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Stochastic Response of Assembled Systems Using Concepts of Domain Decomposition

Abstract: The response evaluation of assembled systems formed by the combination of different sub-components may often prove to be computationally expensive. The problem can escalate in the presence of uncertainties as multiple simulations of the deterministic finite element (FE) code are employed. The present work aims to address this issue efficiently by reducing the physical (deterministic computations) and functional (stochastic computations) domain in evaluating the response of the assembled systems. In doing so, concepts of domain decomposition and the Schur complement are utilized in the context of dynamic sub-structuring by component mode synthesis. This allows division of the overall domain into smaller sub-domains and model reduction of the internal DOFs of the individual sub-components resulting in computational savings. The theoretical formulation of the proposed method is presented followed by a simple numerical example to illustrate its performance. The results reveal good performance of the method and hence the approach is a candidate to solve large-scale practical engineering application problems.

1 Introduction

Significant effort has been made to improve the computational framework of numerical modelling and simulation, facilitated by high performance computing (HPC). Accordingly, numerical algorithms are required to scale with number of processors to realize the maximum capability of the HPC platform. The domain decomposition (DD) method is an example of such an algorithm which exhibits scalable features [1-4]. In general, the finite element (FE) domain is decomposed into a set of sub-domains and each of these are assigned to an individual processor. In a discrete model, representative of a continuous problem, the resulting linear system is recast as a set of smaller linear sub-systems to be solved independently. In doing so, the critical aspect is to ensure that the compatibility and equilibrium conditions are established at the interface DOFs. Thus, such partitioning of the domain by DD through scalable and parallel computing allows zooming in the model resolution with minimizing the computational cost. The increase in model resolution is reasonable as it can significantly reduce the discretization errors in the numerical simulation.

In this work, we aim to exploit the maximum potential of HPC platforms by reducing discretization errors with high-resolution numerical models in conjunction with accounting for uncertainties in the system. The proposed methodology is a combination of a physical domain decomposition in the geometric space and functional decomposition in the stochastic space. For the physical DD, we employ component mode synthesis (CMS) [6], a well-known method for sub-structuring, intelligently coupled with the concept of Schur complement to solve the resulting system of equations efficiently. The functional decomposition quantifies the effect of input uncertainties on the system response in a cost-effective way. For this study, parametric uncertainty modelling has been adopted. Similar work including non-parametric uncertainty modelling can be found in [2, 5]. In short, the primary objective of the present work, is to develop an efficient computational framework which quantifies probabilistic uncertainty in the system by achieving the maximum potential of deterministic high-resolution simulation techniques.

The rest of the paper is organized in the following sequence. In the second section, the established component mode synthesis method has been briefly explained. The proposed methodology has been illustrated in section 3. A numerical study has been presented in section 4. Finally, the work has been summarized at the end.

2 Component Mode Synthesis

CMS methods are quite useful when it comes to analyzing assembled structures which are built-up of several different components [6]. The components are modelled individually and their dynamic models are assembled to yield a reduced model of the entire structure. In this study, CMS would be particularly useful as the deterministic problem is to be solved repeatedly due to the consideration of randomness in the structural systems. Thus, the computational benefit of the reduced order modelling can be achieved with every deterministic simulation.

To be specific, fixed-interface CMS popularly, known as the Craig-Bampton method [6], has been utilized here. The individual substructure models are transformed from physical to component modal coordinates, using a set of chosen basis functions. There are many possible choices for these basis functions including normal modes, found from solving a component eigenvalue problem, and static constraint or attachment modes. The models are assembled

and the global eigenvalue problem of the whole structure is solved. A reduction in size can be achieved by truncating both the component and the global modes. The formulation has been briefly discussed next.

The equation of motion of component α, neglecting damping, is

$$\mathbf{m}^{\alpha}\ddot{\mathbf{u}}^{\alpha} + \mathbf{k}^{\alpha}\mathbf{u}^{\alpha} = \mathbf{f}^{\alpha} \tag{1}$$

where $\mathbf{u}^{\alpha}$ are the physical DOFs, $\mathbf{f}^{\alpha}, \mathbf{m}^{\alpha}, \mathbf{k}^{\alpha}$ are the forces, mass matrix and stiffness matrix of the component, respectively. The physical DOFs can be partitioned into a set of interior DOFs $\mathbf{u}_I$ and a set of interface DOFs $\mathbf{u}_B$ as,

$$\begin{bmatrix}\mathbf{m}_{II} & \mathbf{m}_{IB}\\ \mathbf{m}_{BI} & \mathbf{m}_{BB}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{u}}_I\\ \ddot{\mathbf{u}}_B\end{bmatrix} + \begin{bmatrix}\mathbf{k}_{II} & \mathbf{k}_{IB}\\ \mathbf{k}_{BI} & \mathbf{k}_{BB}\end{bmatrix}\begin{bmatrix}\mathbf{u}_I\\ \mathbf{u}_B\end{bmatrix} = \begin{bmatrix}\mathbf{f}_I\\ \mathbf{f}_B\end{bmatrix} \tag{2}$$

The fixed-interface normal modes of a component are the (mass normalized) eigenvectors of the component with the interface DOFs fixed. The size of the eigenvalue problem equals the number of interior dofs, which is given by

$$\left(\mathbf{k}_{II} - \lambda_j^{fi}\mathbf{m}_{II}\right)\boldsymbol{\phi}_{I,j} = \mathbf{0} \tag{3}$$

The eigenvectors $\boldsymbol{\phi}_{I,j}$ form the columns of the fixed-interface modal matrix. A subset of k modes are retained, reducing the size of the component model.

In the fixed-interface CMS method, the component modal space comprises the kept fixed-interface normal modes and the static constraint modes, which are combined to give the component modal matrix

$$\mathbf{B} = \begin{bmatrix}\boldsymbol{\phi}_{Ik} & -\mathbf{k}_{II}^{-1}\mathbf{k}_{IB}\\ \mathbf{0} & \mathbf{I}_{BB}\end{bmatrix} \tag{4}$$

The constraint modes ensure compatibility of displacements of the components at the interface, improve convergence and also yield the exact static solution. The transformation from physical coordinates $\mathbf{u}$ to component modal coordinates $\mathbf{q}$ is given by

$$\mathbf{u} = \begin{bmatrix}\mathbf{u}_I\\ \mathbf{u}_B\end{bmatrix} = \mathbf{B}\mathbf{q} = \begin{bmatrix}\boldsymbol{\phi}_{Ik} & -\mathbf{k}_{II}^{-1}\mathbf{k}_{IB}\\ \mathbf{0} & \mathbf{I}_{BB}\end{bmatrix}\begin{bmatrix}\mathbf{q}_k\\ \mathbf{q}_c\end{bmatrix} \tag{5}$$

The interior physical coordinates $\mathbf{u}_I$ are transformed into the fixed-interface modal coordinates $\mathbf{q}_k$. The physical interface coordinates $\mathbf{u}_B$ are retained, but will be denoted as constraint coordinates $\mathbf{q}_c$. The component modal mass and stiffness matrices $\boldsymbol{\mu} = \mathbf{B^T}\mathbf{m}\mathbf{B}, \boldsymbol{\kappa} = \mathbf{B^T}\mathbf{k}\mathbf{B}$ have the form

$$\boldsymbol{\mu} = \begin{bmatrix}\mathbf{I}_{kk} & \mathbf{m}_{kc}\\ \mathbf{m}_{kc}^T & \mathbf{m}_{cc}\end{bmatrix}, \boldsymbol{\kappa} = \begin{bmatrix}\boldsymbol{\Lambda}_{kk} & 0\\ 0 & \mathbf{k}_{cc}\end{bmatrix} \tag{6}$$

The matrices $\mathbf{m}_{cc}$ and $\mathbf{k}_{cc}$ are the constraint modal and stiffness matrices for component α, $\mathbf{m}_{kc}$ is a coupling matrix and $\boldsymbol{\Lambda}_{kk}$ is a diagonal matrix of kept fixed-interface modal eigenvalues. The equation of motion for component α thus becomes

$$\boldsymbol{\mu}^{\alpha}\ddot{\mathbf{q}}^{\alpha} + \boldsymbol{\kappa}^{\alpha}\mathbf{q}^{\alpha} = \mathbf{f}_q^{\alpha}, \qquad \mathbf{f}_q^{\alpha} = \mathbf{B}^{\alpha T}\mathbf{f}^{\alpha} \tag{7}$$

Now, the synthesis of two components α and β is considered. The continuity of displacements at their interface, $\mathbf{u}_B^{\alpha} = \mathbf{u}_B^{\beta}$, is transformed into component modal space, so that $\mathbf{q}_c^{\alpha} = \mathbf{q}_c^{\beta}$. A transformation matrix to impose the coupling conditions is such that

$$\mathbf{q} = \begin{bmatrix}\mathbf{q}_k^{\alpha}\\ \mathbf{q}_c^{\alpha}\\ \mathbf{q}_k^{\beta}\\ \mathbf{q}_c^{\beta}\end{bmatrix} = \begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{q}_k^{\alpha}\\ \mathbf{q}_k^{\beta}\\ \mathbf{q}_c\end{bmatrix} \tag{8}$$

The component modal matrices are assembled and the global mass and stiffness matrices become

$$\mathbf{M} = \begin{bmatrix}\mathbf{I}_{kk}^{\alpha} & \mathbf{0} & \mathbf{m}_{kc}^{\alpha}\\ \mathbf{0} & \mathbf{I}_{kk}^{\beta} & \mathbf{m}_{kc}^{\beta}\\ \mathbf{m}_{kc}^{\alpha\,\mathrm{T}} & \mathbf{m}_{kc}^{\beta\,\mathrm{T}} & \mathbf{M}_{cc}\end{bmatrix} \quad \mathbf{K} = \begin{bmatrix}\boldsymbol{\Lambda}_{kk}^{\alpha} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Lambda}_{kk}^{\beta} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{cc}\end{bmatrix} \tag{9}$$

$$\mathbf{M}_{cc} = \mathbf{m}_{cc}^{\alpha} + \mathbf{m}_{cc}^{\beta}, \qquad \mathbf{K}_{cc} = \mathbf{k}_{cc}^{\alpha} + \mathbf{k}_{cc}^{\beta}$$

The global matrices are reduced in size based on the number of fixed-interface modes kept in the component mode matrices $\mathbf{B}^{\alpha}$ and $\mathbf{B}^{\beta}$.

3 Proposed methodology

3.1 Theoretical formulation

Considering the finite element approximation of an arbitrary system reduced by CMS (as shown above) to an n-dimensional modal subspace is given as,

$$\begin{bmatrix} [\mathbf{D}_{II}^{1'}]_{n_1' \times n_1'} & 0 & [\mathbf{D}_{IB}^{1'}]_{n_1' \times n_B} \\ 0 & [\mathbf{D}_{II}^{2'}]_{n_2' \times n_2'} & [\mathbf{D}_{IB}^{2'}]_{n_2' \times n_B} \\ [\mathbf{D}_{BI}^{1'}]_{n_B \times n_1'} & [\mathbf{D}_{BI}^{2'}]_{n_B \times n_2'} & [\mathbf{D}_{BB}^{1} + \mathbf{D}_{BB}^{2}]_{n_B \times n_B} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_I^{1'} \\ \mathbf{q}_I^{2'} \\ \mathbf{q}_B \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_I^{1'} \\ \mathbf{f}_I^{2'} \\ \mathbf{f}_B^1 + \mathbf{f}_B^2 \end{Bmatrix} \quad (10)$$

Note that the internal DOFs of indiviual sub-components have been reduced to n_1' and n_2'.

An illustration of decomposition is shown in Fig.1, where the domain **S** is partitioned into two non-overlapping sub-domains $\mathbf{S}_1$ and $\mathbf{S}_2$.

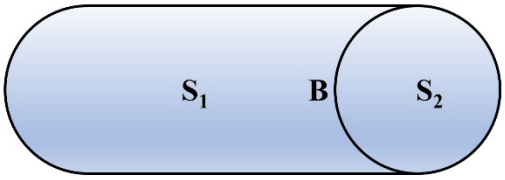


Fig. 1 A representative domain partitioned into two non-overlapping sub-domains.

The above equilibrium equation can be partitioned and rearranged into the following form,

$$[\mathbf{D}_{II}^{1'}]\{\mathbf{q}_I^{1'}\} = \mathbf{f}_I^{1'} - [\mathbf{D}_{IB}^{1'}]\{\mathbf{q}_B\} \quad (11)$$

$$[\mathbf{D}_{II}^{2'}]\{\mathbf{q}_I^{2'}\} = \mathbf{f}_I^{2'} - [\mathbf{D}_{IB}^{2'}]\{\mathbf{q}_B\} \quad (12)$$

$$\left(\underbrace{[\mathbf{D}_{BB}^{1}] - [\mathbf{D}_{BI}^{1'}][\mathbf{D}_{II}^{1'}]^{-1}[\mathbf{D}_{IB}^{1'}]}_{\mathbf{C}_1} + \underbrace{[\mathbf{D}_{BB}^{2}] - [\mathbf{D}_{BI}^{2'}][\mathbf{D}_{II}^{2'}]^{-1}[\mathbf{D}_{IB}^{2'}]}_{\mathbf{C}_2} \right) \{\mathbf{q}_B\} = \left[\underbrace{\{\mathbf{f}_B^1\} - [\mathbf{D}_{BI}^{1'}][\mathbf{D}_{II}^{1'}]^{-1}\{\mathbf{f}_I^{1'}\}}_{\mathbf{F}_1} \right] + \left[\underbrace{\{\mathbf{f}_B^2\} - [\mathbf{D}_{BI}^{2'}][\mathbf{D}_{II}^{2'}]^{-1}\{\mathbf{f}_I^{2'}\}}_{\mathbf{F}_2} \right] \quad (13)$$

Eq. (13) is referred to as the interface problem. The coefficient matrix $\mathbf{C} = \mathbf{C}_1 + \mathbf{C}_2$ is known as the Schur complement matrix. It is noted that the size of **C** is smaller than the coefficient matrix in Eq. (10) and $\mathbf{C}_1$ and $\mathbf{C}_2$ can be constructed in parallel. Thus, once the interface problem is solved, $\mathbf{q}_B$ can be substituted in Eqs. (11) and (12) to obtain the internal DOFs $\mathbf{q}_I^{1'}$ and $\mathbf{q}_I^{2'}$. Also, the solutions to $\mathbf{q}_I^{1'}$ and $\mathbf{q}_I^{2'}$ can be executed in a parallelized manner. To solve the interface problem in which the order of $\mathbf{q}_B$ is high, iterative methods such as the conjugate gradient method can be utilized for computational efficiency.

3.2 Stochastic Domain Decomposition

In the previous sections, the domain decomposition method has been illustrated in the context of deterministic systems. As mentioned previously, parametric uncertainty modelling has been carried out in this work. Thus, the system parameters associated with the internal and interface DOFs are assumed to be random and described by certain probability distribution functions.

To limit the number of eigenvalue analysis to be performed for the reduced model of individual components, functional decomposition is carried out in a high-dimensional correlated polynomial integrated Gaussian Process space, by using Hybrid Polynomial Correlated Function Expansion (H-PCFE) [7]. For implementation, in-house developed codes of H-PCFE have been utilized. The computational framework of the functional decomposition are briefly mentioned as follows.

Let, $\boldsymbol{x} = (x_1, x_2, \dots, x_N)$ be the input parameters, where $\boldsymbol{x} \in D \subset \mathbb{R}^N$. The model response $\boldsymbol{M}$ can be expressed as

$$\boldsymbol{M} = [g_0 + \underbrace{\sum_i \sum_k \alpha_k^{(i)i} \psi_k^i(x_i)}_{\text{first-order}} + \underbrace{\sum_{1 \le i < j \le N} \left(\sum_k \alpha_k^{(ij)i} \psi_k^i(x_i) + \sum_k \alpha_k^{(ij)j} \psi_k^j(x_j) + \sum \sum \alpha_{mn}^{(ij)ij} \psi_m^i(x_i) \psi_n^j(x_j) \right)}_{\text{second-order}} + \cdots \text{up to } M^{\text{th}} \text{order}] + \underbrace{\sigma^2 \boldsymbol{Z}(\boldsymbol{x}, \boldsymbol{\omega})}_{\text{Kriging}} \quad (14)$$

Eq. (9) may be alternatively represented as

$$\boldsymbol{d} = \boldsymbol{\psi}\boldsymbol{\alpha} + \boldsymbol{e} \quad (15)$$

where, $\boldsymbol{d} = \boldsymbol{M} - \boldsymbol{\mu_g}$, $\boldsymbol{M} = (M_1, M_2, \dots, M_m)$ is a vector of observed system response at m training points and $\boldsymbol{\mu_g} = (g_0, g_0, \dots, g_0)^T$ is the vector of mean response (g_0) of length m and $\boldsymbol{\psi} \in \mathbb{R}^{mxp}$. Further, it is assumed that

$$E[e_i] = 0, E[e_i e_j] = \sigma^2 R_{ij} \quad (16)$$

where R_{ij} represents the (i,j)th element of the correlation matrix $\boldsymbol{R}$. $E[\bullet]$ in Eq. (16), denotes expectation operator. The weighted normal equations can be obtained as,

$$(\boldsymbol{\psi}^T \boldsymbol{R}^{-1} \boldsymbol{\psi}) \boldsymbol{\alpha}^* = \boldsymbol{\psi}^T \boldsymbol{R}^{-1} \boldsymbol{d} \quad (17)$$

$$\sigma^2 = \frac{1}{m} (\boldsymbol{d} - \boldsymbol{\psi}\boldsymbol{\alpha}^*)^T \boldsymbol{R}^{-1} (\boldsymbol{d} - \boldsymbol{\psi}\boldsymbol{\alpha}^*) \quad (18)$$

The hyperparameters $\boldsymbol{\theta}$ are a priori unknown, but can be obtained by the Maximum Likelihood Estimate [10]. In order to find the optimal $\boldsymbol{\theta}$, the following objective function is minimized.

$$f_{ML}(\boldsymbol{\theta}) = \frac{1}{N}\log|\boldsymbol{R}(\boldsymbol{\theta})| + \log(\boldsymbol{d}^{\boldsymbol{T}}\boldsymbol{R}(\boldsymbol{\theta})^{-1}\boldsymbol{d}) \quad (19)$$

Eq. (17) can be alternatively represented as,

$$\boldsymbol{B}\boldsymbol{\alpha} = \boldsymbol{C} \quad (20)$$

where $\boldsymbol{B} = \boldsymbol{\psi}^{\boldsymbol{T}}\boldsymbol{R}^{-1}\boldsymbol{\psi}, \boldsymbol{C} = \boldsymbol{\psi}^{\boldsymbol{T}}\boldsymbol{R}^{-1}\boldsymbol{d}, \boldsymbol{\alpha} = \boldsymbol{\alpha}^{*}$.

Since H-PCFE employs extended bases as shown in Eq. (14), some of the equations in Eq. (20) are identical. These equations are redundant and hence can be removed. Removing the redundant from Eq. (20), one obtains

$$\boldsymbol{B}'\boldsymbol{\alpha} = \boldsymbol{C}' \quad (21)$$

where $\boldsymbol{B}'$ and $\boldsymbol{C}'$ correspond to $\boldsymbol{B}$ and $\boldsymbol{C}$, respectively, after removing the redundant equations. The resulting underdetermined set of equations are given in Eq. (21). To solve this set of equations, the homotopy algorithm has been employed [8].

The homotopy algorithm is a regression technique to determine the best solution of an underdetermined system. This algorithm determines the unknown coefficients $\boldsymbol{\alpha}$ by minimizing the L^2 error norm and satisfying an additional condition, defined in terms of an objective function. In this paper, the hierarchical orthogonality of the component functions is considered to be the additional condition to be satisfied.

The matrix $\boldsymbol{B}'$ has dimensions $p \times q$ such that $p < q$. A solution of Eq. (21) is presented in Eq. (22).

$$\boldsymbol{\alpha} = (\boldsymbol{B}')^{-1}\boldsymbol{C}' + (\boldsymbol{I} - (\boldsymbol{B}')^{-1}\boldsymbol{B}')\boldsymbol{v}(s) \quad (22)$$

where $(\boldsymbol{B}')^{-1}$ denotes the generalized inverse of $\boldsymbol{B}'$ satisfying all four Penrose conditions [9]. $\boldsymbol{I}$ represents an identity matrix of dimensions $q \times q$. The algorithm searches for the best solution along the arbitrary vector $\boldsymbol{v}(s)$ and minimizes the predefined objective function, with $s \in [0,\infty)$.

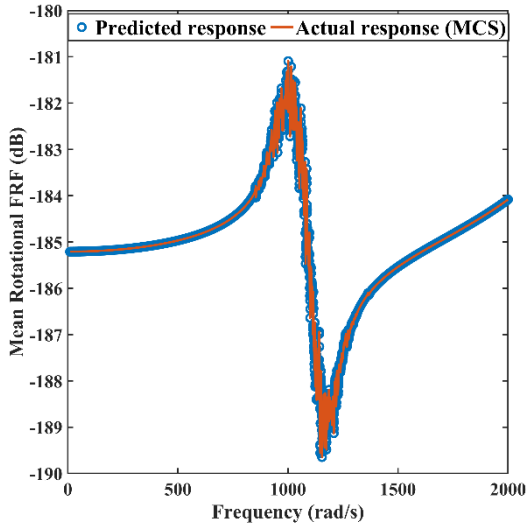
Further details of the computational framework of H-PCFE can be found in [7].

4 Numerical study

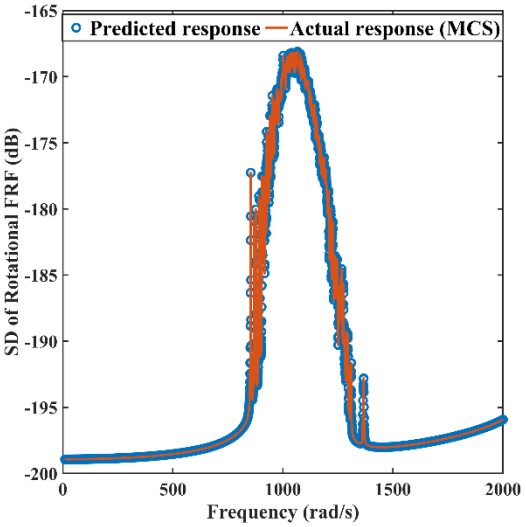
For numerical illustration, an assembled system consisting of the FE models of two cantilever beams connected to each other at their free ends has been considered. Each component has five elements. Thus, there are eight internal DOFs associated with each sub-structure, considering two DOFs per node. Out of these, the first six have been considered to simulate the reduced model of each sub-structure.

The material and geometric properties of the individual components as well as the interface have been considered to be random. Specifically, the density, elastic modulus and the cross-sectional dimensions of each finite element are assumed to be lognormally distributed with 5% variation.

Results presented in Figure 2 show a comparison of the predicted FRFs by H-PCFE and actual FRFs by MCS. To achieve these results, 100 sample points have been generated by Latin hypercube sampling to build the H-PCFE model and 5000 samples of MCS have been generated. The close proximity of the results obtained by H-PCFE and MCS in this example reveal robustness of the former and higher computational savings can be achieved in solving large-scale finite-element problems.



(a)



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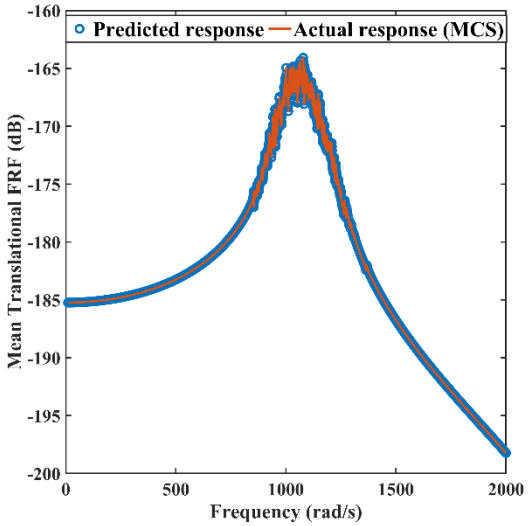

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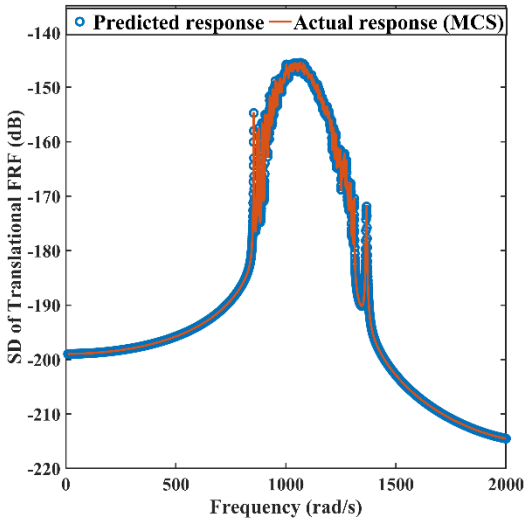

(d)

Fig. 2 Comparison of the FRF statistics of the assembled structural model as obtained by H-PCFE (100 samples) and MCS (5000) at (a), (b) 2nd node from the fixed end of the first beam and (c), (d) interface.

5 Summary

The main contribution of this study lies in developing a computational framework for parametric uncertainty quantification utilizing domain decomposition and CMS in assembled systems. The advantages of the proposed methodology are summarized as follows:

- The internal DOFs of the individual subcomponents can be reduced with the help of CMS and as a result, a reduced-order model can be solved with smaller computational effort as compared to the full system.
- The method performs a physical domain decomposition in the geometric space. Employing the Schur complement helps to reduce the computational cost compared to the usual approach for solving a linear set of equations. Thus, resolution of the numerical models can be magnified to achieve same level of performance with the given computational resources.
- Since the proposed approach lies within the same architecture of domain decomposition (with the only exception that the concept is applied in the eigen space), partitioning of the domain is allowed through scalable and parallel computations. Thus, the maximum potential of HPC platforms can be utilized.
- In addition to the above, the functional decomposition in the stochastic space renders the framework to be computationally feasible for dealing with stochastic systems. This is implemented efficiently by a high-dimensional correlated polynomial integrated Gaussian Process surrogate model (H-PCFE) which is a part of first author's recent work [7].

However, a limitation of the method which could restrict its applicability to large-scale problems is that the formulation contains inverse terms associated with internal DOFs as evident from Eqs. (11)-(13). This may prove to be computationally inefficient for dense FE models exhibiting large numbers of internal DOFs. The problem may further escalate in carrying out the stochastic analysis. This aspect needs to be improved upon in the future.

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