Smart Machines with Flexible Rotors

Arthur W. Lees and Michael I. Friswell

Abstract The concept of smart machinery is of current interest. Several technologies are relevant in this quest including magnetic bearings, shape memory alloys (SMA) and piezoelectric activation. Recently a smart bearing pedestal was proposed based on SMAs and elastomeric O-rings. However, such a device is clearly relevant only for the control of rigid rotors; for flexible rotors there is a need for some modification on the rotor itself. In this paper, equations of motion are developed to describe a rotor with a force generator (for example, a piezoelectric element) mounted on it. It is shown that such a system may be used to compensate for imbalance by inducing a rotor bend. This leads to some questions as to the optimum control strategies, and the paper discusses some of the possibilities.

Keywords Rotordynamics · Control · Imbalance · Rotor bend · Piezoelectric

1 Introduction

Over recent years there have been significant developments in the study of smart structures, but the application of the same approaches to the automatic control of rotating machinery has received significantly less attention. It would appear, however, that this forms a very attractive field of study; if vibration levels were readily controllable, significant constraints on machine design could be eased and some machines could be lighter and more efficient. This lack of attention in this area is a little surprising since many of the requisite technologies are now common place but what is lacking is an integrated approach.

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Magnetic bearings [1] provide one possible component of a smart machine and are, of course, in regular use on certain types of machine. In particular they are readily applied to the control of rigid rotor machines, but their use with flexible rotors is somewhat more complicated. Lees et al. [2] suggested a rolling element bearing supported on a non-linear elastomer whose stiffness can be controlled by an external source. This also is primarily of interest for machines with rigid rotors.

For the control of machines with flexible rotors one is inevitably driven to some form of actuator mounted on the rotor and driven by slip-rings, telemetry or possibly some form of magnetic coupling. Horst and Wolfe [3] used piezoelectric (PZT) patches mounted on the shaft surface. The surface of the shaft was ground to an octagonal cross section to allow for the attachment of the patches and the actuators were connected via slip-rings. Their rotor comprised a single heavy disc overhung at the end of a shaft. The authors demonstrated the feasibility of this type of control and showed the effectiveness in terms of resonant response curves.

Sloetjes and de Boer [4, 5] used a combination of position feedback, integral control and scheduled balancing to give reduced vibration levels. They report on stability issues and show the response of a system during transient operation. Other relevant work in this area is that by Kunze et al. [6].

The current study seeks to extend the work of Lees [7] which analysed the rotordynamic behaviour of a rotor mounted with piezoelectric patches. The concept is that a voltage applied to the patch induces a shaft bend which compensates for the rotor imbalance. The analysis of Lees [7] treats the displacement and bend as two separate variables and derives the appropriate Lagrange equations. In the present study, the moment is applied directly to the rotor. Although the distinction between rotor bends and imbalance is often emphasized, given the ability to control the bend externally, the distinction ceases to be important.

Automatic balancing has been studied by a number of authors [e.g. 8–13]. The approaches described have been applied to specific types of machine but in the current study the aim is to investigate techniques which can be used more generally to compensate for a range of shaft forces, and imbalance is just one of these, albeit the most important.

2 Model of Shaft with Piezoelectric Patches

Piezoelectric patches are compact actuators used in a range of applications requiring precise positional control. The crucial component is a layer of piezoelectric material which undergoes a strain when a voltage is applied across the electrodes. The device is capacitive, a fact which must be considered within the overall system integration. The most convenient form of these patches is the Macro-Fiber Composite (MFC) which has a range of sizes and is conveniently packaged with external electrodes. As a voltage is applied to the electrodes, the strain will change by up to 0.2%. If the piezoelectric composite is bonded to the shaft with appropriate epoxy cement, the overall effect will be to contract the composite, hence applying equal
and opposite forces at either end of the device. If patches are mounted diametrically opposite to each other with appropriate bias and polarities the net effect will be equal and opposite bending moments applied to the shaft. These moments will induce a bend in the shaft which will be used to counteract any intrinsic imbalance.

At this point it is worth considering the attachment of the patches to the shaft. Given that the force is determined by the patch and voltage applied to it, the bending moment can be enhanced by increasing the offset from the shaft center line, perhaps by mounting between two discs. There is however a further difficulty here as the piezoelectric patches will not bend around a tight radius (although MFCs are much better than monolithic piezoelectric plates). This may be overcome by forming flats on an appropriate part of the shaft, an approach used by Sloetjes and de Boer [5]. Another possibility would be to abandon the composites and use PZT fibres directly, but at this investigative stage the MFCs are convenient.

Piezoelectric materials provide one way of implementing some form of rotor mounted actuation, but there are other possibilities which may be used. Piezoelectric materials have the advantage of good frequency response, but do require high voltages and this may be a disadvantage in some applications. An alternative activation could be provided by shape memory alloys. Such activation could operate at a few volts (as opposed to several hundred volts for the piezoelectric devices), and they can sustain higher strains. The disadvantage is the limited frequency response, particularly in the cooling phase, but this may not be too important for machines whose operating regime does not vary too quickly. An intriguing possibility is the use of Magnetic Shape Memory Alloys as this would render the use of slip-rings unnecessary. However, these possibilities are perceived as futuristic and for the present analysis, piezoelectric patches (for example MFCs) are taken as the actuation element.

There has been a significant effort in the modeling of piezoelectric materials as distributed transducers and many review papers have been published (see, for example [14–20]). Analyses range from simple devices such as uniform beams and plates in linear dynamics, to more complicated configurations such as composites under non-linear and non-uniform loading and dynamics (such as helicopter blades and aircraft wings). A majority of the research deals with the modeling of symmetric (bimorph) beams and plates, and a uniform strain model is assumed, where the through thickness variation of strain in the active piezoelectric device is uniform. Erturk and Inman [21] and Leo [20] give further details of the modeling of piezoelectric sensors and actuators integrated with beam structures.

Suppose that piezoelectric layers are added to a beam in a bimorph configuration. Then the moment about the beam neutral axis produced by a voltage \( V \) across the piezoelectric layers [17, 18] may be written as

\[
M_A = \gamma_c V(t)
\]  

where the constant \( \gamma_c \) depends on the geometry, configuration and piezoelectric device. For a bimorph with piezoelectric layers in the 31 configuration, with thickness \( h_c \), width \( b_c \), and connected in parallel, then
\[ \gamma_c = Ed_{31}b_c(h + h_c) \]  

where \( h \) is the thickness of the beam, and \( d_{31} \) is the piezoelectric constant. These expressions assume a monolithic piezoceramic actuator perfectly bonded to the beam; Bilgen et al. [22] considered the effect of the structure of a Macro-Fiber Composite (MFC) on the coupling coefficient, and also the effect of the bond and Kapton layers. The mechanical stiffness and mass density of the piezoelectric layers may also be included in the shaft mass and stiffness.

### 3 Rotating Machine Models and Dynamics

Only a summary is provided here; for further details consult the book by Friswell et al. [23]. Suppose that the machine is modeled in the stationary frame and that the mass and stiffness properties of the shaft are symmetric. Two important points concern the response to unbalance and bends. First, the influence of rotor spin speed on the forces applied to the shaft is different for unbalance and bends; unbalance forces and moments increase with \( \Omega^2 \) whereas the forces and moments due to a bent shaft are independent of the rotor spin speed \( \Omega \). Second, there is no reason why the spatial distribution of unbalance and the bend should be identical, and in general the total excitation force is never zero. One could choose the balance mass distribution for a rotor with an inherent bend; this would balance the bent shaft, but would do so only at a single rotor spin speed. This would also require adding mass at all nodes, which is impractical. However, the response is filtered using the dynamic stiffness matrix and hence adequate balance may be achieved by adding fewer balance masses. This could minimise some average response at all nodes, or achieve a zero response at particular degrees of freedom. A bend could also be used to counteract an inherent imbalance, and the issues are similar.

The key concept for the smart machine described in this paper is the application of moments to the shaft that essentially produce a static bend. Since the requirement is to produce a static bend, a variety of smart materials could be used, for example SMAs or piezoelectric patches. These strain actuators would be arranged in a bimorph configuration and hence may be modeled as a pair of opposing moments on the shaft at the ends of the actuators. They would have to be arranged in pairs of orthogonal bimorphs in order to apply moments to both shaft planes. Although the dynamics and hysteresis of the piezoelectric material can be important, here we assume that the applied moment is proportional to the demanded control input. These applied moments are fixed in the rotating frame, and are easily transformed to the stationary frame. Thus the motion is described by

\[ M\ddot{q} + (\Omega G + C)q + Kq = \mathcal{M}(\Omega^2 b_0 + Kq_{b0} + Bu) e^{j\omega t} \]  

(3)
where $\Omega^2 b_0$ is the vector of forces and unbalance forces and moments in the rotating frame, and $q_{b0}$ denotes the vector of deflections of the stationary rotor due to the permanent bend. $u$ is the vector of control inputs and it is assumed that more than one set of actuators have been distributed on the shaft. The matrix $B$ distributes the control inputs to the correct degrees of freedom. Note that the possibility of an inherent (permanent) shaft bend has also been included in this model. In fact the moment applied by the piezoelectric patches can interpreted as an induced bend in the shaft given by $q_{b0} = -K^{-1}Bu$.

There are two ways of defining the control inputs for each pair of bimorphs. The physically based models require two real numbers that represent the voltage inputs to each bimorph. For steady state analysis in the frequency domain these may be combined as a complex number, and this provides a direct link to the common balancing approaches (for example the influence coefficient method).

4 Control Approaches

There are at least five possible approaches to use the forces arising in the piezoelectric MFC patches, which may be outlined as follows:

- Use proportional feedback in the rotating frame to directly change the natural frequencies. This approach is likely to be limited to small machines and does not directly counteract the inherent unbalance force.
- Apply an axial force to modify the rotor properties and therefore the natural frequencies of the machine. The axial force required is likely to be significant and hence this approach is impractical in most machines.
- Use derivative feedback to control damping rather than stiffness. This will improve the transient response but will not significantly affect the unbalance response, unless the machine is operated close to one of its critical speed.
- Use the patches to induce rotor bends, which can counteract the unbalance. This may be viewed as a standard balancing approach where the moments from the piezoelectric patches replace the forces from the balancing masses.
- Use integral feedback in the rotating frame. Since the unbalance response in the rotating frame can be viewed as a steady state error, applying integral feedback can reduce this error and hence balance the machine.

The first three approaches are unlikely to be practical and hence this paper will concentrate on the last two approaches.

**Balancing by Bending the Shaft.** Essentially the piezoelectric actuators apply moments to the shaft that may be easily controlled. The determination of the best moments to apply is identical to a balancing procedure, with the essential difference that the moments that may be applied take a particular form dictated by the position
of the actuators on the rotor. The great advantage is that the moments applied may be continuously adjusted due to changes in the machine balance or changes in the rotor spin speed. Hence existing methods of balancing and automatic balancing may be used. Methods of automatic balancing based on changing the mass distribution of the rotor do have the advantage that a balanced machine will be balanced at all speeds, although in practice the range of spin speeds where the balance is satisfactory is limited.

**Integral Feedback Control.** Integral feedback is well established to reduce the steady state errors in a wide range of control systems. The error of interest in this case would be the response at the location of the sensors in the rotating frame. Note that this error would have components in both transverse directions and this is counteracted by two orthogonal bimorph actuators. Since the measurements will be taken in the stationary frame, and consist of proximity probe or accelerometer measurements, they must be transformed into the rotating frame, in the directions of the actuators. Most rotating machines will have a phase reference that gives both the rotor spin speed and also the phase correction; this reference enables the transformation matrix to be calculated. If the response in the rotating frame is aligned to the directions of the piezoelectric actuators, then each actuator voltage is simply proportional to the integral of the corresponding response.

Response measurements are assumed in this paper; if accelerations are measured then these must be integrated before transformation, and this is most readily done in the frequency domain by estimating the magnitude and phase of the unbalance response.

The measurements are translational displacements and the actuators apply moment pairs; even if the sensors and actuators are located at the same position on a machine they will not be collocated. Thus there is always a risk of spill-over where some modes, typically high frequency modes, are destabilised by the control system. This is made worse by the transient response excited during a run-up or rundown, and the need to balance the machine at a range of spin speeds where the response is significantly influenced by more than one critical speed.

### 5 Numerical Example

The numerical example will follow that given by Sloetjies and De Boer [5], and is motivated by a helicopter tail drive shaft. The shaft has length 1 m and is a square section aluminium tube with outside width of 10 mm and a thickness of 1 mm. Bearings are located at both ends of the shaft and are modeled as short bearings with stiffness 100 kN/m and damping 1 kNs/m in both transverse directions. The shaft is modeled using 20 beam finite elements [23]. The first critical speed for this machine is at 1771 rev/min. To reduce the calculation time for the simulations the model is projected onto the eight lowest frequency undamped modes of the stationary machine. The unbalance is modeled as a force at the mid-shaft position with
mass unbalance $5.7 \times 10^{-6}$ kgm. The running speed of the machine is 2000 rev/min and the machine is run-up from zero speed in 20 s. Figure 1 shows the response of the machine in stationary and rotating coordinates for the first 30 s, and demonstrates the increased response as the machine runs through the first critical speed. Notice that in the rotating frame the response at the steady running speed is a static displacement.

Only one set of actuators (a pair of bimorph actuators) will be used in this paper, in contrast to the three sets of actuators used by Sloetjes and De Boer [5]. Although the piezoelectric properties may be used to directly estimate the coupling parameter, $\gamma_c$, here we assume the length of the piezoelectric actuators are 100 mm and adjust the coupling parameter to give a mid-shaft displacement of 1.1 mm/V, as given by Sloetjes and De Boer [5]. Thus $\gamma_c = 1.6 \times 10^{-3}$ Nm/V. The stiffness and mass properties of the piezoelectric material could be included in the shaft properties, but are neglected in this paper. Figure 2 shows the root locus when the integral parameter varies from 0 to $10^5$ and shows that the control mainly affects the lowest frequency pole and causes some loss in stability. If we choose the integral control parameter as $5 \times 10^5$, then the closed loop response is shown in Fig. 3, in both the rotating and stationary frames of reference. Note that the controller is switched on at 30 s, after the machine has run-up and reached the steady state. The voltage applied to the piezoelectric patches is shown in Fig. 4. If the controller is switched on at the start of the run-up then the response as the machine runs through the critical speed may be reduced significantly, as shown in Fig. 5.
Fig. 2  The root locus for the integral controller. The *circles* denote the open loop poles.

Fig. 3  Response at the mid-shaft position with the controller switched on at 30 s.

The influence coefficient balancing approach is not simulated. In fact the PZT voltages required to balance the machine are identical to the steady state voltages given in Fig. 4 that clearly produce a zero steady state response at the sensor locations.
Fig. 4 Voltage applied to the piezoelectric bimorphs to control the unbalance response

Fig. 5 Response at the mid-shaft position with the controller switched on at 0 s

6 Conclusion

This paper has demonstrated that rotor imbalance can be controlled by imposing a bend using shaft mounted actuation, and some possible concepts have been outlined. In particular, a simulated case study using piezoelectric actuation has shown that integral feedback control gives promising results. However, further work is needed to develop a robust system, considering issues such as the effective placement of actuators on the shaft, the effect of running through multiple critical speeds, the effect of asymmetries in the shaft and supports, and drift in the control system. Furthermore proportional and derivative control may be added to help improve stability and performance characteristics.
References