Mechanical Systems and Signal Processing (1993) 7(2), 161–172

UPDATING MODEL PARAMETERS BY ADDING AN IMAGINED STIFFNESS TO THE STRUCTURE

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(*Received* 14 *October* 1991, *accepted* 11 *June* 1992)

A mass addition technique for structural parameters updating has recently been described by the authors. It uses eigenvalues of the structure, before and after it is perturbed by adding mass, to adjust selected parameters by sensitivity analysis. The technique avoids the use of noisy mode shape data and overcomes the problem of a non-unique set of parameters when eigenvalues alone are used. However, it is not suitable for a large structure because of the difficulty of adding the necessarily large perturbing mass to the existing structure. This paper presents an alternative technique. An "imagined" stiffness is added to the structure and the FRF of this perturbed structure is obtained from the measured FRF of the original structure by a simple structural modification technique. The eigenvalues of the original and perturbed structures are obtained from the measured and constructed FRFs respectively. These eigenvalues, together with the eigenvalues predicted from an analytical model of the structure, are used to adjust the structural parameters by sensitivity analysis. The technique is demonstrated by a simulated example and by an experiment on an H-frame.

1. INTRODUCTION

Frequency response data and modal data are widely used to update dynamic model parameters. The frequency domain algorithms avoid the additional task of extracting the modal data from the frequency response data and the relatively large modal analysis errors in the mode shape data. Frequency domain algorithms do, however, have limitations. For example, in algorithms based on the equation error formulation [1–4], measurement noise results in dependent and non-zero mean equation errors and produce biased parameter estimates. Furthermore, the response data at the unmeasured co-ordinates must be estimated or the model reduced to the measurement co-ordinates. These limitations are not apparent in algorithms based on the output error formulation although the output error algorithm is time consuming and has convergence problems.

Algorithms using modal data rely heavily on the measured eigenvalues and can produce non-unique sets of parameters [5]. The mode shape data is relatively inaccurate and must be weighted accordingly. One solution is to give some weight to the initial analytically derived parameters. An alternative approach is to increase the amount of more accurate frequency information. The mass or stiffness addition technique, which has recently been presented by the authors [6, 7], can update the parameters using eigenvalues alone. The technique uses eigenvalues of the structure before and after the structure and its theoretical model are equally perturbed by adding known masses. Using this approach, at least as many eigenvalues as the number of unknown parameters can be generated. It has been shown that, by a proper choice of the perturbing co-ordinates, error-free eigenvalues result in parameter convergence to exact simulated values.

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The mass or stiffness addition technique is most practical using additional mass rather than stiffness but the large mass necessary to perturb the eigenvalues of a large or heavy structure sufficiently cannot be easily added to the original structure. This paper presents an alternative approach in which grounded stiffeners are imagined to be added to the structure. The procedure is summarised in Fig. 1.

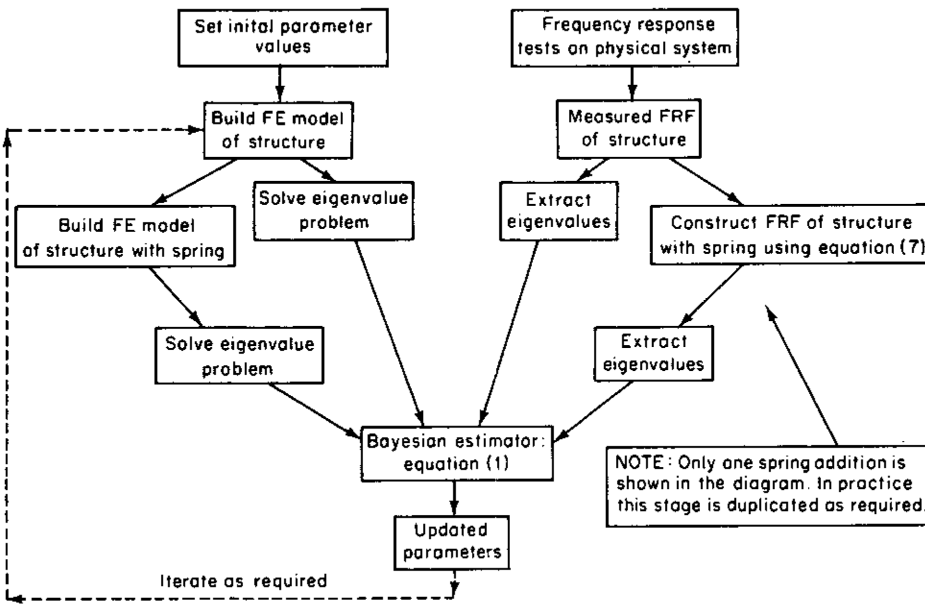


Figure 1. Schematic of updating procedure.

The FRFs of the structure are measured and the eigenvalues obtained by modal analysis from the FRFs. The FRFs of the structure with the stiffeners are then obtained by a simple structural modification technique using the measured point receptances of the original structure at each co-ordinate where a stiffener is added. Thus we are using more data from the original FRFs but at frequencies away from the natural frequencies. The eigenvalues of this perturbed structure are then obtained by modal analysis from these FRFs. Note that, although the FRFs for the perturbed structure are constructed by combining experimental and analytical data, we will refer to the eigenvalues extracted from these FRFs as measured eigenvalues. Finally the measured eigenvalues are compared with the analytical eigenvalues of the corresponding structures derived from an FE model. The mathematical background of this technique and the choice of the perturbing co-ordinates is presented in [7, 8] and is not discussed in this paper. Once the eigenvalues are determined, parameter updates for the current analytical model of a real structure are evaluated using equation (1).

$$\Delta \mathbf{s} = [\mathbf{J}_\lambda^T \mathbf{W}_{\lambda\theta} \mathbf{J}_\lambda + \mathbf{W}_a]^{-1} \{\mathbf{J}_\lambda^T \mathbf{W}_{\lambda\theta} \Delta \lambda + \mathbf{W}_a \{\mathbf{s}_a - \mathbf{s}_{ca}\}\} \tag{1}$$

where: $\mathbf{W}_{\lambda\theta}$, $\mathbf{W}_a$ are the diagonal weighting matrices for the measured eigenvalues and the initial model parameters; $\mathbf{J}_\lambda$ is the Jacobian matrix of eigenvalue sensitivities; $\mathbf{s}_a$ is the vector of parameters of the initial model; $\mathbf{s}_{ca}$ is the vector of the parameters of the current model; $\Delta\lambda$ is the vector of the differences between the measured and analytical eigenvalues and $\Delta \mathbf{s}$ is the vector of increments to update the current parameter estimates.

Equation (1) is an unbiased minimum cost Bayesian estimator which incorporates a constraint of minimum change in the parameters from their initial estimates. If proportional hysteretic damping is assumed, the eigenvalues of the undamped system are the same as the real parts of the eigenvalues of the damped system. The eigenvectors of the undamped and the proportionally damped systems are the same and are orthogonal with respect to the hysteretic damping matrix, **H**, by equation (2).

$$\mathbf{U}_i^{\mathrm{T}}\mathbf{H}\mathbf{U}_j=\mathbf{U}_i^{\mathrm{T}}[\beta_k\mathbf{K}+\beta_m\mathbf{M}]\mathbf{U}_j=\beta_k\omega_j^2+\beta_m=\eta_j \qquad i=j$$
$$=0 \qquad i\neq j \tag{2}$$

where β_m and β_k are the damping proportionality constants with respect to the mass and stiffness matrices respectively. In this case of proportional hysteretic damping, we can use the real parts of the measured eigenvalues to update the undamped model using equation (1). The eigenvalues of the undamped updated model and measured damping factors are used in equation (2) to find a least squares solution of β_m and β_k.

2. EIGENVALUES OF THE STRUCTURE WITH IMAGINED STIFFNESS

2.1. GENERAL APPROACH

Consider a test structure with an additional stiffness k, at a single co-ordinate r and excited by a harmonic force F_j at an arbitrary co-ordinate i as shown in Fig. 2. Let Q_r'' be the force exerted by the structure onto the added stiffness and Q_r' the reaction force exerted on to the structure by the additional stiffness. Q_r' and Q_r'' are the functions of the displacement, q^p of the perturbed structure, and are given by equations (3) and (4) respectively.

$$Q_r'=-kq_r^p \tag{3}$$

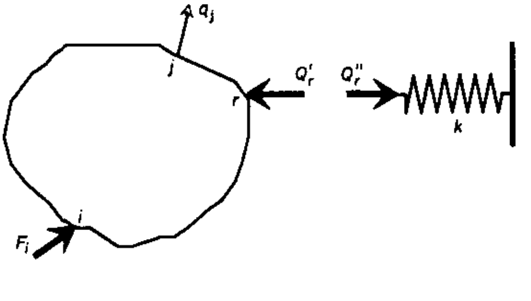
$$Q_r''=kq_r^p. \tag{4}$$

The response at any arbitrary co-ordinate j of the structure with added stiffness, is given by superposition of the responses due to F_i and Q_r', by,

$$q_j^p=\alpha_{ji}F_i+\alpha_{jr}Q_r'$$
$$=\alpha_{ji}F_i-k\alpha_{jr}q_r^p=\alpha_{ji}F_i-k\alpha_{jr}\alpha_{ri}^pF_i \tag{5}$$

where α_{ji} is the receptance relating co-ordinates i and j and the superscript p denotes the perturbed structure. Thus, from equation (5)

$$\alpha_{ji}^p=\alpha_{ji}-k\alpha_{jr}\alpha_{ri}^p. \tag{6}$$



Let the excitation and measurement co-ordinates coincide with the stiffness addition co-ordinate so that $i=j=r$. Equation (6) becomes

$$\alpha^p_{ii}=\frac{\alpha_{ii}}{1+k\alpha_{ii}}. \tag{7}$$

Equation (7) defines the point receptance of the perturbed structure in terms of the point receptance of the unperturbed structure. The eigenvalues of the structure with added stiffness are determined without physical addition of the stiffness, by modal analysis using the point receptance constructed using equation (7). Note, however, that as α^p_{ii} is not linearly related to α_{ii}, measurement errors in α_{ii} with a normal distribution will no longer produce measurement errors in α^p_{ii} with a normal distribution. Thus, while weighting the FRF data to extract the eigenvalues of the unperturbed structure may not be necessary, weighting the FRF data to extract the perturbed eigenvalues may be required. This necessity may be apparent by large randomness in the constructed FRF of the perturbed structure. However, it has been found that it is feasible to measure FRFs with sufficient accuracy to construct the accurate perturbed receptances [8]. An example, in which the experimental eigenvalues were identified by Dobson's method [9] with no weighting in the FRF data, is presented in section 4.

2.2. SIMPLIFIED APPROACH FOR A LIGHTLY DAMPED STRUCTURE

The equation of motion, assuming hysteretic damping and harmonic excitation of frequency ω, at a single co-ordinate i, is given by;

$$[-\omega^2\mathbf{M}+\mathbf{K}+j\mathbf{H}]\alpha=\mathbf{F}_{0i} \tag{8}$$

where $\mathbf{F}_{0i}$ is the forcing vector whose elements are zero except the i-th element which is unity and α is the vector of receptances α_{ij}, $j=1,\ldots,N$. Equation (8) can be rewritten as:

$$[-\omega^2\mathbf{M}+\mathbf{K}+\Delta\mathbf{K}+j\mathbf{H}]\alpha=\mathbf{0}. \tag{9}$$

Where $\Delta\mathbf{K}$ is an N-th order matrix with all elements zero except the i-th diagonal element which is given by $-1/\alpha_{ii}$. But equation (9) is the eigenvalue problem of the perturbed structure where α become the eigenvector and ω the natural frequency of the structure with stiffness k added at co-ordinate i, and where

$$k=-1/\alpha_{ii}. \tag{10}$$

It follows, therefore, any ω^2 in the measurement range can be regarded as the eigenvalue of the structure with added stiffness, where the additional stiffness is given by equation (10). Conversely, for any chosen additional stiffness, we can easily locate the corresponding natural frequencies of the perturbed structure by simply locating the frequencies where equation (10) is satisfied. Unfortunately, for a real additional stiffness, equation (10) is difficult to satisfy because α_{ii} is generally complex due to damping. If damping is light, however, the imaginary parts of the receptances away from the resonance and antiresonance zones are very small and the magnitude of the FRF approaches the FRF of the undamped structure. Thus, we can use equation (10) to approximately locate the natural frequencies of the undamped perturbed structure by treating the magnitudes of the FRF away from the resonance and antiresonance zones as the undamped FRF. This approach to finding the eigenvalues using the FRF of the unperturbed structure is simple and does not require the construction of the FRF of the perturbed structure or modal analysis. Its validity, however, is limited to lightly damped structures.

The general approach of determining the eigenvalues of the structure with added stiffness by constructing the FRF of the perturbed structure and modal analysis (section 2.1), has been successfully implemented and used to update a FE model using simulated error-free data and experimental data [8]. The simplified approach has also been carried out using simulated data (both error-free and with simulated measurement errors) and experimental data. The results are extensively discussed in [8]. This paper presents the results of the implementation by the general approach of constructing the FRF and modal analysis using error-free simulated data and experimental data.

3. SIMULATED EXAMPLE

A free–free beam of length 1·0 m, flexural rigidity $EI = 5000\ \mathrm{Nm}^2$ and mass per unit length $m = 3{\cdot}5$ kg/m was represented by a FE model with 10 dof, composed of four beam elements as shown in Fig. 3. The beam was excited in the transverse direction and longitudinal dof were not included in the model.

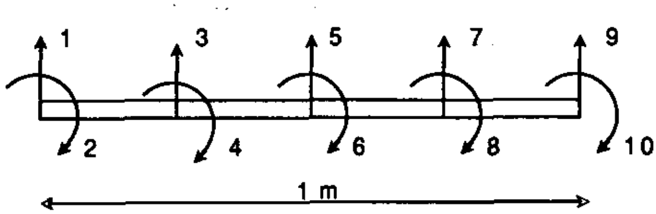


Figure 3. Simulated free beam.

Non-proportional hysteretic damping was simulated in the model by assuming that the proportionality constants between damping and stiffness varied from element to element. The hysteretic damping was included in the model by replacing the real stiffness parameters by the following complex stiffness parameters. Note that element numbers start from the left-hand end in Fig. 3.

$$EI_1 = 5000(1 + j0{\cdot}002)\ \mathrm{Nm}^2 \qquad EI_2 = 5000(1 + j0{\cdot}05)\ \mathrm{Nm}^2$$

$$EI_3 = 5000(1 + j0{\cdot}002)\ \mathrm{Nm}^2 \qquad EI_4 = 5000(1 + j0{\cdot}05)\ \mathrm{Nm}^2.$$

This model was used to simulate the error-free measured FRFs of the non-proportionally damped beam. An initial analytical model was assumed with the same element sub-division but with different set of parameters. It was also assumed that the four elements did not have identical parameters and that damping was zero. The initial mass and real stiffness parameters were:

$$EI_{1a} = 5300\ \mathrm{Nm}^2 \qquad EI_{2a} = 4800\ \mathrm{Nm}^2$$

$$EI_{3a} = 5200\ \mathrm{Nm}^2 \qquad EI_{4a} = 4900\ \mathrm{Nm}^2$$

$$m_{1a} = m_{2a} = m_{3a} = m_{4a} = 3{\cdot}3\ \mathrm{kg/m}.$$

The simulated and the initial analytical model point receptances at co-ordinate 3 are compared in Fig. 4.

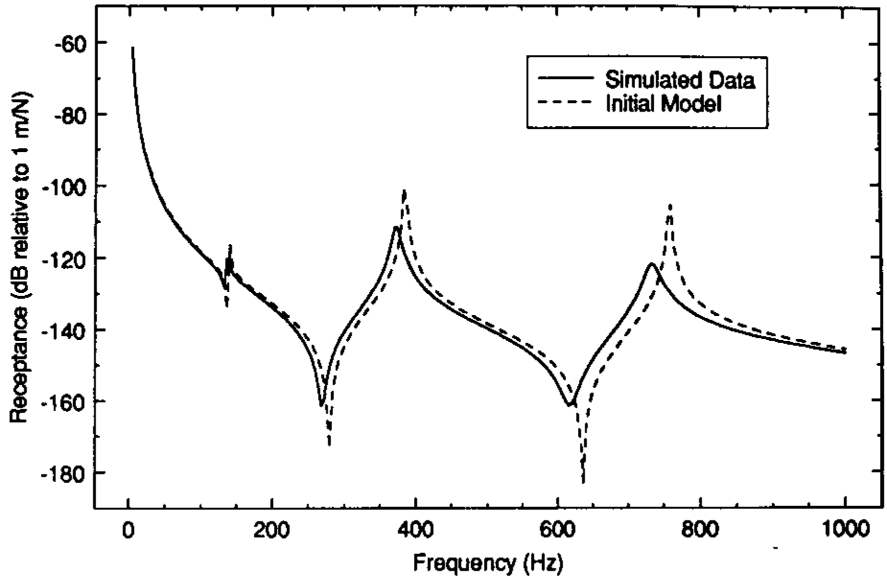


Figure 4. Point receptance of the beam at co-ordinate 3.

The simulated and the initial analytical model were perturbed by adding stiffness of 2×10^6 N/m and 10^7 N/m at co-ordinates 3 and 5. For the simulated model, the point receptances at these co-ordinates were constructed from the FRFs of the original beam using equation (7) and Dobson's modal analysis algorithm was used to determine the eigenvalues of the beam with added stiffness. Only those modes corresponding to the first three elastic modes of the unperturbed beam were used in the updating. The constructed point receptances at co-ordinates 3 for the additional stiffnesses of 2×10^6 N/m and 10^7 N/m are shown in Fig. 5.

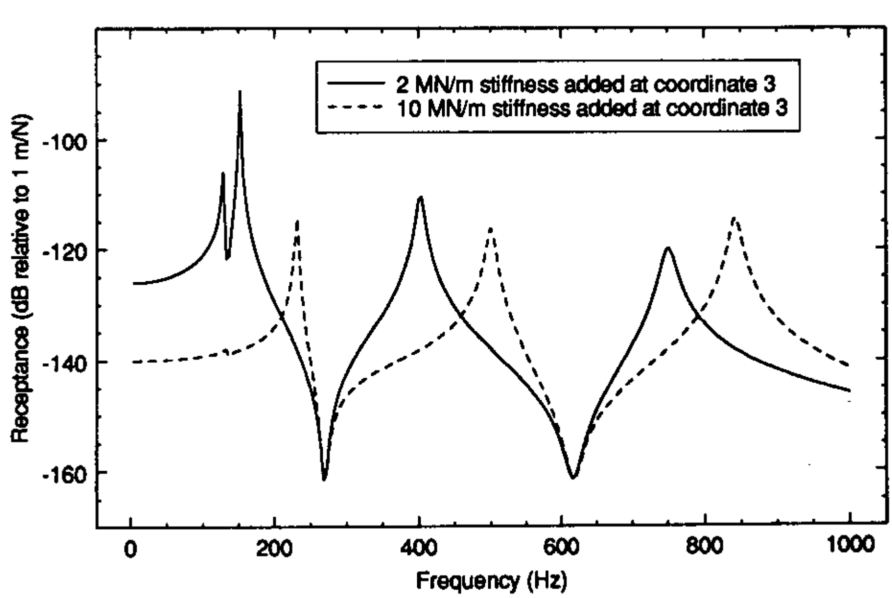


Figure 5. Point receptance at co-ordinate 3 of the beam.

Table 1 shows the analytical natural frequencies of the initial model and the natural frequencies and damping factors of the simulated beam with and without the added stiffnesses.

TABLE 1

Natural frequencies and damping factors (in parentheses) of the simulated beam and its initial model

Added stiffness ($\times 10^6$ N/m)		Natural frequencies (Hz) and damping factors					
		Simulated beam			Initial model		
	Co-ordinate	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
—	—	134·77 (0·0259)	373·39 (0·0259)	733·26 (0·0259)	138·76 —	385·53 —	758·45 —
2·0	3	151·39 (0·0071)	402·40 (0·0236)	750·35 (0·0242)	156·44 —	414·78 —	776·30 —
	5	215·62 (0·0093)	373·39 (0·0259)	755·55 (0·0243)	222·14 —	385·53 —	781·42 —
10·0	3	231·20 (0·0139)	501·04 (0·0218)	842·59 (0·0162)	239·34 —	514·91 —	871·63 —
	5	371·34 (0·0105)	373·39 (0·0259)	858·01 (0·0178)	382·73 —	385·65 —	886·92 —

The eigenvalues were used to update the initial parameters by sensitivity analysis. Since measurement errors were not simulated in the FRF, an unconstrained optimisation, without any weighting, was used in this simulation example. Thus, $\mathbf{W}_{\lambda\theta}$ and $\mathbf{W}_a$ in equation (1) were set to the identity matrix and the zero matrix respectively. The updated parameters, after five iterations, are given below. Although very accurate they are not exact because the identified eigenvalues, although very accurate, are not exact.

$$EI_1 = 4997(1 + j0{\cdot}0020)\ \mathrm{Nm}^2 \qquad EI_2 = 5005(1 + j0{\cdot}0493)\ \mathrm{Nm}^2$$

$$EI_3 = 4994(1 + j0{\cdot}0023)\ \mathrm{Nm}^2 \qquad EI_4 = 5002(1 + j0{\cdot}0502)\ \mathrm{Nm}^2$$

$$m_1 = 3{\cdot}499\ \mathrm{kg/m} \qquad m_2 = 3{\cdot}501\ \mathrm{kg/m}$$

$$m_3 = 3{\cdot}500\ \mathrm{kg/m} \qquad m_4 = 3{\cdot}500\ \mathrm{kg/m}.$$

4. EXPERIMENTAL EXAMPLE

4.1. THE STRUCTURE AND ITS FE MODEL

An H-frame was made by bolting together three aluminium beam members. The beams were of uniform cross-section of 50 × 25 mm. The frame was modelled with free-boundaries using 17 elements, with a total of 37 dof. The analytical model included proportional hysteretic damping, ignored axial flexibility of the beam elements and assumed perfectly rigid joints in the frequency range of interest, 0–600 Hz. The FE model is shown in Fig. 6.

The mass per unit length, m_a, and flexural rigidities, EI_a, of the beam elements, dimensions and material data, and the first five natural frequencies of the FE model are given in Table 2.

4.2. VIBRATION TESTING

The frame was suspended by soft springs at co-ordinates 3 and 24 and excited at co-ordinate 14 using random excitation. The response was measured at co-ordinates 14 and

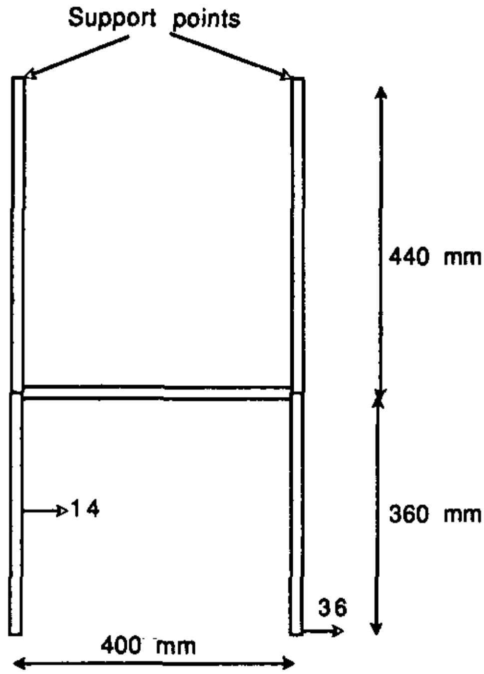


Figure 6. H frame.

TABLE 2

Natural frequencies and the parameters of the initial H frame model

Parameter		Natural frequencies (Hz) of the initial model				
EI_a (Nm^2)	m_a (kg/m)	f_{1a}	f_{2a}	f_{3a}	f_{4a}	f_{5a}
4557	3·4	54·0	119·3	133·8	187·1	500·9

36 of the frame and the FRF at co-ordinate 14 was compared with the analytical model as shown in Fig. 7.

The measured FRF showed five resonances in the frequency range 50–500 Hz with the natural frequencies and damping factors given in Table 3.

These resonances are close to the first five elastic modes of the analytical model. In addition a mode at 23 Hz was found in the measured FRF. This mode probably resulted from an imperfect boundary condition. While a free boundary condition was sought, the shaker was fixed and the pushrod which connected it to the frame had a high, but finite, transverse flexibility. This mode was ignored and the analytical model was updated by assuming a free boundary condition.

4.2. PERTURBING THE STRUCTURE BY ADDING STIFFNESS

The FE model was perturbed by adding grounded stiffeners, in turn, at co-ordinates 14 and 36 of the FE model. FRFs for the structure with grounded stiffeners were constructed from the measured FRFs at 14 and 36 using equation (7). Four stiffnesses were added at

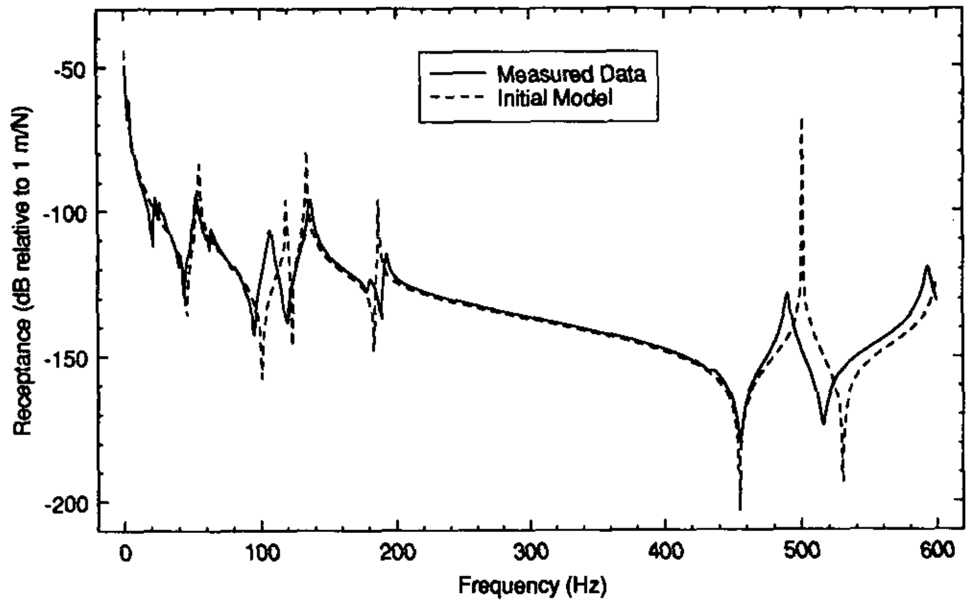


Figure 7. Initial point receptance of H frame at co-ordinate 14.

TABLE 3

Measured natural frequencies and damping factors of the H frame

	f_1	f_2	f_3	f_4	f_5
Natural frequency (Hz)	52·6	106·9	136·7	192·1	489·8
	η_1	η_2	η_3	η_4	η_5
Damping factor	0·0142	0·0237	0·0038	0·0085	0·0051

each of the two co-ordinates as follows:

Co-ordinate 14: 5×10^5 N/m	Co-ordinate 36: 5×10^5 N/m
2×10^6 N/m	1×10^6 N/m
1×10^7 N/m	1×10^7 N/m
5×10^7 N/m	5×10^7 N/m.

Since the technique requires only of a few measured modes, modes which are generated as a result of an imperfect boundary condition and modes which are grossly degraded by noise should not be used. Thus, it was decided, in this case, to use only those modes which correspond to the modes in the frequency range 100–500 Hz of the unperturbed frame, that is the second to fifth modes inclusive. Table 4 shows the measured natural frequencies for these modes for the frame with the additional stiffnesses at co-ordinates 14 and 36. The natural frequencies were identified from the FRFs using Dobson's modal analysis algorithm.

By assuming proportional damping, the identified natural frequencies of the frame with and without the added stiffnesses were used to update the parameters of an undamped FE

TABLE 4

Measured natural frequencies of the H frame with added stiffness

Added stiffness ($\times 10^6$ N/m)	Co-ordinate	Measured natural frequencies (Hz) f_2	f_3	f_4	f_5
0·5	14	109·8	147·8	193·4	490·0
	36	110·2	161·9	236·2	484·0
2·0	14	114·7	175·9	209·6	490·9
1·0	36	111·0	166·1	288·3	488·0
10·0	14	117·9	188·4	334·2	494·6
	36	112·4	169·5	445·2	543·5
50·0	14	118·6	188·7	428·9	505·2
	36	112·4	169·8	454·8	553·5

model. The choice of the undamped parameters to update was made as follows.

EI_1, m_1 Stiffness and mass parameters of elements of the legs and cross beam, away from the joints.

EI_2, m_2 Stiffness and mass parameters of elements of the vertical legs, next to the joints.

EI_3, m_3 Stiffness and mass parameters of elements of the cross beam, next to the joints.

For the purpose of computing the weighting matrices $\mathbf{W}_{\lambda\theta}$ and $\mathbf{W}_a$ (inverse of diagonal variance matrices) for use in the Bayesian estimator (1), the following standard deviations were assumed:

$$\text{STD } EI_1 = \text{STD } EI_2 = 100 \text{ Nm}^2$$

$$\text{STD } EI_3 = 200 \text{ Nm}^2$$

$$\text{STD } m_1 = \text{STD } m_2 = \text{STD } m_3 = 0{\cdot}05 \text{ kg/m}$$

$$\text{STD } f = 0{\cdot}5 \text{ Hz (for all natural frequencies)}.$$

Using the minimum cost Bayesian estimator, the parameters of the undamped model converged in four iterations. To find the damping proportionality constants, measured damping factors, η_2 to η_5, and eigenvalues of the updated undamped model were used in a least squares solution of equation (2). Table 5 gives the updated parameters, natural frequencies and damping factors.

TABLE 5

Updated parameters of the H frame

Updated mass (kg/m) and stiffness (Nm^2) parameters					
EI_1	EI_2	EI_3	m_1	m_2	m_3
4370	4912	2841	3·295	3·139	4·421

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Natural frequency	48·4	105·8	137·9	187·6	490·0
Damping	0·0520	0·0142	0·0105	0·0078	0·0050
Damping proportionality constants			$\beta_k = 0{\cdot}0046$	$\beta_m = 4400$	

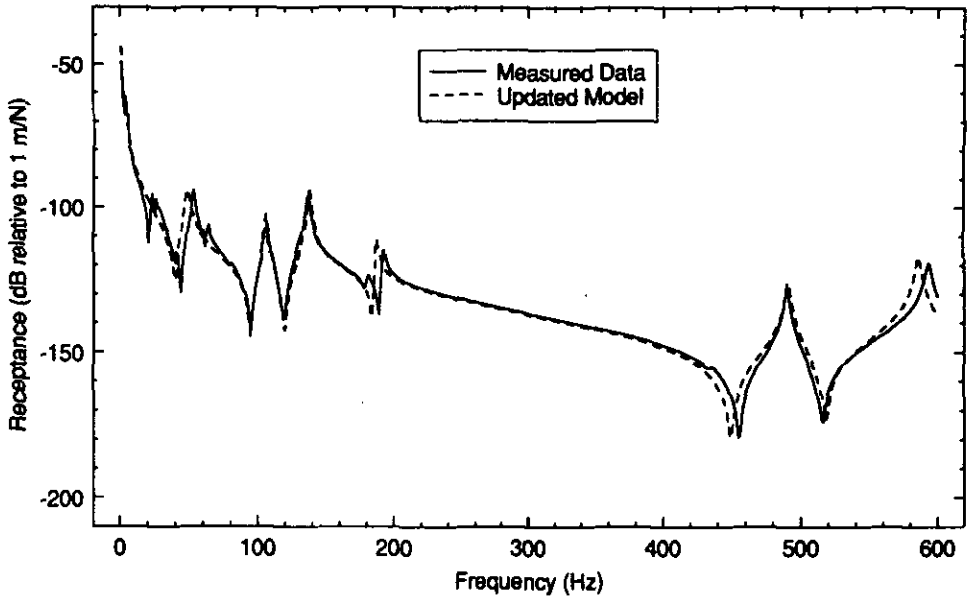


Figure 8. Updated point receptance of H frame at co-ordinate 14.

Figure 8 compares the measured receptance and the receptance based on the updated FE model with proportional damping. Excitation and response are at co-ordinates 14.

5. CONCLUSION

This paper considered the use of a simple technique for model parameter adjustment. The general approach is to construct the point receptances of a perturbed or modified structure at each modification co-ordinate and extract the eigenvalues from the constructed FRFs using any well known modal analysis algorithm. Whilst measurement noise can degrade the quality of the constructed FRFs, and hence the identified eigenvalues, the examples considered so far indicate that construction of a reasonably good FRF of the perturbed structure is quite feasible. The question of computation time has not been addressed in this paper. An eigensolution has to be performed for the full FE model of the structure and the structure with each added stiffness and this must be repeated for each iteration of the parameter values. Thus it would appear that the computation time might be large. However, only a small number of eigenvalues are required and this would suggest the use of subspace iteration. The starting vector for the subspace iteration would be the eigenvector for the system with the previous set of parameter values.

REFERENCES

1. H. G. Natke and N. Cottin 1986 *Ingenieur Archiv* **56**, 106–113. On the parameter identification of elastomechanical systems using weighted input and modal residuals.
2. H. G. Natke and N. Cottin 1985 *Proceedings of the 2nd International Symposium on Aeroelasticity and Structural Dynamics*, 625–631. Updating mathematical models on the basis of vibration and modal test data: A review of experience.
3. M. I. Friswell 1989 *Proceedings of the 12th Biennial Conference on Mechanical Vibration and Noise, Montreal, Canada* **18-4**, 393–400. Updating physical parameters from frequency response function data.

4. C. P. Fritzen 1986 *Journal of Vibration, Acoustics, Stress and Reliability in Design* **108**, 9–16. Identification of mass, damping, and stiffness matrices of mechanical systems.
5. A. Berman 1989 *Proceedings of the 7th IMAL* 355–359. Non-unique structural system identification.
6. M. I. Friswell, N. G. Nalitolela and J. E. T. Penny 1990 *Proceedings of the 15th International Seminar on Modal Analysis, Katholieke Universiteit, Leuven, Belgium,* 173–188. Updating the parameters of a finite element model.
7. N. G. Nalitolela, J. E. T. Penny and M. I. Friswell 1992 *International Journal of Analytical and Experimental Modal Analysis.* **7**, 157–168. A mass or stiffness addition technique for structural parameter updating.
8. N. G. Nalitolela 1991 *PhD Thesis, University of Aston in Birmingham, UK.* System identification of linear vibrating structures.
9. B. J. Dobson 1987 *Mechanical Systems and Signal Processing* **1** 29–40. A straight line technique for extracting modal properties from frequency response data.