasible region, strength, buckling and practical design constraints.

4.1.3.1. Lamination parameters feasible region. The lamination parameters feasible region is extracted from Ref. [30]. Thus,

\[
2|\frac{\zeta_{1}^{A,D}}{\zeta_{2}^{A,D}}| - \frac{\zeta_{1}^{A,D}}{\zeta_{2}^{A,D}} - 1 \leq 0,
\]

\[
2|\frac{\zeta_{1}^{A,D}}{\zeta_{2}^{A,D}}| + \frac{\zeta_{1}^{A,D}}{\zeta_{2}^{A,D}} - 1 \leq 0,
\]

\[
(\zeta_{i}^{A} - 1)^{4} - 4(\zeta_{i}^{D} - 1)(\zeta_{i}^{A} - 1) \leq 0,
\]

\[
(\zeta_{i}^{A} + 1)^{4} - 4(\zeta_{i}^{D} + 1)(\zeta_{i}^{A} + 1) \leq 0,
\]

\[
(2\zeta_{i}^{A} - \zeta_{2}^{A} - 1)^{4} - 16(2\zeta_{i}^{D} - \zeta_{2}^{D} - 1)
\]

\[
(2\zeta_{i}^{A} + \zeta_{2}^{A} + 1)^{4} - 16(2\zeta_{i}^{D} + \zeta_{2}^{D} + 1)
\]

\[
(2\zeta_{i}^{A} - \zeta_{2}^{A} + 3)^{4} - 16(2\zeta_{i}^{D} - \zeta_{2}^{D} + 3)
\]

\[
(2\zeta_{i}^{A} + \zeta_{2}^{A} + 3)^{4} - 16(2\zeta_{i}^{D} + \zeta_{2}^{D} + 3)
\]
where $l$ and $x$ are given by

\[
(2\xi_1^A + \xi_2^A - 3)^4 - 16(2\xi_1^D + \xi_2^D - 3)
\times (2\xi_1^A + \xi_2^A - 3) \leq 0,
\] (24)

\[
(2\xi_3 - \xi_2^A + 1)^4 - 16(2\xi_3^D - \xi_2^D + 1)\times (2\xi_3^3 - \xi_2^3 + 1) \leq 0,
\] (25)

\[
(2\xi_3 - \xi_2^A - 1)^4 - 16(2\xi_3^D + \xi_2^D - 1)\times (2\xi_3^3 + \xi_2^3 - 1) \leq 0,
\] (26)

\[
(2\xi_3^3 - \xi_2^3 - 3)^4 - 16(2\xi_3^D - \xi_2^D - 3)\times (2\xi_3^3 + \xi_2^3 + 3) \leq 0,
\] (27)

\[
(2\xi_3^3 + \xi_2^3 + 3)^4 - 16(2\xi_3^D + \xi_2^D + 3)\times (2\xi_3^3 + \xi_2^3 + 3) \leq 0,
\] (28)

\[
(\xi_1^D - \xi_2^D - 1)^4 - 4(\xi_3^D - \xi_2^D - 1)(\xi_1^D - \xi_2^D - 1) \leq 0,
\] (29)

\[
(\xi_1^D + \xi_2^D + 1)^4 - 4(\xi_3^D + \xi_2^D + 1)(\xi_1^D + \xi_2^D + 1) \leq 0,
\] (30)

\[
(\xi_1^D - \xi_2^D + 1)^4 - 4(\xi_3^D - \xi_2^D + 1)(\xi_1^D - \xi_2^D + 1) \leq 0,
\] (31)

\[
(\xi_1^D + \xi_2^D - 1)^4 - 4(\xi_3^D + \xi_2^D - 1)(\xi_1^D + \xi_2^D - 1) \leq 0.
\] (32)

The above constraints are imposed on the skin, stiffener flange and web laminates, respectively. Further details on these constraints can be found in Refs. [11,29].

4.1.3.2. Strength constraints. Strength constraints are introduced by limiting the laminate in-plane strains longitudinally, transversely and in shear, for both tension and compression. CLT is used to calculate the laminate strains under the applied in-plane loads. Hence,

\[
\left\{\varepsilon^i\right\} = \left[A^{-1}\right]\{N\}.
\] (33)

The strength load factor is given by the ratio between the allowable and applied strain. Hence,

\[
\lambda_i^j = \frac{\varepsilon_{ai}^j}{\varepsilon_{ai}^i}, \quad i = T, C; \quad j = x, y, xy,
\] (34)

where $\varepsilon_{ai}$ is the allowable strain, $\varepsilon_{ai}^0$ is the applied strain, $x$, $y$ and $xy$ represent the longitudinal, transverse and shear directions, respectively. Note that $T$ and $C$ denote tension and compression.

For both the tension and compression cases, strength constraints are given by

\[
\frac{1}{\lambda_i^j} - 1 \leq 0, \quad i = T, C; \quad j = x, y, xy.
\] (35)

These constraints are applied to the skin, stiffener flange and stiffener web laminates.

4.1.3.3. Buckling constraints. An energy method (Rayleigh-Ritz) and CF solutions have been employed to evaluate buckling constraints. Both local and global buckling have been addressed. Local buckling assesses

the individual element failure whereas global buckling considers the failure of the stiffened panel as a whole. Local buckling of the stiffened panel comprises buckling of the skin between the stiffener flanges, the skin–stiffener flanges, and the stiffener web. The local skin–stiffener interaction is partially accounted for by considering the effect of the stiffener flanges over the skin. In this particular case, the stiffener flanges will act as a reinforcement stiffening up the skin. Global buckling of the stiffened panel is assessed by considering the column and overall shear interaction.

Note that the CF solutions used in this paper provide a good degree of accuracy for flat anisotropic plates with aspect ratios greater than three.

4.1.3.3.1. Buckling of the skin. The skin between the stiffener flanges is assumed to be a long flat plate simply supported along the edges under normal and shear load. In this case, the length of the plate is $a$, and the effective width of the plate is the difference between the stiffener pitch ($b$) and the stiffener flange width ($b_f$). Weaver [8,9] recently provided a comprehensive set of CF solutions for long flexural anisotropic plates under compression and shear loading. Additionally, Weaver [8] detailed a procedure to identify exactly the critical uniaxial compression load.

Normal buckling: Weaver [8] approximated the critical buckling load of a long anisotropic plate with simply supported conditions along the edges and under normal loading as follows:

\[
N_{cr}^x = K_x \frac{\pi^2}{b^2} \sqrt{D_{11}D_{22}},
\] (36)

where $K_x$ is a non-dimensional buckling coefficient calculated by an iteration scheme.

Shear buckling: The critical shear buckling load is taken from Ref. [9] and has the following expression:

\[
N_{cr}^{xy} = K_{xy} \frac{\pi^2}{b^2} \sqrt{D_{11}D_{22}},
\] (37)

where $K_{xy}$ is the non-dimensional shear buckling coefficient. In the case of negative shear the shear-buckling coefficient is calculated assuming that the sign of each ply angle is reversed.

Normal–shear-buckling interaction: The following expression [33] is used to address the interaction:

\[
RF_{ph} = \frac{1}{N_{cr}^x/N_{cr}^{xy} + (N_{cr}^x/N_{cr}^{xy})^2}.
\] (38)

The constraint for the local buckling of the skin, is given by

\[
1 - RF_{ph} \leq 0.
\] (39)

4.1.3.3.2. Buckling of the skin–stiffener flanges. The skin and stiffener flanges are assumed to behave as a flat plate consisting of three contiguous strips with simply supported conditions along the external edges. Fig. 4 shows the loading, material axis and cross-section geometry of this arrangement.
Capey [34] considered the effect of the thickness variation across the width on the longitudinal buckling load. Analytical solutions for isotropic materials were provided for the practical cross-section shown in Fig. 4.

In this paper and as in Ref. [34], the practical cross-section was idealised by a symmetric cross-section as depicted in Fig. 5, assuming that the neutral axis passes through the centre of each of the plate’s strips. Furthermore, since the laminates at the strip edges will present a certain degree of unsymmetry, smeared properties are assumed and the reduced bending stiffness approach [35] is taken. Thus,

\[ l_{sme} = l + t_{sf}, \]

\[ b_{sme} = b_{sf}, \]

\[ [D]_{sme} = [D^*] - [B^*][A^*]^{-1}[B^*] \]

with

\[ [A^*] = [A]_{skin} + [A]_{sf}, \]

\[ [B^*] = \frac{t_{sf}}{2} [A]_{skin} - \frac{t}{2} [A]_{sf}, \]

\[ [D^*] = [D]_{skin} + [D]_{sf} + \frac{t_{sf}^2}{4} [A]_{skin} + \frac{t^2}{4} [A]_{sf}, \]

where \([A]_{skin}\) is the membrane stiffness matrix of the skin, \([A]_{sf}\) is the membrane stiffness matrix of the stiffener flange, \([D]_{skin}\) is the bending stiffness matrix of the skin, and \([D]_{sf}\) is the bending stiffness matrix of the stiffener flange.

The Rayleigh-Ritz (RR) method [35] is used to perform the local buckling analysis. The RR method is based on the principle of minimum potential energy. The potential energy of a system has at equilibrium an extremal value [33]. For the neutral equilibrium the potential energy due to bending \((V_T)\) is balanced by a factor \((\lambda)\) of the work done by the external loads \((W_T)\). Hence,

\[ V_T - \lambda W_T = 0. \]  

(46)

The potential energy due to bending is given by

\[ V_T = \frac{1}{2} \int_{\text{Area}} \left[ D_{ijkl} \frac{\partial^2 w}{\partial x_i \partial x_j}^2 + 2D_{ikl} \frac{\partial^2 w}{\partial x_i \partial y} \frac{\partial^2 w}{\partial x_k \partial y} + D_{ik} \frac{\partial^2 w}{\partial x_i \partial x_k} \right] \, dy \, dx, \]

(47)

where \(D_{ij}\) are the bending stiffness terms, and \(w\) is the out-of-plane displacement.

The work done by the external loads is given by

\[ W_T = \frac{1}{2} \int_{\text{Area}} \left[ N_x \left( \frac{\partial w}{\partial x} \right)^2 + N_y \left( \frac{\partial w}{\partial y} \right)^2 \right. \]

\[ + 2N_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] \, dy \, dx. \]  

(48)

For the solution procedure, the out-of-plane displacement shape is represented by a double sine Fourier series, since it satisfies the simply supported boundary conditions at the external edges. Thus,

\[ w = \sum_{i=1}^{m} \sum_{j=1}^{n} A_{mn} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right), \]  

(49)

where \(A_{mn}\) are undetermined coefficients.

The critical buckling load is given by the lowest value or critical factor \((\lambda^c)\), which is obtained by minimising (46) with respect to the \(A_{mn}\) coefficients. Hence,

\[ \frac{\partial}{\partial A_{mn}} (V_T - \lambda W_T) = 0. \]  

(50)

This provides an eigenvalue problem in \(\lambda\). The smallest non-zero solution is the critical factor. Therefore, the critical buckling load is given by

\[ \{N^c\} = \lambda^c \{N\}. \]  

(51)

In this case, as the plate consists of three strips (two outer and one central), the potential energy and external work are given by the sum of the potential energy and external work of each of the plate’s strips. Note that the bending stiffness of the strips at the edges and at the central strip, are given by \([D]_{sme}\) and \([D]_{skin}\) respectively.

Once the critical buckling factor is identified, the skin–stiffener flanges buckling constraint is expressed as

\[ 1 - \lambda_{sf}^c \leq 0. \]  

(52)
4.1.3.3.3. Buckling of the stiffener web. The stiffener web is assumed to be a long flat plate simply supported along the short edges and one long edge free at the other long edge under normal loading. The critical buckling load per unit length [36] is given by

$$N_{sw}^c = \frac{12}{h_{sw}} \left( D_{66} - \frac{D_{26}^2}{D_{22}} \right).$$

(53)

The buckling load factor for the stiffener web is given by the ratio between the critical and applied load. Hence,

$$\lambda_{sw}^c = \frac{N_{sw}^c}{N_{sw}},$$

(54)

where $N_{sw}$ is the normal load applied at the stiffener web per unit length.

The stiffener web buckling constraint is calculated as follows:

$$1 - \lambda_{sw}^c \leq 0.$$  

(55)

4.1.3.3.4. Column buckling. The stiffened panel is assumed to behave as a wide column with pinned ends. The critical buckling load accounting for the shearing force is given by

$$P_{cr} = \frac{P_e}{1 + (P_e/(AswG_{sw}^{xy}))},$$

(56)

where $P_e$ is the Euler buckling load for a column and $G_{xy}^{sw}$ is the shear modulus of the stiffener web.

The Euler load expression for a column is given by

$$P_e = \frac{\pi^2 EI_c}{a^2},$$

(57)

where $EI_c$ is the longitudinal bending stiffness of the stiffened panel. Following Ref. [38], it can be demonstrated that

$$EI_c = \frac{b}{d_{sk}^{pl}} + A_{sk}E_{sk}^{pl} z_{cg}^2 + \frac{h_{sf}}{d_{sf}^{pl}} + A_{sf}E_{sf}^{pl} \left( z_{cy} - \frac{t}{2} \right)^2$$

$$+ E_x \left( \frac{1}{12} t_{sw} h_{sw}^3 + A_{sw} \left( z_{cg} - \frac{h_{sw}}{2} - \frac{t}{2} \right)^2 \right),$$

(58)

in which, $d_{sk}^{pl}$ and $d_{sf}^{pl}$ are terms of the bending stiffness compliance matrix of the skin and the stiffener flange, $E_{sk}^{pl}$, $E_{sf}^{pl}$ and $E_x$ are the Young’s modulus of the skin, stiffener flange and stiffener web, respectively, and $z_{cg}$ is the centroid of the super-stiffener section.

4.1.3.3.5. Overall shear buckling. The stiffened panel is assumed to be infinitely long with simply supported conditions along the long edges. The critical shear load is taken from Ref. [30]. Hence,

$$N_{sh}^c = K_{sh} \frac{\pi^2}{a^2} \sqrt{\frac{D_{33}^c D_{22}}{D_{11}^c}}.$$  

(59)

where $D_{ij}$ is the longitudinal bending stiffness ($EI_c$) per unit width and $K_{sh}$ is the non-dimensional shear buckling coefficient.

4.1.3.3.6. Column-overall shear buckling interaction. An interaction formula [33] is used to evaluate the global buckling. Thus,

$$RF_{cs} = \frac{1}{F_c / P_{cr} + (N_{sw}^c/N_{sh}^c)^2},$$

(60)

where $F_c$ is the longitudinal force applied at the super-stiffener centroid.

The constraint for the global buckling of the stiffened panel is given by

$$1 - RF_{cs} \leq 0.$$  

(61)

4.1.3.4. Practical design constraints. Practical design rules are taken from Ref. [30]. The design constraints considered are the percentages of ply angles, skin–stiffener flange Poisson’s ratio mismatch and skin gauge.

4.1.3.4.1. Percentages of ply angles. Niu [39] suggested that in composite design at least 10% of each ply angle should be provided. The maximum and minimum percentages of the ply angles for the skin, stiffener flange and stiffener web are limited.

4.1.3.4.2. Skin–stiffener flange Poisson’s ratio mismatch. The reduction of the Poisson’s ratio mismatch is critical in composite bonded structures [39]. The difference between the skin and the stiffener flange Poisson’s ratio is limited by a small number $\zeta$ to reduce the mismatch. An acceptable value of $\zeta$ is assumed to be 0.05.

The Poisson’s ratio mismatch design constraint between the skin and the stiffener flange is given by

$$\left| \nu_{xy}^{sk} - \nu_{xy}^{sf} \right| - \zeta \leq 0.$$  

(62)

The skin and stiffener flange Poisson’s ratios are calculated using CLT. Thus,

$$\nu_{xy}^{sk} = a_{k}^{sk} \frac{a_{11}^{sk}}{a_{11}^{sf}}, \quad k = skin, sf,$$

(63)

where $a_{ij}$ ($i = l; j = 1,2$) are the terms of the in-plane compliance matrix for the skin and stiffener flange laminates, respectively.

4.1.3.4.3. Skin gauge. Ref. [39] stated that the minimum skin gauge is determined by the danger of a puncture due to lightning strike. It suggested that a minimum skin thickness of 3.81 mm should be used.

4.2. Second step—GA-based optimisation

As in Ref. [30], a standard GA [19,20,40] is employed at this step to solve the discrete lay-up optimisation problem. The optimum lamination parameters obtained in the first step are targeted to identify the actual lay-ups for the skin, stiffener flange and web, respectively. The structure of a standard GA is well reported in the literature (e.g. Ref. [20]). A typical structure of a GA consists of: generation of a population, evaluation, elitism, crossover, reproduction and mutation. Note that at this step the GA is applied separately to the skin, stiffener flange and web.
4.2.1. Fitness function

The sum of the squared differences between the optimum and actual lamination parameters, is used as a fitness function. Extra penalty terms are added in the fitness function to account for ply contiguity constraints. Hence,  

\[
f(j\beta) = \sum_{i=1}^{3} w_{f_i}^{A} (\varepsilon_{i} - \varepsilon_{i_{opt}})^2 + \sum_{i=1}^{3} w_{f_i}^{D} (\varepsilon_{i} - \varepsilon_{i_{opt}})^2 + \frac{4}{k=1} \Theta_k,  
\]

where \( j\beta \) is the design variable vector or gene representing the laminate stacking sequence, \( w_{f_i}^{A,D} \) are the weighting factors for the lamination parameters and \( \Theta_k \) are the penalty terms to limit the number of plies of the same orientation stacked together. The value of \( \Theta_k \) is 1, when more than 4 plies of the same orientation are stacked together [13], otherwise it is 0.

4.2.2. Design variables-genes

The design variables are the thicknesses and the 0°, 90°, 45° and -45° ply angles that constitute the laminate stacking sequences for the skin, stiffener flange and web. Those variables are modelled as chromosomes in genes within the GA. The corresponding encoded chromosomes to ply angles are: 1, 2, 3, 4, 5, 6 and 7 for ±45°, 90°, 0°, 45°, -45°, 90° and 0°, respectively.

Fig. 6 shows the modelling of the skin gene. The total skin thickness is given by \( b \), the encoded ply angle is \( \theta \) and \( n \) corresponds to half or half plus one plies depending on whether the skin laminate is symmetric or mid-plane symmetric.

Fig. 7 illustrates the modelling of the genes for the stiffener flange and web, depending on the stiffener type. The variables \( t_o \) and \( t_w \) are defined in Table 1, \( \psi \) and \( \phi \) are the encoded ply angles for the stiffener flange and web, respectively, and \( m \) and \( q \) are half or half plus one plies depending on whether the stiffener flange and web laminates are symmetric or mid-plane symmetric.

5. Numerical examples

Ref. [30] is used to compare results obtained with the two-step optimisation approach herein presented. Material properties are described in Table 2. The length and stiffener pitch of the stiffened panel are 762 and 203.2 mm, respectively. The stiffened panel is under combined loading. The stiffened panel normal (compression) and shear loads are 3502.54 and 875.63 N/mm, respectively.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>AS4/3502 material properties as in Ref. [30]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{11} ) (N/mm²)</td>
<td>127553.8</td>
</tr>
<tr>
<td>( E_{22} ) (N/mm²)</td>
<td>11307.47</td>
</tr>
<tr>
<td>( G_{12} ) (N/mm²)</td>
<td>5990.48</td>
</tr>
<tr>
<td>( v_{12} )</td>
<td>0.3</td>
</tr>
<tr>
<td>( \rho ) (kg/mm³)</td>
<td>1.578 \times 10^{-6}</td>
</tr>
<tr>
<td>( t_p ) (mm)</td>
<td>0.132</td>
</tr>
</tbody>
</table>

Two optimum designs corresponding to stiffener type b under buckling and ply contiguity constraints were taken from Ref. [30] to perform a comparison. Those are detailed in Table 3. The masses of the continuous (\( M_c \)) and discrete (\( M_d \)) optimisation as well as the critical buckling load factors (\( \lambda_{cr}^2 \)) are provided.

Firstly, the selected designs from Ref. [30] were evaluated using the buckling methodology described in Section 4.1.3.3. 225 terms (\( m = n = 15 \)) were considered to provide enough accuracy and used in the double sine series for the RR method. A trade study showed a small error (approx. 0.8%) between 225 and 400 terms in the double sine series. However, when using 225 terms in the series, the computational efficiency increased significantly (approx. 4 times). Results are presented in Table 4. It is clearly observed that results from Ref. [30] obtained using CF solutions to carry out the buckling analysis, show higher load bucking factors than those produced using FE. The main reason for these differences lies in the fact that the CF solutions used in this paper do not fully account for the interaction between the skin and the stiffener. It is well known that the stiffener flange and stiffener web will have an impact on the local and global buckling capabilities of the superstiffener. When a local buckling mode occurs, the skin and stiffener will usually share the same number of longitudinal half wavelengths. This phenomenon is normally associated with a lower energy state than the one resulting from the individual buckling of the skin or the stiffener web. In addition, the stiffener flange might act as a reinforcement increasing locally the stiffness of the skin, therefore improving its resistance to buckling. Considering the buckling of the skin or the stiffener web in isolation implies that either the skin or the stiffener web presents high stiffness and therefore has no contribution to the local
stiffened panel. In contrast, the stiffener shows a high buckling load carrying capability of the skin and hence the no 0 solution the skin laminate presents flexural anisotropy and hence the laminate anisotropy is used to improve the stiffness. This assumption generally leads to conservative designs and hence higher buckling load factors. The CF solutions used in this paper address the effect of the stiffener flange on the local buckling of the skin, when it acts as a reinforcement.

Secondly, the optimisation approach herein described was applied considering buckling, ply contiguity constraints and locating at least one set of ±45° plies at the outer surface of the skin and stiffener laminates. The first step was set up using stiffener type b and 225 terms (m = n = 15) in the double sine series for the RR method. The stiffener flange width was fixed to 60.96 mm as in Ref. [30]. The lower bound for the stiffener web height was set up to 75 mm. At the second step, a GA code was used with a population of 40, 200 generations, a 0.7 probability of crossover, a 0.05 probability of mutation and assuming that all weighting factors for the lamination parameters were equal to 1. Table 5 details the optimum designs obtained.

It is clearly seen that the optimum design in Table 5 is lighter than the CF design from Ref. [30], and offers a potential mass saving of approximately 11%. However, if it is compared to the FE design from Ref. [30], a mass penalty of approximately 5% is observed in contrast to the 18% associated with the standing alone CF solutions used in Ref. [30]. This suggests that the combination of CF solutions and an energy method (RR) employed in this paper to assess buckling can produce results close to FE. Note that in this case although ply contiguity constraints were not included at the first optimisation step, they can still be met at the second step with a negligible mass penalty (0.1%). As in Ref. [30] it is observed that for the optimum solution the skin laminate presents flexural anisotropy and no 0° plies. Laminate anisotropy is used to improve the buckling load carrying capability of the skin and hence the stiffened panel. In contrast, the stiffener shows a high percentage of 0° plies and no 90° plies. As one might expect, the skin loses stiffness in the longitudinal direction whilst simultaneously improving its buckling resistance. This effect is compensated by increasing the stiffness of the stiffener in the longitudinal direction to prevent global buckling failure. An FE model was set up in MSC/NASTRAN [41] following Ref. [30] to evaluate the design buckling performance. The critical load factor is shown in brackets in Table 5. For this design, the buckling failure mode is a local–global combination. This suggests that the FE technique used in Ref. [30] might provide conservative results if the global buckling failure is close to the driving mode of failure. The thicknesses and lamination parameters for the first and second optimisation steps are given in Table 6. A good correlation is seen between the lamination parameters at both steps.

Finally, the effect of the stiffener type on the optimum design under strength, buckling and practical design rules was evaluated. Optimum designs using CF solutions were taken from Ref. [30], and shown in Table 7. In addition to the masses of the continuous and discrete optimisation, the critical buckling and strength (\(\lambda_{0}^{cr}\)) load factors are provided. For this case, at the first step, the stiffener flange width was freed and considered as a design variable. As previously stated, 225 terms (m = n = 15) were used in the double sine series for the RR method. The lower bounds for the stiffener flange width and web height were set as 60 and 70 mm, respectively. Common aerospace design strain levels of 3600 micro-strains (\(\mu\)) in both tension and compression and 7200 \(\mu\) in shear were imposed. Stacking sequence constraints such as ply contiguity and at least one set of ±45° plies at the outer surface of the skin and stiffener laminates were added at the second step. Table 8 shows the optimum designs obtained using this two-step optimisation approach.

In this case, the optimum designs obtained do not differ significantly from those found in Ref. [30]. Potential mass savings are found in both the continuous and discrete steps (max. approx. 2%). In addition, there is a redistribution of the material between the skin and the stiffener. This is thought to be due to the stiffening effect of the stiffener flanges over skin, which is included in the optimisation. As stated in Ref. [30], the driving design constraint is strength. It is also seen that the stiffener type has an impact on the design. Designs with stiffener type c are the lightest whereas designs with stiffener type b are the

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### Table 3
Optimum superstiffener type b designs with buckling and ply contiguity constraints

<table>
<thead>
<tr>
<th>Method</th>
<th>(M_{l}/M_{d}) (kg)</th>
<th>(\lambda_{0}^{cr})</th>
<th>(h_{sw}) (mm)</th>
<th>Lay-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE</td>
<td>2.45/2.51</td>
<td>0.99</td>
<td>74.77</td>
<td>Skin (33 plies) [±45/90/45/90/45/±45/−45]MS</td>
</tr>
<tr>
<td>CF</td>
<td>2.94/2.96</td>
<td>0.99</td>
<td>75</td>
<td>Skin (45 plies) [±45/45/90/45/90/45]MS</td>
</tr>
</tbody>
</table>

### Table 4
Comparison of buckling results (FE buckling load factors are shown in brackets)

<table>
<thead>
<tr>
<th>Ref. [30]</th>
<th>(\lambda_{0}^{cr})</th>
<th>(RF_{cs})</th>
<th>(\lambda_{0}^{cr})</th>
<th>(RF_{pb})</th>
<th>(\lambda_{0}^{cr})</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE</td>
<td>(0.99)</td>
<td>1.83</td>
<td>1.23</td>
<td>0.74</td>
<td>0.67</td>
</tr>
<tr>
<td>CF</td>
<td>0.99 (1.32)</td>
<td>1.83</td>
<td>2.55</td>
<td>2.31</td>
<td>0.99</td>
</tr>
</tbody>
</table>
heaviest. The difference between these two optimum designs is approximately 6%. Note that in the cases of stiffener types *c* and *d*, the stiffener flange minimum thickness was considered to be at least 4 plies. It is observed that for these two stiffener types the thickness of flanges tended to a minimum. This suggests that, in this case, the stiffener flanges might not be needed. However, if T-shaped stiffeners are used the stiffener flanges have to provide a certain degree of integrity to the joint with the skin.

As previously stated, an FE model was set up in MSC/NASTRAN following Ref. [30] to evaluate the buckling performance of the designs. The critical load factors are shown in brackets in Table 8. It is clearly seen that no buckling failure occurs. Table 9 collects the thicknesses and lamination parameters for the first and second optimisation steps. Adequate to good agreement is found in all cases.

### 6. Conclusions

A method to perform initial sizing optimisation for anisotropic composite panels with T-shaped stiffeners was developed. The optimisation problem was divided into two steps. At the first step, the stiffened panel was optimised using MP techniques and lamination parameters accounting for their membrane and flexural anisotropy. The stiffened panel was assumed to be long, wide and composed of several superstiffener elements. The panel was subjected to a combined loading under strength, buckling, stiffener manufacturability and practical design constraints. The skin and stiffener laminates were assumed to be symmetric, or mid-plane symmetric laminates with 0°, 90°, 45°, or −45° ply angles. Dimensions and lamination parameters for an optimum superstiffener design were obtained. At the second step, a GA code was used to target the optimum lamination parameters to find the
actual lay-ups for both the skin and the stiffener. Manufacturing requirements for the stiffener were considered.

The combination of CF solutions and an energy method (RR) for buckling analysis presented in this paper was able to capture the behaviour of the skin when the stiffener

<table>
<thead>
<tr>
<th>Stiffener type</th>
<th>$M_k/M_d$ (kg)</th>
<th>$\zeta^*_b$</th>
<th>$\zeta^*_l$</th>
<th>$b_{of}$ (mm)</th>
<th>$h_{sw}$ (mm)</th>
<th>Lay-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2.90/3.01</td>
<td>1.27 (1.20)</td>
<td>0.98</td>
<td>60.01</td>
<td>70</td>
<td>Skin (52 plies) ${\pm45/45}/{\pm45/90/(50/0)}/{\pm45}$</td>
</tr>
<tr>
<td>b</td>
<td>2.94/3.02</td>
<td>1.06 (1.35)</td>
<td>0.99</td>
<td>60</td>
<td>70</td>
<td>Skin (52 plies) ${\pm45/45/90/45/(45/0)}/{\pm45/90}$</td>
</tr>
<tr>
<td>c</td>
<td>2.74/2.86</td>
<td>1.02 (1.03)</td>
<td>0.99</td>
<td>60</td>
<td>70</td>
<td>Skin (59 plies) ${\pm45/(45/0)}/{\pm45}$</td>
</tr>
<tr>
<td>d</td>
<td>2.75/2.97</td>
<td>1.12 (1.13)</td>
<td>0.99</td>
<td>60</td>
<td>70</td>
<td>Skin (57 plies) ${\pm45/45/90/(45/02)}/{\pm45/(45/0)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 8</th>
<th>Optimum superstiffener designs under buckling, strength and practical design constraints (FE buckling load factors are shown in brackets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffener type</td>
<td>$h$ (mm)</td>
</tr>
<tr>
<td>----------------</td>
<td>--------</td>
</tr>
<tr>
<td></td>
<td>$\xi_A$</td>
</tr>
<tr>
<td>a</td>
<td>6.8340</td>
</tr>
<tr>
<td>b</td>
<td>6.8640</td>
</tr>
<tr>
<td>c</td>
<td>5.1024</td>
</tr>
<tr>
<td>d</td>
<td>5.5440</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 9</th>
<th>Thicknesses and lamination parameters for optimum superstiffener designs under buckling, strength and practical design constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffener type</td>
<td>$h$ (mm)</td>
</tr>
<tr>
<td></td>
<td>$\xi_A$</td>
</tr>
<tr>
<td>a</td>
<td>7.7157</td>
</tr>
<tr>
<td>b</td>
<td>7.8880</td>
</tr>
<tr>
<td>c</td>
<td>8.3352</td>
</tr>
<tr>
<td>d</td>
<td>8.5800</td>
</tr>
</tbody>
</table>
flanges acted as a reinforcement. Considering this effect in the optimisation showed an improvement in performance when compared with other work (Ref. [30]). The optimised stiffened panel obtained under buckling and ply contiguity constraints with this combination of CF solutions and RR was approximately 11% lighter than the optimised solution reported in Ref. [30] using just CF solutions. It was seen that this optimum was close to the one produced with FE (approx. 5% in contrast to 18% using stand alone CF solutions as in Ref. [30]). The inclusion of membrane and flexural anisotropy in the optimisation procedure showed that elastic tailoring could be used to an advantage.

When the stiffened panel was optimised under strength, buckling and practical design constraints, potential mass savings (max. approx. 2%) was found when compared to those reported in Ref. [30]. It was observed that a redistribution of the material between the skin and the stiffener took place. It was also seen that the design was driven by strength constraints and the stiffener manufacture had an impact on mass (approx. 6%).

In general terms, good agreement was found between the lamination parameters obtained at the first and second steps. Sometimes the lamination parameters at both steps did not match completely; nevertheless good designs could still be produced. Note that the designs at the first step will always be lighter than the second step designs since at the latter step a rounding process occurs.

The benefits of using CF solutions are that they provide an insight into the buckling problem, increase computational efficiency and can provide good initial starting points for more computational demanding optimisations (FE driven).

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