Model updating and validation of a dual-rotor system

H. Miao¹, C. Zang¹, M. Friswell²

¹Jiangsu Province Key Laboratory of Aerospace Power System, College of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
²College of Engineering, Swansea University, Singleton Park, Swansea SA2 8PP, UK

e-mail: miaohui@nuaa.edu.cn

Abstract

Finite element (FE) modeling of structures with three-dimensional (3D) solid elements is often used in order to improve the model fidelity. However, in some cases the model required is extremely complicated, for example the whole engine modeling (WEM) of an aero engine in the design process. A refined WEM including static casings and rotor systems may have a huge number of FE elements with tens of millions degrees of freedom. The dynamic analysis of such models is not only time consuming and expensive but difficult to implement even on the most high performance computers. A simplified one dimensional (1D) model of the rotor system that significantly reduces the size of the model is traditionally used instead of the 3D model, but the analysis precision of such models is generally very poor. In this paper, model updating and validation of the one-dimensional (1D) model of a dual rotor system is introduced using a refined three-dimensional (3D) solid model as a reference. The major errors of the simplified 1D model are determined by comparison of modeling errors between the 1D and 3D models, and the regions of the 1D model required to be updated are selected. Then, first-order based optimization is implemented to update the simplified 1D model using the data simulated from the refined 3D model so that its dynamic prediction agrees with those from the 3D model analysis. Finally, the updated 1D model of the dual rotor system is further used to predict the critical speeds and unbalanced responses. Comparison of the predicted results from the updated 1D model with those calculated from the refined 3D model, shows that the updated 1D model has good accuracy and high efficiency. Therefore, this 1D model can replace the 3D model of the dual rotor system in the WEM dynamic analysis and optimization in the design process to significantly reduce the modeling size and the expenses of computing cost and time.

1 Introduction

With the rapid growth of computer resources, finite element model-based dynamic analyses have been widely used to predict the dynamic characteristics of structures characterized by complex geometries and boundary conditions. Especially for rotating machinery such as aero-engines, the need to understand the system response to design predictable, low maintenance, cost-effective machines has made the modeling of rotor systems become more and more refined and complex. The models contain more intricate geometric complexities, bearings, seals, and attached components such as disks, blades, fans, and couplings. The finite element method has become available for the dynamic analysis of the complex rotor systems after 30 years of development.

Rotordynamics plays a crucial role in identifying critical speeds, and to ultimately design rotating structures that tolerate extremely high vibrations. Modeling techniques with finite element modeling have progressed and changed since it was first utilized for rotor dynamics in the 1970’s. Now there are three main FE modeling approaches for rotor dynamic analysis, namely the 1D beam, the 2D axisymmetric and the 3D solid models. The beam-based models were initially used to model the rotor systems for FEM analysis [1,2], and were further developed via Bernoulli-Euler [3] and Timoshenko beam theory [4,5] to take advantage of the axis symmetry of the rotor in which attached components were constructed as
lumped masses. Beam models are still in use these days for rotor dynamic analysis and have also integrated several variations to account for more complicated geometries such as hollow sections, tapered beam sections [6], conical sections [7-8], and the inclusion of axisymmetric element representations of disks to a beam rotor [6]. However, there are some disadvantages of 1D beam modeling. For example, real life rotors are not one-dimensional and the influence of disks on shafts is not accounted. With advances in computing resources, 2D axisymmetric elements and 3D solid elements for modeling were developed [9-14]. The 2D FE model, as with the 1D model, can model specific phenomena related to gyroscopic effects, attached components, and varying geometry sections of rotor systems by considering only symmetric geometry and components. Engineers have used axisymmetric [12], cyclic [15-18] models for analysis in order to obtain smaller models. However, the axisymmetric and cyclic assumptions usually fail to model the response of bladed rotor systems with unsymmetric loading scenarios due to system unbalance, fluid-structural interactions, and blade-out scenarios. Moreover, those assumptions cannot represent some structures that exploit a complex geometry. Thus, the 3D solid model is needed to account for complex nonsymmetric systems with complex rotor geometry, nonsymmetric attached components such as bladed disks that are commonly found in turbines and aero-engines. The modeling and analysis of 3D solid rotor models have progressed rapidly in the last few years. The 3D solid models have been used to look at complex geometries, attached component connections, and their response, to accurately model gyroscopic and nonlinear effects that were not captured in a linear formulation [9, 18, 11-13, 18-22], to investigate large coupled rotor-structural systems [23], and for the global and large scale modeling of rotating turbomachine assemblies [18]. However, these models can easily reach high orders of dimensionality as seen in the literature [21] where just a bladed disk can easily reach 150000+ DOF in a turbomachine or aero-engine. There are often multiple disks and even multiple shafts, and the dimensionality can grow rapidly. Thus, the use of 3D solid models of rotor systems presents expensive computation and requires high effective computing resources. Thus there is a need to reduce the size of these complex systems so that they can be efficiently and accurately analyzed.

The FE modeling methods for the rotor systems play an important role in the efficiency and accuracy of the rotor dynamic analysis. For instance, the accuracy of the 1D model is less than that of the 3D solid model, but its computing efficiency is much higher. Generally, to some extent, the 1D model may be used to predict the characteristics of the rotor systems if the predicting errors between 1D and 3D models can be minimized using results from the 3D model as the reference data. This process is also termed model updating. Model updating [24] is a very useful tool for the design process of a mechanical system. Generally speaking, model updating methods can be broadly classified into two groups, namely, direct and the iterative methods. Direct methods are based on updating the stiffness and matrices of elements directly without any iteration, and so it is very fast. However, the updated model generally has no physical meaning and the updated matrices cannot always maintain the structural connectivity. In contrast, the iterative methods update design parameters which indirectly update the stiffness and mass matrices. Therefore, the structural connectivity can be easily maintained and the parameters of updated model can be physically interpreted [25]. This approach is also considered as an optimization of errors between the analytical prediction and the reference data.

In this paper, the 1D beam-based model of a dual-rotor system is updated using the modal data simulated from the refined 3D solid model. First, updating parameters and regions are determined by the analysis of the modeling errors between the simplified 1D and the refined 3D models. Then, the first-order based optimization method is implemented to update the simplified 1D model. Finally, the updated 1D model is used to predict the critical speeds and unbalance responses. The results demonstrate the successful validation of the 1D model using the results of the refined 3D solid model of a dual-rotor system.
2 Dynamic equation and model updating

2.1 Rotor dynamic equation

In rotor dynamics, the dynamic equation has additional contributions from the gyroscopic effect comparing with the general dynamic equation. The dynamic equation in a stationary reference frame can be written as

\[
[M][\dot{u}] + ([C] + \omega[G])[\dot{u}] + [K][u] = \{f\}
\]

(1)

where \([M], [C]\) and \([K]\) are the mass, damping and stiffness matrices, and \(\{f\}\) is the external force vector, \([G]\) is the gyroscopic matrix and the rotational velocity is \(\omega\).

Critical speeds can be determined from the Campbell diagram, by identifying the intersection points between the frequency curves and the excitation lines. Critical speeds can also be determined directly by solving a new eigen problem.

For an undamped rotor, the dynamics equation (1) can be rewritten as

\[
[M][\dot{u}] + \omega[G][\dot{u}] + [K][u] = 0
\]

(2)

A solution is sought in the form

\[
\{u\} = \{\phi\} e^{i\lambda t}
\]

(3)

where \(\{\phi\}\) is the mode shape and \(\lambda\) is the natural frequency. When the natural frequency is equal to the synchronous excitation frequency,

\[
\lambda = \omega
\]

(4)

Substituting equations (3) and (4) into equation (2) gives the new eigen problem

\[
([K] - \lambda^2[M])\{\phi\} = 0
\]

(5)

where \([\bar{M}] = [M] - j[G]\). Using QR or Lanczos methods, the eigenvalues and eigenvectors for equation (5) can be solved. The critical speeds of the undamped rotor system equal the natural frequencies in equation (5).

2.2 Modal correlation

The FE model can be used to predict the dynamic response of the actual structure only if it is proved reliable. The measured modal model can be used to validate the reliability of the FE model. It is assumed that the mode shapes of the FE model and the test model use the same normalization method, so the equivalent condition of the two models within the same frequency range is that the matched frequencies and mode shapes should be equal and coincident, respectively. The relative error of the matched frequencies between the models is defined as

\[
\text{RE}(\%) = \left| f_e - f_{\text{FEM}} \right| / f_e \times 100\%
\]

(6)

where RE is relative error, \(f_e\) is the measured natural frequency, \(f_{\text{FEM}}\) is the natural frequency calculated by the FE model.

The Modal Assurance Criterion (MAC) is a measure of the squared cosine of the angle between two mode shapes. The MAC between an analytical and experimental mode shape is calculated as

\[
\text{MAC}_y = \frac{\phi^T_y \phi_u}{\sqrt{\phi^T_y \phi_y \phi^T_u \phi_u \phi^T_u \phi_u}}
\]

(7)
Where $\phi_i$ is the $i$ th experimental mode shape, and $\phi_i$ is the $i$ th analytical mode shape. The MAC between all possible combinations of analytical and test modes are stored in the MAC-matrix. The off-diagonal terms of the MAC-matrix provide a mean to check linear independence between modes. Two mode shapes with a MAC value of 1 indicate identical modes. In this paper, the 1D beam-based model is updated by the data of the refined 3D solid model instead of the experimental data, but the correlation adopted is the same as mentioned in this section.

### 2.3 Finite element model updating

The main purpose of FE model updating is to adjust parameters in the FE model to minimize the errors between the analytical and reference models in order that the predictions of the dynamic characteristics from the FE model match the data in the frequency range of interest. The model updating problem is essentially an optimization, implemented by minimizing the prediction error given by:

Minimize

$$g(x) = \| W R(x) \|^2$$

Subject to

$$x_i^L \leq x_i \leq x_i^U \quad i = 1, 2, ..., n \quad (9a)$$

$$s_i^L(x) \leq s_i(x) \leq s_i^U(x) \quad i = 1, 2, ..., m \quad (9b)$$

where $g(x)$ is the objective function, $W$ is the weighting matrix, and $R$ is the residual vector that can be expressed as $R(x) = f_r - f_a(x)$, in which $f_r$ and $f_a$ denote vectors of the reference and predicted dynamic properties, respectively. The vector expressed as $x = [x_1, x_2, x_3, ..., x_n]^T$ represents the design variables, and each variable has been specified upper and lower bounds. $s_i(x)$ is the state variable in relation to the design variable. The first order optimization method is used to update the FE model with the reference data. By this method, the constrained problem expressed in Equations (8) and (9) is transformed into an unconstrained problem via penalty functions. An unconstrained version of the problem is formulated as

$$F(x, q_k) = \frac{g(x)}{g_0} + \sum_{i=1}^{n} P_x(x_i) + q_k \sum_{i=1}^{m} P_s(s_i)$$

where $F$ is a dimensionless, unconstrained objective function. $P_x$ is the exterior penalty function applied to the design variables $x_i$. $P_s$ is the extended-interior penalty function applied to the state variables $s_i$. $g_0$ is the reference objective function value that is selected from the current design set. $q_k$ is a response surface parameter that control the constraint satisfaction. For more details about these parameters, refer to the literature [26].

Derivatives are formed for the objective function and the state variable penalty functions, leading to a search direction in design space. The steepest descent and conjugate direction searches are performed during each iteration until convergence is reached. Convergence is assumed by comparing the current iteration design set ($j$) to the previous ($j-1$) set and the best ($b$) set. Thus we require

$$\|f^{(j)} - f^{(j-1)}\| \leq \tau \quad (11a)$$

and

$$\|f^{(j)} - f^{(b)}\| \leq \tau \quad (11b)$$

where $\tau$ is the objective function tolerance.
3 Rotor modeling and dynamic analysis

3.1 Dual-rotor system model

The dual-rotor system investigated here is shown in Figure 1. The whole system consists of an inner-rotor, an outer-rotor and four bearings in which bearing 3 is an inter-rotor bearing. It is obvious that the inner-rotor is slender and flexible. The dual-rotor system has coupled vibration due to the effect of the inter-rotor bearing. The bearing stiffnesses are assumed to be symmetric, and the stiffness cross-coupling terms for all of the bearings are neglected. Table 1 shows the numerical properties of the bearings used in the calculations. The damping of the bearings has a small influence in modal analysis of the system, and is neglected. There are two bladed disks on each shaft, and each disk has 36 blades which have complex geometry. The whole rotor system is cyclically symmetric, and the geometric properties are shown in Table 2. The elastic modulus of all components is 210GPa, the Poisson’s ratio is 0.3, and the mass density is 7830kg/m³.

![Figure 1: Dual-rotor system model](image)

<table>
<thead>
<tr>
<th>Bearing</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kxx=Kyy</td>
<td>1.6×10⁷</td>
<td>1.4×10⁷</td>
<td>1.4×10⁷</td>
<td>1×10⁷</td>
</tr>
</tbody>
</table>

Table 1: Mass and rotary inertia of bladed disks

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Length, m</th>
<th>Inner diameter, m</th>
<th>Outer diameter, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer shaft</td>
<td>0.32</td>
<td>0.032</td>
<td>0.048</td>
</tr>
<tr>
<td>Inner shaft</td>
<td>0.6</td>
<td>0</td>
<td>0.024</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disk</th>
<th>Inner diameter, m</th>
<th>Outer diameter, m</th>
<th>Thickness, m</th>
<th>Distance from bearing 1, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disk 1</td>
<td>0.024</td>
<td>0.15</td>
<td>0.04</td>
<td>0.1</td>
</tr>
<tr>
<td>Disk 2</td>
<td>0.048</td>
<td>0.15</td>
<td>0.04</td>
<td>0.22</td>
</tr>
<tr>
<td>Disk 3</td>
<td>0.048</td>
<td>0.15</td>
<td>0.04</td>
<td>0.38</td>
</tr>
<tr>
<td>Disk 4</td>
<td>0.024</td>
<td>0.15</td>
<td>0.04</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 2: Geometric properties of the dual-rotor system
3.2 3D solid model and 1D beam-based models

An accurate rotor dynamic analysis needs solid finite elements to account for the more complex geometry, especially for large complex structures found in many rotors. Rotor dynamics can now be accomplished using solid element models [9, 13] and eliminate the beam model disadvantages mentioned above. The dual-rotor system modeled in ANSYS using Solid 185 elements and Combin14 elements is shown in Figure 2. Solid185 is used for the 3D modeling of solid structures such as shafts, disks, and blades. The element has eight nodes with three degrees of freedom at each node: translations in the nodal x, y, and z directions. Combin14 has longitudinal or torsional capability in 1D, 2D, or 3D applications. The longitudinal spring-damper option is a uniaxial tension-compression element with up to three degrees of freedom at each node: translations in the nodal x, y, and z directions. The bearings are simulated by Combin14 elements. The total number of elements in this model is 92810 and the number of nodes is 126804.

Figure 2: Solid FE model of the dual-rotor system

Figure 3 shows a 1D beam-based model of the dual-rotor system. The 1D model is constructed using Beam188 elements with two nodes each having three degrees of freedom of translation and three degrees of freedom of rotation. The effects of rotary inertia, continuous mass, gyroscopic moments and shear deformations are also included in this model. The bearings are simulated by Combin14 elements and the stiffness coefficients of each bearing are given in Table 1. The bladed disks are modeled as lumped mass by Mass 21 elements which take account of mass and rotary inertia. The values of mass and rotary inertia of each bladed disk are shown in Table 3. The total number of elements and nodes in the 1D model are 44 and 33, respectively, which are significantly reduced compared to the solid model.

Figure 3: Beam-based model of the dual-rotor system
The first six mode frequencies and mode shapes of the 3D and 1D models were solved using ANSYS tools and the results are shown in Table 4. Comparison of these results indicates that the relative errors range from 2% to 9.14% for the first six frequencies. The errors in the 3rd and 4th mode frequencies are the largest. Figure 4 gives the 1st, 3rd and 5th mode shapes of the 3D solid model. From Figure 4, it is obvious that the inner and outer rotors are in coupled vibration. The MAC values, as shown in Table 4, are calculated in order to compare the mode shapes between the 3D model and the 1D model. The correlation between the 3D model and the 1D model is good since all six MAC values are above 90%. The maximum MAC value is 97.9% and the minimum MAC value is 91.2%.

<table>
<thead>
<tr>
<th>Property</th>
<th>Bladed disk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Mass, kg</td>
<td>6.75</td>
</tr>
<tr>
<td>$J_p$, kg·m$^2$</td>
<td>2.85×10$^{-2}$</td>
</tr>
<tr>
<td>$J_d$, kg·m$^2$</td>
<td>1.52×10$^{-2}$</td>
</tr>
</tbody>
</table>

Table 3: Mass and rotary inertia of the bladed disks

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D solid model, Hz</td>
<td>1st</td>
</tr>
<tr>
<td>50.315</td>
<td>50.348</td>
</tr>
<tr>
<td>48.145</td>
<td>48.145</td>
</tr>
<tr>
<td>Relative error, %</td>
<td>4.31</td>
</tr>
<tr>
<td>MAC, %</td>
<td>97.9</td>
</tr>
</tbody>
</table>

Table 4: Natural frequencies of the dual-rotor system in the static (non-rotating) condition

(a) 1st mode shape, 50.315Hz  (b) 3rd mode shape, 137.25Hz  (c) 5th mode shape, 240.75Hz

Figure 4: Dual-rotor system mode shapes in the static (non-rotating) condition

### 3.3 Critical speed calculation

There are two possible modes of operation in a dual-rotor system. One is with both rotors co-rotating in the same direction and the other has the rotors counter-rotating with respect to each other. In this paper, the counter-rotating dual rotor system is investigated. The counter-rotating dual-rotor system in aero-engines has some advantages, since the gyroscopic moment, reaction forces at the supports and the forces conveyed to the aircraft are reduced. The rotor speed relation of the dual-rotor system, shown in Figure 1, is $\omega_2 = 1.5\omega_1$, where $\omega_2$ denotes the outer-rotor speed, $\omega_1$ is the inner-rotor speed. Modal analyses corresponding to different angular velocities are performed to generate a Campbell diagram showing the evolution of the natural frequencies. The large gyroscopic effects mean that the whirl frequencies tend to separate significantly, and may cross or interact with the other frequency curves as the speed increases. The critical speeds correspond to the intersection points between frequency curves and the line $\omega_n = k\Omega$ (where $k$ represents the slope, and for synchronous excitation, $k=1$). Usually the forward critical speeds of the rotor system are most important because the rotor responds in synchronous forward precession due to the unbalance excitation.

The critical speeds are obtained from the Campbell diagram of the 3D model and the 1D model. Figures 5 and 6 show the Campbell diagrams generated by inner-rotor excitation and the outer-rotor excitation,
respectively. Figures 5 and 6 show that the variation of the whirl frequency curves between the 3D model and the 1D model are small. In other words, the gyroscopic effect has little influence on the simplification of the 1D model. Also it can be seen that the second forward and backward whirl frequency curves between the 3D and 1D models have larger deviation than the other frequencies. Given the synchronous excitation line \( \omega_n = \Omega \) of the inner-rotor, as shown in Figure 5, the critical speeds excited by inner-rotor can be found from the crossing points. The errors in the 2\(^{nd}\) critical speeds are larger than the other speeds. The relative errors of the 2\(^{nd}\) critical speeds excited by the inner-rotor and outer-rotor are 9.33\% and 8.89\%, respectively.

![Figure 5: Campbell diagram of dual-rotor speeds excited by the inner-rotor](image)

![Figure 6: Campbell diagram of dual-rotor speeds excited by the outer-rotor](image)

<table>
<thead>
<tr>
<th>order</th>
<th>Excited by inner-rotor/RPM 3D model</th>
<th>1D model</th>
<th>Relative error, %</th>
<th>Excited by outer-rotor /RPM 3D model</th>
<th>1D model</th>
<th>Relative error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3129</td>
<td>2988</td>
<td>4.51</td>
<td>2950</td>
<td>2825</td>
<td>4.24</td>
</tr>
<tr>
<td>2</td>
<td>8534</td>
<td>7738</td>
<td>9.33</td>
<td>8111</td>
<td>7390</td>
<td>8.89</td>
</tr>
<tr>
<td>3</td>
<td>13712</td>
<td>13479</td>
<td>1.70</td>
<td>15136</td>
<td>14719</td>
<td>2.76</td>
</tr>
</tbody>
</table>

Table 5: Comparison of critical speeds between the 3D and 1D models
4 Updating of the 1D beam model

4.1 Updating for static model

The results shown in Tables 4 and 5 indicate that the 1D model has errors, compared with the 3D model, because the coupling effect between the shafts and bladed disks are neglected in the 1D model. This effect increases the bending stiffness of the disk and shaft interfaces due to the thickness of the disks, and so the 3D model is stiffer than the 1D model. From the results mentioned above, it is concluded that the accuracy of the 1D model can be improved by updating the stiffness of the regions between the shafts and the disks. The elastic moduli of the regions shown in Figure 7, is selected to update the coupling stiffness of the shafts and the disks.

![Figure 7: Updating regions of the 1D model](image)

4.2 Optimization for model updating

In the 1D model, the gyroscopic effects of the bladed disks are included in the rotary inertia of the lumped mass elements. The Campbell diagrams shown in Figures 5 and 6 indicate that the split in the whirl frequency curves caused by the gyroscopic effects have small differences between the 3D model and the 1D model. In addition, the correlation of these two models is in good agreement as the MAC values of the first six mode shapes of both models are above 0.9. So based on the results of the 3D model, the frequencies of the 1D model can be updated using the first-order optimization method. The objective function is formulated using the first 6 natural frequencies, defined as

\[ g(x) = \sum_{i=1}^{6} W_i \left( 1 - \frac{\omega_i^{1D}}{\omega_i^{3D}} \right)^2 \]  

where \( \omega_i^{3D} \) and \( \omega_i^{1D} \) are natural frequencies of the 3D model and the 1D model, respectively. The weights \( W_i \) are usually set to 1.

The objective function tolerance \( \tau \) in equations (11a) and (11b) is set to 0.01\( g_0 \), where \( g_0 \) is the reference objective function value, obtained from the initial values of the material properties. Here, the elastic modulus of the updating regions shown in Figure 7 are all initially set to 210GPa, the density of the material is set to 7930kg/m\(^3\), and the Poisson’s ratio is 0.3. It is supposed that variations of elastic moduli of the updating regions are from 1 to 10 times the initial values. The state variables, which are the first 6 natural frequencies, may vary within a range of 3% compared to the reference natural frequencies of each corresponding mode. The updating problem of the dual-rotor system can be formulated as

\[
\begin{align*}
\text{minimize} & \quad g(x) \quad x = (E_1, E_2, E_3, E_4)^T \\
\text{subject to} & \quad E^0 \leq E_i \leq E^0 \times 10 \quad i = 1, 2, \ldots, 4 \\
& \quad \omega_j^0 \times 0.97 \leq \omega_j^{1D} \leq \omega_j^0 \times 1.03 \quad j = 1, 2, \ldots, 6 
\end{align*}
\]
where $E_1$, $E_2$, $E_3$ and $E_4$ are the elastic moduli of the four updating regions, respectively. $E_0^0$ denotes the initial value of elastic moduli of four updating regions. $\omega_j^{1D}$ is the $j$ th natural frequency of the 1D model. $\omega_j^0$ denotes the initial value of the $j$ th natural frequency of the 3D model.

### 4.3 Results and comparison to the 3D solid model

The 1D beam-based model shown in Figure 7 is updated using the first-order optimization method. The convergence of the updating parameters during the updating process is shown in Figure 8 and the corresponding objective function values are plotted in Figure 9. It is obvious that the updating process starts to converge after 16 iterations and stops completely after 30 iterations. Table 6 gives the converged results of the updating parameters, and the elastic moduli of the updating regions in the inner-rotor are 872.31GPa, 781.79GPa, and in the outer-rotor are 314.12GPa and 211.75GPa respectively. This indicates that the coupling effect between shaft and bladed disk has more influence in the inner-rotor than in the outer-rotor, since the updated values in the inner-rotor are larger. Table 7 gives the updated results of the 1D model of the dual-rotor system. The relative errors of natural frequencies between the initial 1D model and the 3D model are significantly reduced, with the relative errors of the first 6 frequencies reduced to less than 0.5%, and the largest error in mode 4 reduced to 0.37% from 9.14%. The MAC values of the first 6 modes between the updated 1D model and the 3D model also have improved, especially in modes 3 and 4, where the MAC values have increased from 94.5% to 99.5% and 94.6% to 99.4%, respectively.

![Figure 8: Convergence of the updating parameters (Elastic modulus)](image_url)

![Figure 9: Convergence of the objective function](image_url)
### 5 Model validation and discussion

#### 5.1 Critical speed prediction

The critical speeds of the updated 1D model were predicted to further validate the accuracy of the updated 1D model. Table 8 compares the predictions before and after updating, and shows that the relative errors have been greatly reduced after updating. The largest error in the critical speeds excited by the inner-rotor is reduced from 9.33% to 0.63%. All the relative errors of the first 3 critical speeds are reduced to within 1.07% after updating. The results show that the predictions of the updated 1D model are very close to the data of the 3D model. It is also concluded that the updating method is reasonable for the 1D model which could replace the 3D model in further rotor dynamic analyses.

<table>
<thead>
<tr>
<th>Order</th>
<th>Excited by the inner-rotor, RPM</th>
<th>Relative error, %</th>
<th>Excited by the outer-rotor, RPM</th>
<th>Relative error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>3129</td>
<td>4.51</td>
<td>2950</td>
<td>4.24</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>8534</td>
<td>9.33</td>
<td>8111</td>
<td>8.89</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>13712</td>
<td>1.70</td>
<td>15136</td>
<td>2.76</td>
</tr>
</tbody>
</table>

Table 8: Comparison of critical speeds between the 3D model and updated 1D model

#### 5.2 Unbalance response calculation

The unbalance responses of the updated 1D model are predicted using harmonic analysis by ANSYS. A mass unbalance situated at the center of disk 4 of the inner-rotor is set to $7 \times 10^5$ kgm and the structural damping is set to 0.001. Figures 10 and 11 compare the unbalance responses of the 3D, initial 1D and updated 1D models, for an operating speed range up to 240 Hz. Because the bearings are symmetric, only the forward modes are excited. The unbalance responses of the updated 1D model are much closer to the 3D model than those of the initial 1D model. The 2<sup>nd</sup> critical speed is excited by the inner-rotor, and the predicted response has been significantly improved after updating. Figures 12 and 13 compare unbalance responses excited by the outer-rotor, for an operating speed range up to 300 Hz. The 4<sup>th</sup> critical speed of...
the updated 1D model has a good match with that of the 3D model, and so the updated 1D model is valid even in this increased frequency range.

Figure 10: Unbalance responses excited by the inner rotor at a specific node of disk 1

Figure 11: Unbalance responses excited by the inner rotor at a specific node of disk 2

Figure 12: Unbalance responses excited by the out rotor at a specific node of disk 1
5.3 Performance benefits of the updated model

Table 9 compares the computing requirements, such as in-core memories and CPU time, of the 3D model and the updated 1D mode. The requirements of the 3D solid model are much larger than those of the 1D updated model, highlighted by the increased number of elements, nodes and equations. The 3D model requires much more in-core memory and CPU time than the updated 1D model. Especially for harmonic analysis, the CPU time of the 3D model is about 2496 times of that of the 1D model. Therefore, the updated 1D model may be used instead of the 3D model in dynamic analysis of the dual-rotor system with significant computational advantages.

<table>
<thead>
<tr>
<th>Analysis type</th>
<th>No. of elements</th>
<th>No. of nodes</th>
<th>No. of equations</th>
<th>In-core memory, MB</th>
<th>CPU time, Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Campbell diagram analysis</td>
<td>92810</td>
<td>126804</td>
<td>380411</td>
<td>1931</td>
<td>2098</td>
</tr>
<tr>
<td>Unbalance response analysis</td>
<td></td>
<td></td>
<td></td>
<td>6415</td>
<td>208500</td>
</tr>
<tr>
<td>Updated 1D model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Campbell diagram analysis</td>
<td>58</td>
<td>51</td>
<td>239</td>
<td>0.734</td>
<td>12.60</td>
</tr>
<tr>
<td>Unbalance response analysis</td>
<td></td>
<td></td>
<td></td>
<td>0.232</td>
<td>83.52</td>
</tr>
<tr>
<td>Ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Campbell diagram analysis</td>
<td>1600</td>
<td>2486</td>
<td>1592</td>
<td>2631</td>
<td>167</td>
</tr>
<tr>
<td>Unbalance response analysis</td>
<td></td>
<td></td>
<td></td>
<td>27651</td>
<td>2496</td>
</tr>
</tbody>
</table>

Table 9: Comparison of performance between the 3D model and the updated 1D model

6 Conclusions

A dynamic analysis of the dual-rotor system was performed with a 3D solid model and a 1D beam model. Some errors in the dynamic behavior prediction of the 1D model exist due to the modeling simplification. The updated 1D beam model has very high accuracy in dynamic prediction and is able to replace the 3D solid model in the dynamic analysis of the dual-rotor system in order to overcome the time-consuming problem with the refined 3D model analysis.
In the process of updating the dual-rotor system, the first 6 frequencies of the 1D model were updated using the simulated data from the refined 3D solid model. The elastic modulus of the overlap regions between the shaft and disk were selected as updating parameters, and the results showed that the selections of updating parameters and regions were reasonable and the stiffness of the updating regions of the inner-rotor had more influence than that of the outer-rotor on the critical speeds of the studied dual-rotor system. The relative errors of the first 6 frequencies between the updated model and 3D model were all below 0.5%. The updated 1D model was further validated via critical speed and unbalance response prediction. The results revealed that the dynamic characteristics of the updated 1D model agreed well with the 3D model. All of the relative errors of the first 3 critical speeds were reduced within 1.07% after updating. The updated 1D model not only significantly improved the prediction accuracy, but also greatly reduced the computing scale and time. The updated model could be used instead of the 3D model in the modeling and dynamic analysis of the dual-rotor system of aero-engines, and would improve design efficiency and reduce costs.

Acknowledgements

The financial supports of the National Natural Science Foundation of China (Project No. 51175244, 11372128), the Collaborative Innovation Center of Advanced Aero-Engine, and the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD) are gratefully acknowledged.

References


Model updating and correlation


