Comparison of the Lagrangian approach for modelling the heel-to-toe contact in passive dynamic walkers with the discrete pivot point method


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ABSTRACT

Traditional biped walkers based on passive dynamic walking usually have flat or circular feet. This foot contact may be modelled with an effective rocker - represented as a roll-over shape - to describe the function of the knee-ankle-foot complex in human ambulation. Mahmoodi et al. [1] has modelled this roll-over shape as a discretized set of pivot points. In this paper, Lagrangian mechanics are used to formulate non-discrete ordinary differential equations for the stance phase that conserve mechanical energy. Qualitative insight can be gained by studying the bifurcation diagrams of gait descriptors such as average velocity, step period, mechanical energy and inter-leg angle for different gain and length values for the feet, as well as different mass and length ratios. The results from both approaches are compared and discussed. This research is not only useful for understanding the stability of bipedal walking, but also for the design of prosthetic feet.

Key Words: Passive walking; Gait analysis; Roll-over shape; Prosthetic foot; Bifurcation diagrams

1. Introduction

An unpowered mechanical biped walker can walk down an inclined plane with a steady, symmetric gait comparable to human walking [2]. These ‘compass-like’ passive dynamic walkers are usually preferred because of their simplicity and may be used as a tool to analyse efficient bipedal locomotion. They are commonly modelled with flat or curved/circular feet; however it has been shown that foot kinematics have a direct influence on the stability of a bipedal robot [1]. This contact may be modelled using an effective rocker to describe the function of the knee-ankle-foot complex in human walking. This effective rocker can be obtained from the physiological roll-over shape defined as the trajectory of the centre of pressure in the local co-ordinate system aligned with the stance leg [3]. This roll-over shape can be determined experimentally from motion capture systems and ground reaction plates and does not change appreciably with walking speed [4], with shoe heel height [5] or when carrying extra weight [6]. Modelling the physiological knee-ankle-foot system can give a better understanding of its functions during able-bodied gait and can improve the stability of designs for ankle-foot prosthesis and orthoses.
2. Implicit method for rolling contact

Mahmoodi et al. [1] modelled this rolling contact as a discretized set of pivot points as shown in Fig. 1. In this model the stance leg pivots about pivot point 1 with inverted pendulum dynamics until pivot point 2 makes contact with the floor. At this point a transition occurs that conserves the angular momentum of the walker using the initial conditions that consists of virtual leg lengths, initial angular velocities and initial angular displacements. This approach was done in order to overcome the inability to model the complex non-circular geometry of roll-over shapes. Before this, only point contact or curved feet could be used. From this study, an interest was gained in the qualitative analysis using a complex roll-over shape in an inverted pendulum passive dynamic model. The drawbacks of using this model is that this approach is unable to conserve energy through the infinitesimal jumps that occur as the walker rolls over from one pivot point to another throughout the stance phase, as seen in Fig. 3. The approach proposed in this paper uses a Lagrangian method to model the roll-over contact of the stance leg. This provides more accurate results by conserving mechanical energy throughout the stance phase and hence gives a greater insight into the dynamics of bipedal locomotion.

3. Governing Equations

Further details of the equations of motion for the pivot point model are discussed in another publication [1]. Euler-Lagrange equations [7] are used in order to formulate the ordinary differential equations that determine the dynamics of the walker (see Fig. 2). The Lagrangian function \( L = K - \Pi \) (the difference between kinetic and potential energies) can be determined in terms of angles \( \theta_1 \) and \( \theta_2 \) used here as the generalized co-ordinates,

\[
L = \frac{1}{2} \Theta_1 (\theta_1) \dot{\theta}_1^2 + \Theta_2 (\theta_2) \dot{\theta}_2^2 + \frac{1}{2} \Theta_1 (\theta_1) \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} \Theta_2 (\theta_2) \dot{\theta}_2^2 - \Pi (\theta_i) \quad i = 1, 2.
\]

Then from Lagrange’s equations \( \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i} = 0 \) the following ODEs can be immediately obtained

\[
\Theta_1 \ddot{\theta}_1 + \Theta_2 \ddot{\theta}_2 + \frac{\partial \Theta_1}{\partial \theta_1} \dot{\theta}_1^2 + \frac{\partial \Theta_1}{\partial \theta_2} \dot{\theta}_1 \dot{\theta}_2 + \left( \frac{\partial \Theta_1}{\partial \theta_2} - \frac{1}{2} \frac{\partial \Theta_2}{\partial \theta_1} \right) \dot{\theta}_2^2 = -\frac{\partial \Pi}{\partial \theta_1},
\]

\[
\Theta_2 \ddot{\theta}_1 + \Theta_2 \ddot{\theta}_2 + \left( \frac{\partial \Theta_2}{\partial \theta_1} - \frac{1}{2} \frac{\partial \Theta_1}{\partial \theta_2} \right) \dot{\theta}_1^2 + \frac{\partial \Theta_1}{\partial \theta_2} \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} \frac{\partial \Theta_2}{\partial \theta_2} \dot{\theta}_2^2 = -\frac{\partial \Pi}{\partial \theta_2}.
\]

Further details are being written as a journal publication [8].

4. Bifurcation diagrams

Bifurcation diagrams can be used to show gait factors such as inter-leg angle, step period and average velocity as a function of mass and leg length ratios. A mass ratio is defined as the ratio of hip mass, \( m_H \) to leg mass, \( m \), while a length ratio is the ratio of upper leg length, \( b \), to lower leg length, \( a \). Refer to Fig. 1. Different curvatures for the feet can also be compared to see how this effects the dynamics of human locomotion. The curvature of the foot is characterised by the gain, where detailed definitions are explained in an earlier paper [1]. Figure 4 shows bifurcation diagrams as a function mass ratio and leg length ratio. Unless stated, the values for mass ratio, length ratio, slope angle, hindfoot length, forefoot length and roll-over gain are 2, 1, 2°, 16cm, 16cm and 0.8 respectively.
Figure 3: Mechanical energy difference in stance phase time. The blue line shows the mechanical energy difference in the pivot point model, while the red line shows that the analytical solution has constant mechanical energy throughout the stance phase.

Figure 4: Bifurcation diagrams for inter-leg angle and step period at variations of mass ratio and length ratio. The pivot point model results are shown as blue dots, while the Lagrangian method is shown as red dots.

5. Discussion

By comparing the gait descriptors inter-leg angle and step period for each model, there can be seen to be a slight difference in values for the step period and inter-leg angle up until bifurcation occurs and the walker demonstrates two-period walking. The inter-leg angle in the pivot-point model was overpredicted by 0.66% and 0.70% for the length ratio and mass ratio respectively, while the step period was overpredicted by 3.51% and 3.39%. At bifurcation the walker displays asymmetric walking as one step takes longer than the next. Shown in Fig. 4a and Fig. 4b, bifurcation occurs earlier in the pivot point model at a length ratio of 2.17, while the Lagrangian approach it occurs at 2.53. The qualitative trends, however,
appear to be the same. The differences between these initial results is assumed to be because of the error in mechanical energy at the collisions at each pivot point in the rolling contact. In further work, it will be interesting to compare both models until the transition to a chaotic region, however, most decisions requiring optimisation of prosthetic feet are in the range before any bifurcation occurs as this is the region of symmetric walking. Hence within this range the accuracy of both models is comparable but computationally the Lagrangian model is approximately 100 times faster.

6. Conclusions

An analytical approach for modelling a rolling contact in a biped walker with a roll-over shape has been presented. This approach conserves mechanical energy throughout the stance phase and can be used to more accurately predict gait descriptors such as average velocity, step period, mechanical energy and inter-leg angle for different gain and length values for the feet, as well as different mass and length ratios. The next task is to compare results with unbalanced mass distributions in order to explore prosthetic design applications. Future work can also include adding a linear or torsional spring in order to emulate the muscle contractions in human walking and compare the ground reaction forces with experimental data. This research is not only useful in order to improve stability and correct gait for the design of prosthetic feet, but also for rehabilitative devices such as ankle-foot orthoses.

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References


