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STEPPED SINE TESTING USING RECURSIVE ESTIMATION

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Measuring frequency response functions (FRFs) of structures using stepped sine excitation is an accurate but time-consuming process. At each frequency step an adequate time must be allowed for transients to die away before the force and acceleration signals are sampled and processed. Most stepped sine analysers require the user to specify a global time delay and sampling time for all frequencies at the start of a test.

This paper describes how the time required to perform a stepped sine test may be reduced by recursively estimating the FRF value at a given excitation frequency. Once this value has converged the next frequency may be tested. Thus the time taken to measure the FRF at each frequency is not fixed, but is the time required to predict the FRF value to the selected accuracy. The critical choice in the recursive algorithms is the rate at which the initial transient signal is forgotten. This must be a compromise between speed of convergence and noise rejection properties. The technique is demonstrated using simulated and experimental data.

1. INTRODUCTION

Measuring frequency response functions (FRFs) of structures using stepped sine excitation is a mature technique. The resulting measurements have a high signal to noise ratio and the frequency spacing may be optimised, during testing, if necessary. Stepped sine testing is also the best method to measure the behaviour of non-linear systems. The disadvantage of the stepped sine testing method is the time required to perform a test. In a standard test the structure is excited by each frequency in turn and a (hopefully!) short period of time is required to allow the transient signals to die away. Once a steady state is reached the force and acceleration signals may be sampled and then processed. The time required to reach steady state is determined, predominantly, by the damping in the structure and the closeness of the excitation frequency to a structural natural frequency. Normally a global time delay and sampling time has to be specified for all frequencies at the start of a test.

This paper describes how the time required to perform a stepped sine test may be reduced by recursively estimating the FRF value at a given excitation frequency. Once this value has converged the next frequency may be tested. Thus the time taken to measure the FRF at each frequency is not fixed, but is the time required to predict the FRF value to the selected accuracy. A 'forgetting factor' is included so that the initial transient signals are gradually forgotten and convergence is faster. The measured data may be included in a parameter estimation scheme in two ways. Each point may be incorporated into the estimation individually as it is measured. Alternatively the measurements may be processed in blocks consisting of a single period of the excitation signal sinusoid. Processing the data in these blocks simplifies and speeds up the calculation, but requires a reasonable number

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TABLE 1

The modal model for the simulated example

Mode no.	Frequency (Hz)	Damping (%)	Residue
1	27·8	1·88	1
2	29·9	0·71	2
3	29·9	1·54	−1
4	35·1	0·65	1
5	36·3	1·54	−3
6	41·3	1·24	−1
7	42·9	0·18	2
8	46·9	1·42	1
9	53·7	0·52	4
10	55·4	1·13	−3
11	57·3	1·19	−1

of full periods of the sinusoid to confirm convergence. This paper describes the theory of recursive stepped sine testing and also demonstrates the technique using simulated and experimental data.

2. TIME REQUIRED FOR STEPPED SINE TESTING

In a standard stepped sine test the structure is excited by a sinusoidally varying force at a fixed frequency. After allowing for transient motion in the structure to become negligible the structure will respond in a steady state. For a linear system the steady state response will only be at the excitation frequency. Non-linear systems will also respond at harmonics of the excitation frequency. The time required to undertake the test is dictated by two factors: the time required for the transient motion to decay, which depends on the structure's damping, and the time necessary to estimate the FRF parameters, which depends on the measurement noise and the required accuracy of the FRF. In any structure the excitation frequency significantly effects the extent to which the transient motion is excited. In lightly damped systems these transients will also require a considerable time to become negligible.

To demonstrate the potential time savings, the transient decay time for a typical test scenario will be computed. The natural frequencies, damping ratios and residues for this example are given in Table 1. The frequencies and damping ratios are for a space shuttle payload mounted on a pallet [1]. The residues were chosen arbitrarily. Figure 1 shows the resulting FRF between 20 and 70 Hz. Suppose a stepped sine test consisting of 100 steps was performed to obtain the FRF given in Fig. 1. Figure 2 shows the time required for the magnitude of the transients to decay to 1% of the maximum magnitude of the FRF.

TABLE 2

Comparison of testing time required

No. of frequencies	Test time (s) variable delay	Test time (s) fixed delay	Ratio
100	143	694	4·9
200	267	2048	7·7
300	378	3129	8·3
400	487	3988	8·2
500	591	4747	8·0

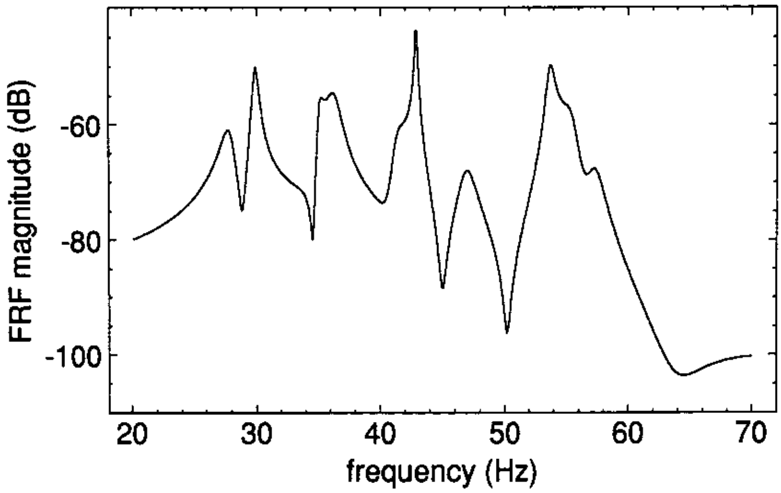

Figure 1. FRF magnitude used in transient decay example.

To gain some idea of the length of a test the times given in Fig. 2 may be added together. To this the measurement time must be added. Assuming that 1 s is sufficient to measure the response parameters at each frequency then the total test time for this example is 143 s. If the delay before measurements are taken is fixed for all frequencies at the largest time required then the equivalent test time is approximately 694 s. Table 2 shows the estimated test times for different numbers of test frequencies.

These results show that considerable time savings may be obtained by varying the time allowed for transients to decay as the test progresses. Another important advantage is that accuracy of the resulting FRF may be specified rather than an arbitrary time delay. This paper considers using a recursive estimator to determine when the transient becomes negligible. Other methods to achieve a similar effect involve checking the coherence of the measured FRF or computing a moving average. Only one recursive estimator is considered in this paper. Friswell and Penny [2, 3] consider a second recursive estimator.

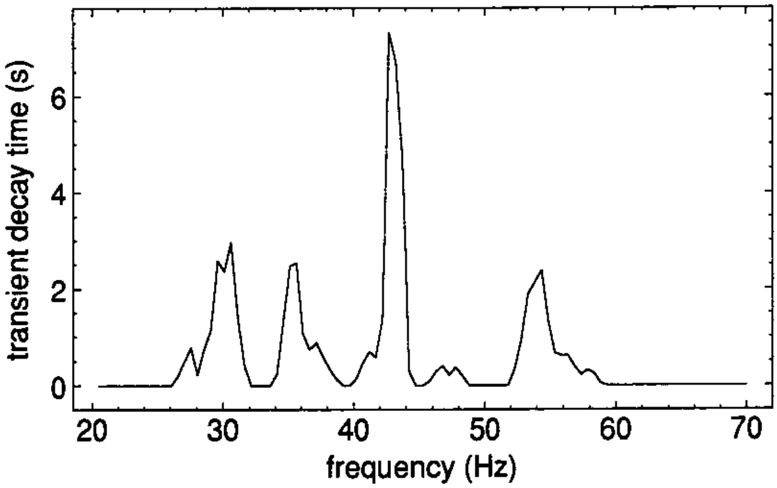

Figure 2. Time for transients to decay to 1% of maximum FRF value.

3. BLOCK RECURSIVE ESTIMATION

To obtain the FRF the response at the fundamental excitation frequency must be extracted. The higher order FRFs may also be obtained if required, but are not considered further in this paper. The force and acceleration signals are assumed to be of the form

$$x(t) = a \cos \omega t + b \sin \omega t + c + \text{harmonics} + \text{noise} \tag{1}$$

where a, b and c are constants fixed for a particular signal at a given frequency, ω. The standard method for extracting the a and b coefficients is derived from Fourier series analysis. The constant c may be estimated but should be zero for a linear structure and is not considered further. For sampled signals from an experiment the coefficients may be estimated by

$$a_n = \frac{2}{np} \sum_{i=1}^{np} x_i \cos(\omega t_i), \qquad b_n = \frac{2}{np} \sum_{i=1}^{np} x_i \sin(\omega t_i) \tag{2}$$

where: p is the number of samples in time period T; $t_i = i\delta t$, is the ith time sample; $x_i = x(t_i)$, is the ith response sample; $\delta t = T/p$, is the time interval between samples; $T = 2\pi/\omega$, is the time period of the force excitation; n is the number of time periods used.

Equation (2) assumes that the sampling interval is an integer multiple of the excitation frequency. This assumption is reasonable since to satisfactorily output a sinusoid generated digitally, through an analogue to digital converter, the output sample frequency must be an integer multiple of the excitation frequency. Should the hardware available not be able to output at arbitrary time intervals then the closest time interval is chosen and the output frequency noted accordingly.

3.1. STANDARD LEAST SQUARES ESTIMATION

The standard method for calculating a_n and b_n uses all the data at once. Equation (2) may be split to calculate estimates of the parameters after each time period. The following algorithms are developed for a_n only. The algorithms to calculate b_n are analogous. The estimate for a_n given in equation (2) may be split as follows for $n \geqslant 1$ with $a_0 = 0$,

$$a_n = \frac{1}{n} \left\{ \frac{2}{p} \sum_{i=1}^{p(n-1)} x_i \cos(\omega t_i) + \frac{2}{p} \sum_{i=p(n-1)+1}^{np} x_i \cos(\omega t_i) \right\} = \frac{n-1}{n} a_{n-1} + \frac{1}{n} A_n \tag{3}$$

where

$$A_n = \frac{2}{p} \sum_{i=1}^{p} x_{n,i} \cos(\omega t_{n,i})$$

and

$$t_{n,i} = t_{p(n-1)+i} \quad \text{and} \quad x_{n,i} = x(t_{n,i}).$$

Thus $t_{n,i}$ and $x_{n,i}$ are the ith time and response samples of the nth time period. Hence the parameter estimate based on the first n time periods, a_n, is a weighted sum of the parameter estimate based on the first $n-1$ time periods, a_{n-1}, and the estimate based on the nth time period alone, A_n. Notice that the weights depend on the number of time periods already used in the parameter estimates. As n becomes large then the current time period contributes very little to the parameter estimate. The estimate of the parameter a given by equation (3), assuming that $a_1 = A_1$, is the mean of the first n values of A_i.

In a stepped sine test, during the change in excitation frequency the force signal is forced to be continuous, although its derivative is often discontinuous. The displacement and velocity response of the structure will be continuous. The motion resulting after a change in frequency consists of the steady state response at the new excitation frequency and a transient response due to the change in steady state gain and phase. The next section deals with this in more depth. Thus the initial part of a signal is likely to contain unwanted frequencies that are not harmonics of the excitation signal and will lead to incorrect answers. Hence some method is required to gradually reduce the influence of the past signal on the parameter estimates.

3.2. THE RECURSIVE ALGORITHM

One method to gradually forget past measurements is to use fixed weights in equation (3) rather than the weights involving the number of time periods used. Thus, for $n \geqslant 1$,

$$a_n = \lambda a_{n-1} + (1 - \lambda) A_n \tag{4}$$

where λ is the forgetting factor which is between 0 and 1. In equation (4) a_0 is often taken to be the estimate of a for the previous frequency. When $\lambda = 0$ the past values of the parameter estimate are ignored and the parameter estimate is generated from the current time period only. With $\lambda = 1$ the parameter estimate is not changed at all. Applying equation (4) recursively gives

$$a_n = (1 - \lambda)(A_n + \lambda A_{n-1} + \cdots + \lambda^{n-2} A_2 + \lambda^{n-1} A_1) + \lambda^n a_0. \tag{5}$$

Hence the estimate of a is a weighted sum of the first n values of A_i, and henceforth this approach will be referred to as the weighted mean parameter estimation (WMPE) algorithm. The value of the forgetting factor has two important effects; the speed at which data is effectively forgotten and the noise rejection properties. Equation (5) shows that the smaller the value of λ the faster the information is forgotten because more recent data is weighted more heavily than past data. With small values of λ the parameter is effectively estimated with a smaller data set and the algorithm can follow fast changes in the underlying model. Unfortunately if measurement noise is present the algorithm will produce erratic parameter estimates as the estimates try to follow the noise. This is demonstrated using a simulated example in the next section. Friswell and Penny [3] describe an alternative recursive estimator and compare its performance with the estimator described above.

The algorithm may be stated in one of two ways; either the estimates for the previous frequency may be used, on the assumption that they will be reasonably close to the new estimates, or a standard least squares used for an initial fixed number of time periods. Both methods will be demonstrated in the next section.

4. POINT BY POINT RECURSIVE ESTIMATION

The estimation method given in the previous section only changes the parameter estimates once per period of the excitation force signal. The alternative is to update the estimates of a and b every time the data is sampled. Generally the block estimation will be the best method for two reasons: the computation involved in updating the parameter estimate after every sampled point is very much greater and parameters may be adversely affected by harmonics of the excitation frequency in the measured data. The effect of harmonics in the measured signals are integrated out over the time period of the excitation force. Estimating the parameters after every sample will not integrate out the harmonics. Estimating the parameters on a point by point basis may be possible for low frequency signals, where sufficient time is available for the extra computation. If a structure is being excited at a low frequency then it contains interesting dynamics at these frequencies. The structure must still be allowed to reach a steady state, which is determined by the damping ratio, not the excitation frequency.

5. A SIMULATED EXAMPLE

The recursive stepped sine testing algorithm will be applied to the response of a simulated 1 dof mass, spring and damper system. With sinusoidal forcing, the equation of motion of such a system is given by

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f(t) = \sin \omega t \tag{6}$$

where ω_n is the natural frequency, ζ is the damping ratio and $f(t)$ is the force per unit mass. Without loss of generality the mass is assumed to be unity in the following examples. Assume that for $t < 0$ the forcing is at frequency ω_1 and for $t > 0$ the forcing is at frequency ω_2. Also assume that before the frequency change at $t = 0$ the system is in steady state so that the response is at frequency ω_1 and is given by

$$x(t) = \frac{-2\xi\omega_n\omega_1 \cos \omega_1 t + (\omega_n^2 - \omega_1^2) \sin \omega_1 t}{(\omega_n^2 - \omega_1^2)^2 + (2\zeta\omega_n\omega_1)^2}. \tag{7}$$

From equation (7) the displacement and velocity at $t = 0$ may be calculated and used as the initial conditions for the repsonse for $t > 0$, when the system is excited with the second frequency ω_2. The form of the displacement is

$$x(t) = \frac{-2\zeta\omega_n\omega_2 \cos \omega_2 t + (\omega_n^2 - \omega_2^2) \sin \omega_2 t}{(\omega_n^2 - \omega_2^2)^2 + (2\zeta\omega_n\omega_2)^2} + e^{-\zeta\omega_n t}(P \cos \omega_d t + Q \sin \omega_d t) \tag{8}$$

where $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ and P and Q are constants that are calculated from the initial conditions obtained from equation (7). The first term in equation (8) is the steady state response and the second term the transient response due to the change in frequency. If the FRF is changing rapidly, for example near a resonance, then to produce a continuous response and velocity at $t = 0$, the level of the transient signal must increase. The 1 dof example will be used to demonstrate the effect of the excitation frequency on the excitation of transients and how this effects the identification of the steady state response. The effect of measurement noise will also be demonstrated. The simulated system has a natural frequency, ω_n, of 200 Hz and a damping ratio, ζ, of 1%.

In the following examples only parameter a_n will be plotted. Similar results are obtained for the sine coefficient, parameter b_n.

5.1. EXCITATION AWAY FROM RESONANCE

Away from the resonant frequency of the system the change in the FRF is small and should not excite a large transient response. Consider changing from an excitation frequency of 99 to 100 Hz. The transient response is very small and in practical cases is likely to be smaller than the measurement noise. The coefficients of the sine and cosine terms in the signal, a and b, are identified, using as initial values the exact parameters at 99 Hz. The convergence of these estimates is shown in Fig. 3 for different values of forgetting factor. As expected convergence is rapid because the transient signal is negligible and the change in parameters required is small. The smaller the forgetting factor is, the faster the convergence since past results are disregarded more quickly.

5.2. EXCITATION FREQUENCY CLOSE TO RESONANCE

When the structure is excited close to resonance the change in frequency produces a large transient response. The initial parameter values were taken to be the parameters for the response at 199 Hz. The parameters now have to change considerably and Fig. 4 shows the convergence of parameter a for different forgetting factors. Since the transient response is larger the convergence is much slower than the 100 Hz case, shown in Fig. 3. Also this

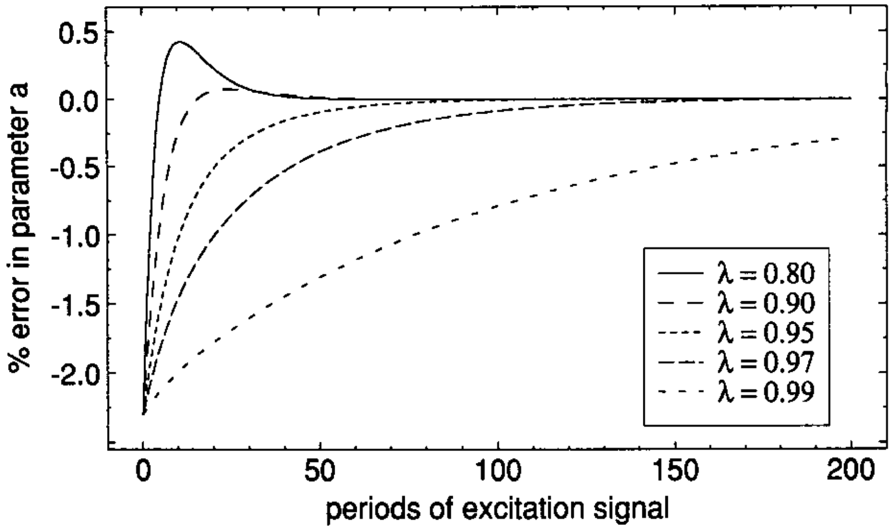

Figure 3. Parameter convergence for different forgetting factors (excitation away from resonance).

transient response is at the damped natural frequency of the system, 199·99 Hz, which is very close to the excitation frequency. Therefore the algorithm cannot readily distinguish between the steady state and the transient response and accurate identification is not possible until damping has made the transient response negligible.

5.3. THE EFFECT OF SYSTEM DAMPING

Increased damping produces a transient response that decays more rapidly. Figure 5 shows the convergence of parameter a for three different damping ratios with a forgetting factor of 0·8. The parameter values are given as a percentage error since the true parameter values are different in each case. The higher the damping, the faster the transient response

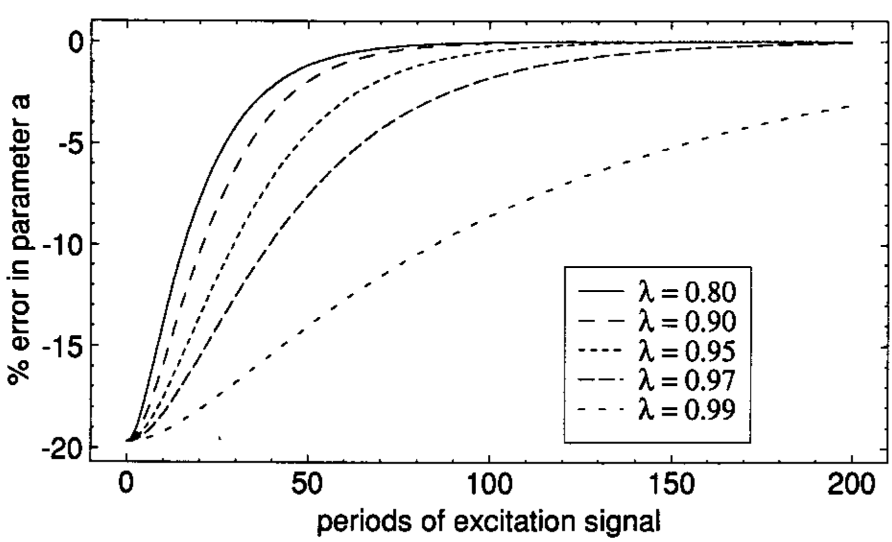

Figure 4. Parameter convergence for different forgetting factors (excitation near resonance).

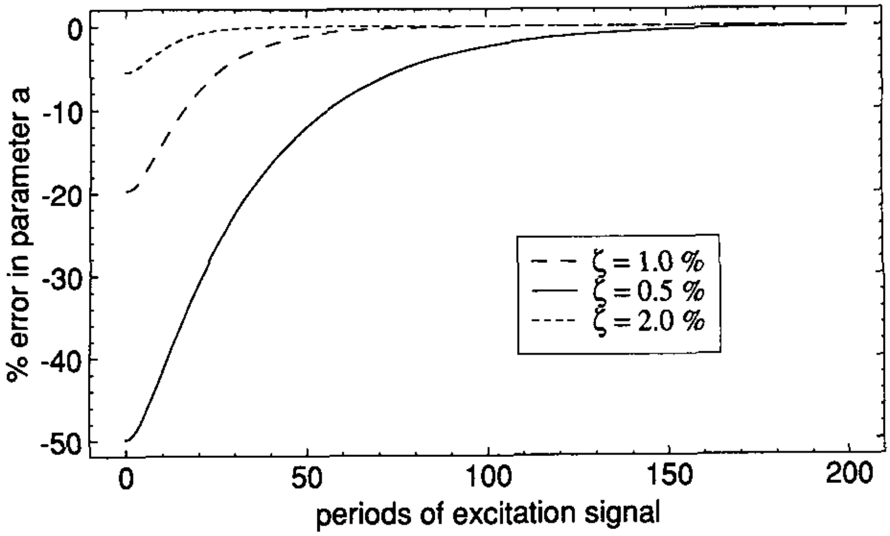

Figure 5. The effect of damping on parameter convergence.

decays and the smaller the change in the FRF. Thus the higher the damping is, the faster the parameter convergence.

5.4. THE CHOICE OF STARTING PARAMETER

The starting value for the parameters may be the parameters derived for the previous frequency or they may be derived from a least squares fit to an initial number of time periods. Using the results of the previous frequency is probably the best procedure apart from when the change in frequency straddles a resonance. Then the phase of the force signal changes by approximately 180° producing an inaccurate initial estimate for parameter b. For a damping ratio of 1% Fig. 6 shows the convergence of parameter a using a different number of time periods for the initial least squares estimation. Also shown in Fig. 6 is the convergence of parameter a using the parameter of the previous frequency initially. Providing the starting parameters are reasonably accurate this seems to provide

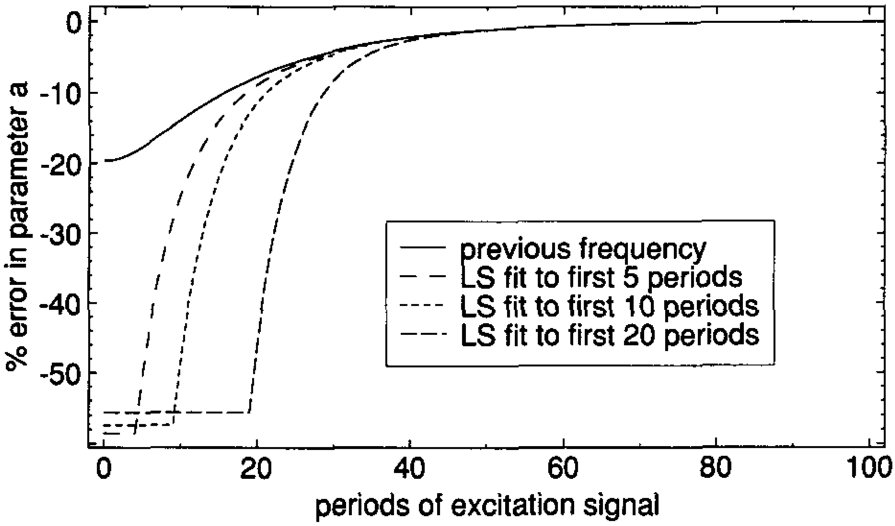

Figure 6. The effect of starting value on parameter convergence.

the best choice. Naturally, because of the forgetting factor, all the estimates based on different starting values converge to the same parameter values eventually.

5.5. THE EFFECT OF MEASUREMENT NOISE

The examples so far have not included measurement noise. The basic trend is that the smaller the forgetting factor the more sensitive the parameter values are to measurement noise. In the worst case the values of the parameters of the steady state response may be totally masked by the measurement noise. To demonstrate these effects, zero mean measurement noise, with a variance of 10% of the maximum response, has been added to the time signals. Figure 7 shows the resulting parameter convergence for different forgetting factors. Quite obviously the smaller the forgetting factor the higher the variance of the converged parameter estimates. Measurement noise is considered further in the next section.

5.6. THE EFFECT OF HARMONICS

Any frequencies in the response signal that is a harmonic of the excitation signal will be integrated out. Sub-harmornics present in the response will affect the parameter estimation. For example, the simulated system above has a natural frequency at approximately 200 Hz. If the system were excited at 1 kHz then the transient response would only integrate out every five time periods. The only difference between the errors produced by such a sub-harmonic and the general transient signal is that the error in the estimated steady state parameters will be structured. Figure 8 shows the parameter convergence when the system with 1% damping is excited by a signal changing from 990 Hz to 1 kHz. The 200 Hz sub-harmonic is quite evident from the oscillation with a time period of five times the force excitation time period. This oscillation seems large in this case because the FRF magnitude is so small.

6. PARAMETER VARIANCES AND CONVERGENCE CRITERIA

The choice of a constant forgetting factor and speed of convergence is dependent on the level of measurement noise and the convergence criteria used. The simulated example

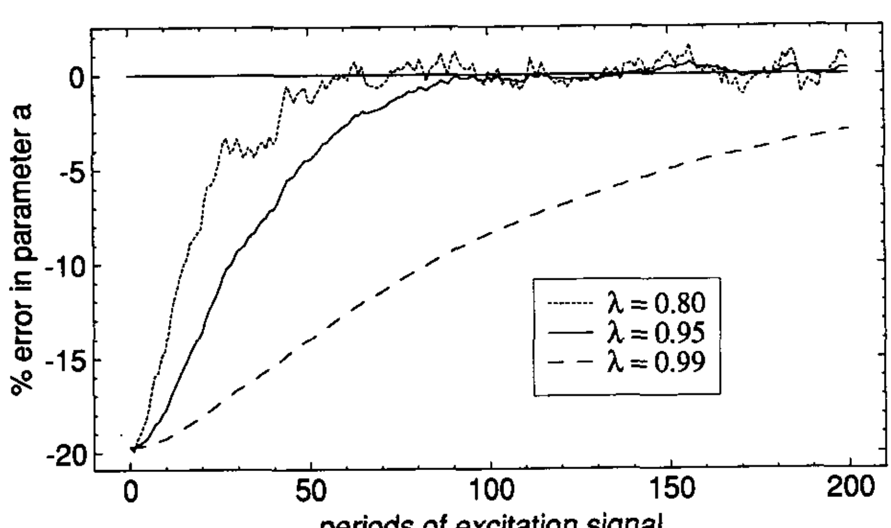

Figure 7. The effect of measurement noise on parameter convergence.

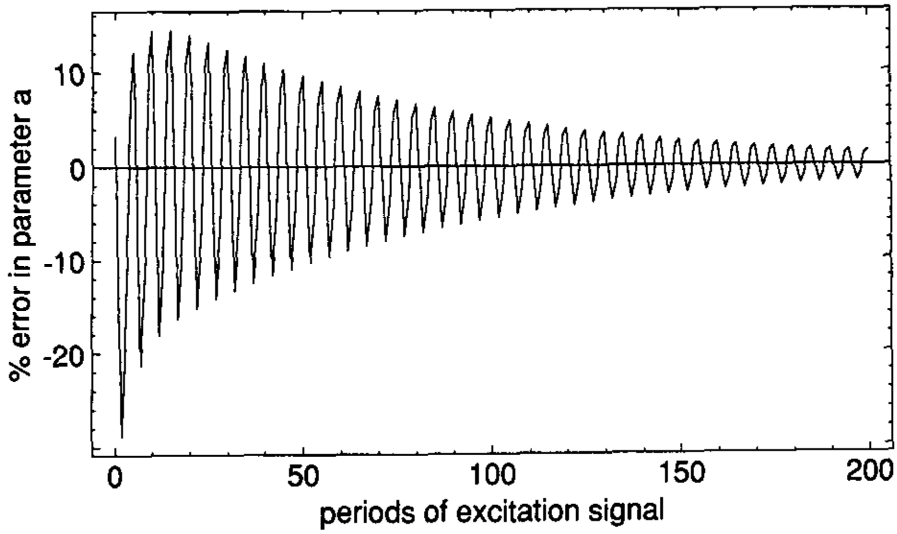

Figure 8. The effect of sub-harmonics on parameter convergence.

showed that if insufficient averaging is performed then the parameter estimates do not converge to a steady value but follow the noise to a certain extent. The forgetting factor should be chosen so that the variance in the identified parameters due to the measurement noise still allows the convergence criteria to be applied. The analysis below assumes a constant forgetting factor using the weighted mean estimation algorithm. For large n, that is when parameter convergence is being investigated, the minimum weighted error approach will be very similar.

6.1. CALCULATING THE VARIANCE OF THE PARAMETER ESTIMATES

Suppose the measurement noise, ϵ, has zero mean and variance σ_ϵ^2. Initially the measurement noise must also take account of the free vibration of the structure contaminating the steady state response, producing a non-zero mean. As the transient response decays then the noise is random with zero mean. Let the exact parameters corresponding to a and b be a^e and b^e. Suppose that the structure has been excited for a sufficient time for the transient response to be negligible. Then the signal based on these exact parameters will be x^e and a^e is given by

$$a^e = \frac{2}{p} \sum_{i=1}^{p} x_{n,i}^e \cos(\omega t_{n,i}) \tag{9}$$

for any time period n, where $x_{n,i}^e = x^e(t_{n,i})$. The following argument will be developed for the parameter a but the same argument will apply to parameter b. The actual signal, x, contains measurement noise and may be written as

$$x(t_{n,i}) = x_{n,i} = x_{n,i}^e + \epsilon_{n,i} \tag{10}$$

where $\epsilon_{n,i}$ is the measurement noise at time $t_{n,i}$. Then the mean, or expected value, of A_n defined in equation (3) is simply a^e. The variance of A_n is given, assuming that the measurement noise at different time samples is independent, by

$$E[(A_n - a^e)^2] = E\left[\left(\frac{2}{p} \sum_{i=1}^{p} \epsilon_{n,i} \cos(\omega t_{n,i})\right)^2\right] = \frac{2\sigma_\epsilon^2}{p} \tag{11}$$

where $E[\,]$ denotes the expected value. If the mean of a_{n-1} is the exact parameter a^e then, from equation (4), the mean of a_n will also be a^e.

Let the variance of a_n be denoted by σ_n^2. Then the variance of a_n may be written in terms of the variance of a_{n-1}, using equations (4) and (11) and assuming the noise at different time samples is uncorrelated, as

$$\sigma_n^2 = E[(a_n - a^e)^2] = \lambda^2 \sigma_{n-1}^2 + (1-\lambda)^2 \frac{2\sigma_\epsilon^2}{p}. \tag{12}$$

Recursively applying equation (12) gives the variance of a_n in terms of the variance of the initial parameter estimate a_0. Thus

$$\begin{aligned} \sigma_n^2 &= \lambda^{2n}\sigma_0^2 + (1-\lambda)^2(\lambda^{2(n-1)} + \lambda^{2(n-2)} + \cdots + \lambda^2 + 1)\frac{2\sigma_\epsilon^2}{p} \\ &= \lambda^{2n}\sigma_0^2 + \frac{1-\lambda}{1+\lambda}(1-\lambda^{2n})\frac{2\sigma_\epsilon^2}{p}. \end{aligned} \tag{13}$$

As n becomes large, assuming that $\lambda \neq 1$ so that the measured data is used to some extent, the variance of the parameters converges to

$$\sigma_\infty^2 = \frac{1-\lambda}{1+\lambda}\frac{2\sigma_\epsilon^2}{p}. \tag{14}$$

Thus for the parameters to converge with a specified variance the value of the forgetting factor must satisfy

$$\lambda > \frac{2\sigma_\epsilon^2 - p\sigma_\infty^2}{2\sigma_\epsilon^2 + p\sigma_\infty^2}. \tag{15}$$

Equation (14) shows that the use of the recursive identification scheme effectively smooths the data. The variance of the parameter estimates are less than the variance of the parameter estimates taken one period at a time, A_n. Unfortunately calculating the forgetting factor using equation (15) requires that the variance of the measurement noise is known *a priori.*

6.2. PARAMETER CONVERGENCE

So far the properties of the recursive identification algorithm have been investigated but the criteria to determine when convergence has been achieved has not been mentioned. It is very important to determine accurately when the parameters have converged. The usual test for convergence in analytical algorithms is to test if adjacent parameter estimates are close. This has two major problems. Due to the measurement noise two subsequent parameter estimates may satisfy the criteria even though convergence has not been achieved. Thus the criterion should be satisfied for more than two points. Secondly, after convergence the variance of the parameter estimates, demonstrated in Fig. 7, may be too high for a chosen convergence criterion ever to be satisfied. Difficulties arise because the measurement noise variance is unknown for a given excitation frequency. One solution would be to estimate the variance of the converged parameters at every frequency. Using equation (14) the measurement noise variance may be estimated and an optimum value for the forgetting factor calculated for the next frequency.

The above criterion uses absolute errors and variances. Often it is preferable to specify a required error in terms of a percentage of the parameter value. This is easily incorporated into the algorithm provided a minimum value is specified for zero parameters. Rather than consider a percentage of each individual parameter, taking the percentage of the signal magnitude would be preferable.

7. A VARIABLE FORGETTING FACTOR

Equation (15) suggests that we require to specify the variance of the parameter estimates. Then, from a knowledge or estimate of the measured noise variance, we may calculate a forgetting factor to produce our specified parameter variance. This suffers from two problems: knowing the measurement noise variance and estimating the required parameter variance. Equation (15) may still be used to give a good guide to the values of the forgetting factor.

An alternative approach is to change the value of the forgetting factor as the estimation proceeds. The value of the forgetting factor should be increased to help the smoothing process. During the initial transient, when the estimated variance is high, the value of the forgetting factor should be chosen to forget the contaminated data as quickly as possible. Fortescue *et al.* [4] suggest the error between the parameter estimate based on the previous data and the parameter estimate based on the data in the current time period provides a good basis to calculate the forgetting factor. If this error is high then the parameter is changing rapidly and so the forgetting factor should be low. If the error is low then the parameter is likely to have settled down, the error is likely to be noise and the forgetting factor should be high. The simplest variable forgetting factor for the weighted mean algorithm is

$$\lambda_n = 1 - \frac{(a_{n-1} - A_n)^2}{\Sigma_0} \tag{16}$$

where Σ_0 is chosen based on the speed of convergence required and the measurement noise. Using a variable forgetting factor, such as equation (16), could produce values outside the interval [0, 1] and so limits must be placed on the value of the forgetting factor.

The minimum weighted error approach suggested by Fortescue *et al.* [4] and Friswell and Penny [2, 3] may have some advantage in this application as the 'first order filter' effect of the intermediate variable will smooth out the effective forgetting factor, [3], Figure 9

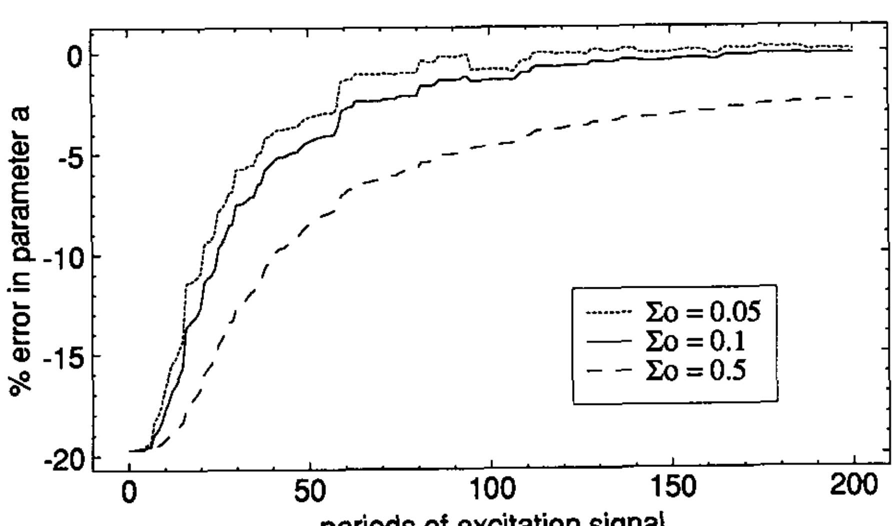

Figure 9. The effect of a variable forgetting factor on parameter convergence.

shows the convergence of the parameter a for the case of 1% damping and an excitation frequency changing from 199 to 200 Hz. Here Σ_0 is given by 0·05, 0·1 and 0·5 times the magnitude squared of the initial parameter values, $a_0^2 + b_0^2$. Notice that the choice of Σ_0 is critical to the satisfactory convergence of the parameters. Using the variable forgetting factor has had little benefit since we still have to choose a value for Σ_0. Figure 10 shows the value of the forgetting factor for the middle value of Σ_0.

8. FRF ESTIMATION

So far this paper has discussed the estimation of the fundamental Fourier coefficients of time signals. To estimate the FRF of a structure at a particular frequency both the force into the structure and the resulting acceleration must be analysed. On convergence the FRF may be estimated as

$$\alpha = \frac{a_a - jb_a}{a_f - jb_f} \tag{27}$$

where a_a and b_a are the parameters of the acceleration signal and a_f and b_f are the parameters of the force signal. In general the response of the structure will contain more transient signals than the force measurement. The convergence criterion must be satisfied by both signals and there is no real advantage in considering the convergence of the FRF itself.

9. EXPERIMENTAL RESULTS

The algorithms outlined in this paper were tested on a free–free aluminium alloy beam measuring 25 × 50 × 800 mm. The beam was forced and the response measured at approximately the beam mid-point using a CED 1401 + data acquisition system. The first structural natural frequency of the beam is at approximately 200 Hz.

The first test involved an excitation frequency change from 199 to 200 Hz. Figure 11 shows the convergence of the estimate of parameter a for the force signal using the weighted mean method with forgetting factors of 0, 0·8 and 0·95. The zero forgetting

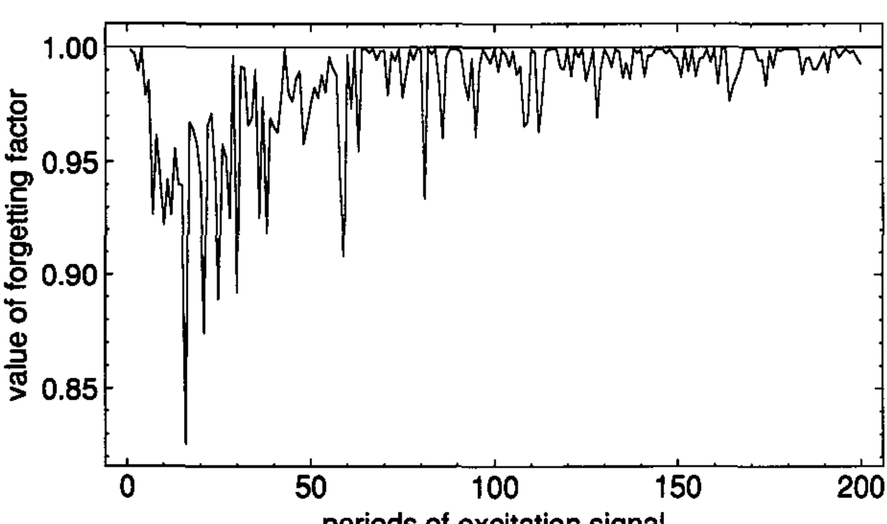

Figure 10. The variation of the variable forgetting factor.

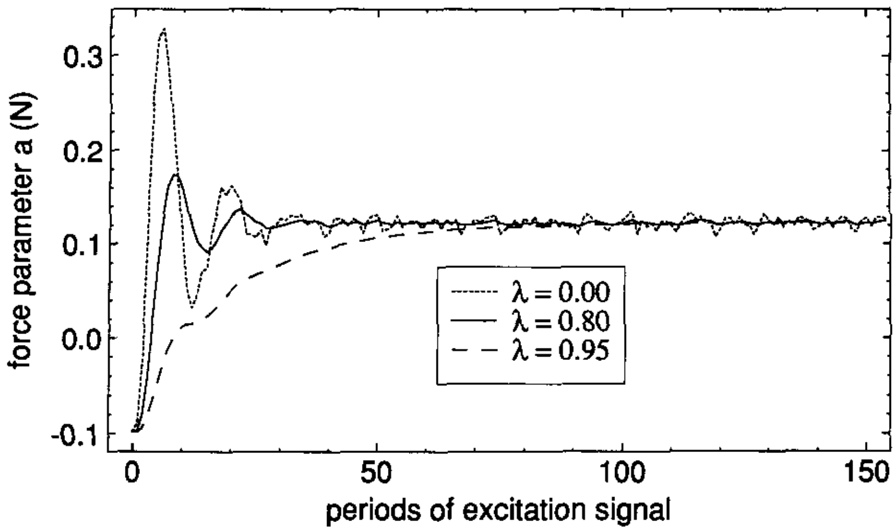

Figure 11. An experimental example of parameter convergence (excitation near resonance).

factor produces the raw estimated data per time period. Although the other parameters could be plotted the results are similar to Fig. 11. The oscillation over approximately 12 time periods in the raw data shows that there is a rigid body mode at approximately 17 Hz. Although the oscillation could be eliminated using a more carefully designed experiment, it is instructive to retain it. Figure 12 shows the convergence of the corresponding parameter estimates after an excitation frequency change from 99 to 100 Hz using the same values of forgetting factors. The measurement noise and the smoothing effect of the forgetting factor are clear.

10. CONCLUSION

This paper has introduced the possibility of recursively estimating the Fourier coefficients of the excitation frequency during a stepped sine test. Incorporating such an

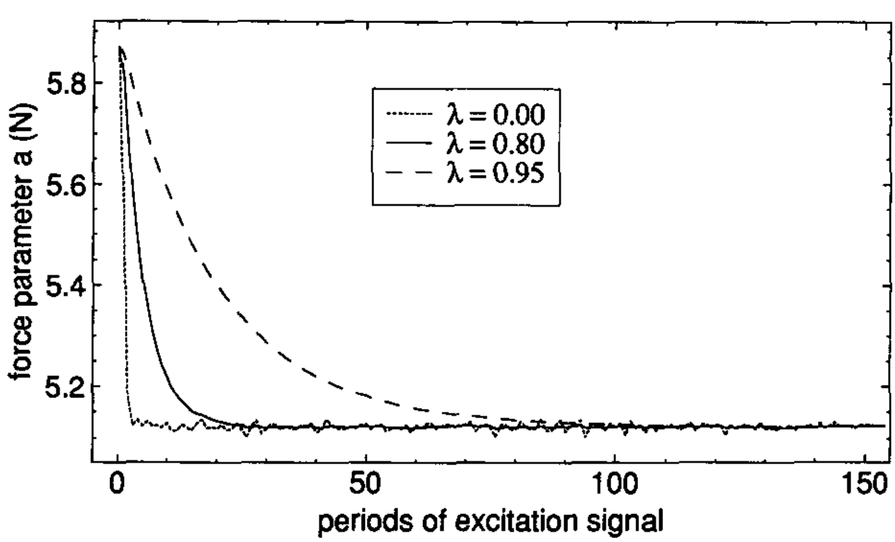

Figure 12. An experimental example of parameter convergence (excitation away from resonance).

estimator should reduce the time to complete a test and the user should be more confident in the accuracy of the results. The critical choice in the recursive algorithms is the value of the forgetting factor, which must be a compromise between speed of convergence and noise rejection properties. Using a variable forgetting factor only marginally improves the situation as a parameter must still be specified to account for measurement noise. Research is continuing on different recursive algorithms that automatically adjust to account for measurement noise and the specification of convergence criteria.

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