OPTIMAL DESIGN OF ELASTOMER COMPOSITES FOR MORPHING SKINS

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ABSTRACT
Morphing aircraft concepts aim to enhance the aircraft performance over multiple missions by designing time variant wing configurations. The morphing concepts require wing skins that are flexible enough to allow large in-plane stretching and high bending stiffness to resist the aerodynamic loads. In this study, an optimization problem is formed to enhance the in-plane flexibility and bending stiffness of wing skins modeled as composite plates. Initially, the optimal fiber and elastomer materials for highly flexible fiber reinforced elastomer laminates are studied using materials available in the literature. The minor Poisson’s ratio of the laminate is almost zero for all the fiber and elastomer combinations. In the next stage, the effects of boundary conditions and aspect ratio on the out-of-plane deflection of the laminate are studied. Finally, an optimization is performed to minimize the in-plane stiffness and maximize the bending stiffness by spatially varying the volume fraction of fibers of a laminate. The optimization results show that the in-plane flexibility and bending stiffness of the laminate with a variable fiber distribution is 30-40% higher than for the uniform fiber distribution.

NOMENCLATURE
δ In-plane displacement of the plate.
Δ Out-of-plane displacement of the plate.
$u_{x,y,z}$ Displacements in the $x$, $y$, and $z$ directions.
$\theta_{x,y,z}$ Rotations with the $x$, $y$, and $z$ axis.

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INTRODUCTION
In recent years, considerable research has focussed on designing aircraft wings which can reconfigure their initial wing shape to the optimal shape of a specific flight regime. The aircraft wings capable of such a reconfiguration in shape in flight, are termed morphing wings. The possible morphing wing abilities include variable sweep, dihedral position, chamber change, wing chord change and wing span change. One of the key challenges in developing a successful morphing wing is the development of morphing skin that is a continuous layer of material that would stretch over the morphing structure to form a smooth aerodynamic surface. Most research on morphing skin technology [1] can be broadly classified under three major areas: Compliant structures, shape memory polymers (SMPs) and anisotropic elastomeric skins.

Compliant structures rely on the internal structure of aircraft wing to allow small amounts of trailing edge camber change [1]. This morphing technology requires only small deformations and therefore, conventional metal or resin-matrix-composite skin materials can be used to carry aerodynamic loads. Bi-stable laminated composite and corrugated skins are few examples of complaint structures.

SMP skin materials have received considerable attention for morphing aircraft wing concepts, and are suited to span-extension type morphing applications. Shape memory polymers exhibit an order of magnitude reduction in modulus and up to 200% strain capability when heated past a transition temperature, yet return to their original modulus upon cooling. However,
the SMPs have following disadvantages for a large scale wing morphing [2]: 1) Electrical heating of the SMP skin to reach transition temperature can be difficult. 2) The time required for heating SMP material to transition can be ill-suited to dynamic control morphing objectives. 3) A temperature dependent morphing mechanism in aircraft wings which are filled with fuels could be catastrophic.

Elastomeric materials can be ideal candidates for the morphing skin applications [3]. High strain capability, lower degree of risk due to their passive operation, elastic recovery with nominal strain values and a smooth aerodynamic surface of elastomers are advantageous for morphing skins. These materials include thermoplastic polyurethanes, copolyester elastomer, and woven materials made from elastane yarns. A very few studies have investigated the elastomeric skins tailored specifically for span-morphing applications with a suitable supporting substructure to withstand aerodynamic loads.

Peel et al. [3] developed a wing skin, actuator, and actuator attachment for a simple morphing wing. Upper and lower wing skins were fabricated with carbon fiber/polyurethane elastomer laminates. Three pneumatic rubber muscle actuators were used to provide the required actuation. An elastic camber down of 25° at the nose and 20° at the tail were achieved. Bubert et al. [2] studied the properties of various silicone elastomers for the morphing skins. The most promising silicone elastomer was Rhodorsils V-330 CA-45 which had the right combination of low viscosity and long work time to enable easy and effective fiber-reinforced elastomer composite (FMC) manufacture, and demonstrated high maximum elongation and tear strength.

Kikuta [4] investigated the mechanical properties of elastomers that could be used as a skin for morphing wings. The materials tested were thermoplastic polyurethanes, copolyester elastomer, woven materials and shape memory polymers. The materials which strained well and required less force, could not sustain high pressure loads. The materials that did not strain well and required more actuation force were able to handle a larger sustained pressure load. Finally, the study suggested that Tecoflex 80A was the best elastomer for morphing skin applications.

Another key area of research is the design and optimization of laminates for morphing applications. With the conflicting objectives of low in-plane stiffness, high out-of-plane bending stiffness and zero Poisson’s ratio, the nature of design variables selected for optimization has to be different from that of conventional composite structures. For example, the conventional composite laminates are made of plies with the fibers being straight, parallel and uniformly spaced. However, for a morphing skin application, the curvilinear fiber-format, termed as variable stiffness laminates, can be beneficial as it can decouple the conflicting requirements of morphing skins. Gurdal et al. [5] demonstrated that the variable stiffness concept provides flexibility to the designer for trade-offs between overall panel stiffness and buckling load, in that there exist many configurations with equal buckling loads yet different global stiffness values, or vice versa.

Similar to the curvilinear fiber orientations, another important parameter is the volume fraction of fibers in a composite laminate. Martin and Leissa [6] investigated the plane stress problem of a composite laminate with spatially varying volume fractions of fibers. The fiber redistribution increased the buckling load by as much as 38%. Benatta et al. [7] demonstrated the improvements in the structural properties of beams by creating functionally graded materials (FGM) in the form of a symmetric composite whose fiber volume fraction varies through the thickness. Kuo and Shiau [8] studied the buckling and vibration of composite laminated plates with variable fiber spacing using the finite element method. The results indicated that more fibers distributed in the central portion of the plate can efficiently increase the buckling load and natural frequencies. Most of these studies have focussed on the vibration and buckling performance of laminates. However, the same design parameters of curvilinear fibers and variable volume fraction of fibers can also be used to achieve some of the design requirements of morphing skins, and is the aim of this study. In this work, two essential features of morphing skins are studied. In the first part, the optimal combination of fiber and elastomer matrix available for morphing wing skins are studied. The second part involves an optimization process to find the optimal distribution of volume fraction of fibers in a laminate to minimize the in-plane stiffness and maximize the out-of-plane stiffness, simultaneously.

**ELASTOMER MATERIAL SELECTION**

The wing span extension type morphing mechanisms require the in-plane stretching of wing skins to be more than 25% strain without failure. Conventional fiber reinforced epoxy laminates have tensile failure strains in the range of 1 to 2% in the fiber direction and transverse tensile failure strains of around 0.5% [9]. The elastomers are one of the few materials that can undergo strains more than 25% without failure. In this section, the lamina material properties are studied with various elastomers as the matrix and fibers available in the literature.

The lamina properties are evaluated with a micro-mechanical model based on the rule of mixtures [10]. This study finds optimal material properties and highlights issues related to the design of fiber reinforced elastomer laminates. The fibers and elastomer material properties collected from the literature are given in Tables 1 and 2. The graphite and kevlar fibers are transversely isotropic and the glass, boron, and silicon fibers are isotropic. However, the elastomer laminate stiffness, similar to the epoxy laminates, are controlled by the longitudinal modulus of the fibers rather than its transverse modulus properties. The combination of longitudinal modulus and Poisson’s ratio can play a major role in selecting the fibers for a morphing skin. The elastomeric material properties are collected from
the references [3, 4]. Kikuta [4] gave the mechanical properties of tecoflex, riteflex and arnitel elastomers which are linear for strains up to 25% and exhibit non-linear properties after that. The properties of these materials corresponding to the linear range are given in Table 2. However, the urethane material properties collected from reference [3] do not mention its strain behavior.

These preliminary results shows that the FRE composites satisfy the stiffness and Poisson’s ratio requirements desirable for the morphing skins. However, the factors that can control the selection of elastomers are the non-linear stress-strain property, the elastic recovery, the fiber-elastomer bonding and the thermal properties.

![FIGURE 1. FIBER REINFORCED ELASTOMER LAMINA](image)

The longitudinal Young’s modulus, lateral modulus, major and minor Poisson’s ratio of the fiber reinforced elastomer (FRE) lamina as shown in Fig. 1 is calculated with the rule of mixtures and are shown in Figs. 2 and 3. The longitudinal Young’s modulus of the lamina which is the main parameter for out-of-plane bending strength, varies from 20 to 220 GPa. The lateral Young’s modulus, a parameter corresponding to the in-plane stretching of the fiber reinforced elastomer skins, varies from 10 to 400 MPa, which is of the order of $10^3$ less than the longitudinal modulus. The shear modulus also varies in a similar way to the lateral Young’s modulus. The major Poisson’s ratio of the FRE lamina varies from 0.2 to 0.55 as shown in Fig. 3(a). Peel has shown that the in-plane Poisson’s ratios of FRE can reach higher than 32 and less than -60 [11]. However, for a wing span extension type morphing mechanism, the major Poisson’s ratio does not play a major role whereas the minor Poisson’s ratio has to be small. The minor Poisson’s ratio of the FRE lamina shown in Fig. 3(b) is of the order of $10^{-3}$ for any fiber and elastomer combination. This is expected for a highly orthotropic material as the minor Poisson’s ratio depends on the ratio of transverse to longitudinal modulus, as given in Eqn. (1), which is of the order of $10^{-3}$ for the FRE composites considered.

$$v_{21} = \frac{E_2}{E_1} v_{12}$$  \hspace{1cm} (1)

### TABLE 1. FIBER PROPERTIES

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_1$ (GPa)</th>
<th>$E_2$ (GPa)</th>
<th>$v_{12}$</th>
<th>$G_{12}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kevlar (29)</td>
<td>61</td>
<td>4.2</td>
<td>0.35</td>
<td>2.9</td>
</tr>
<tr>
<td>Glass</td>
<td>71</td>
<td>71</td>
<td>0.22</td>
<td>30</td>
</tr>
<tr>
<td>Kevlar (49)</td>
<td>154</td>
<td>4.2</td>
<td>0.35</td>
<td>2.9</td>
</tr>
<tr>
<td>Graphite (AS)</td>
<td>224</td>
<td>14</td>
<td>0.2</td>
<td>14</td>
</tr>
<tr>
<td>Saffil</td>
<td>300</td>
<td>300</td>
<td>0.2</td>
<td>126</td>
</tr>
<tr>
<td>Graphite (HMS)</td>
<td>385</td>
<td>6.3</td>
<td>0.2</td>
<td>7.7</td>
</tr>
<tr>
<td>$Al_2O_3$</td>
<td>385</td>
<td>385</td>
<td>0.3</td>
<td>154</td>
</tr>
<tr>
<td>SiC</td>
<td>406</td>
<td>406</td>
<td>0.2</td>
<td>169</td>
</tr>
<tr>
<td>Boron</td>
<td>420</td>
<td>420</td>
<td>0.2</td>
<td>170</td>
</tr>
</tbody>
</table>

### TABLE 2. MATRIX PROPERTIES

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_1$ (MPa)</th>
<th>$v_{12}$</th>
<th>$G_{12}$ (MPa)</th>
<th>@ Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicone rubber</td>
<td>0.916</td>
<td>0.50</td>
<td>0.3053</td>
<td>-</td>
</tr>
<tr>
<td>Tecoflex 80A</td>
<td>1.07</td>
<td>0.75</td>
<td>0.3057</td>
<td>21.3</td>
</tr>
<tr>
<td>RP6410 urethane</td>
<td>1.65</td>
<td>0.50</td>
<td>0.549</td>
<td>-</td>
</tr>
<tr>
<td>RP6442 urethane</td>
<td>6.095</td>
<td>0.50</td>
<td>2.034</td>
<td>-</td>
</tr>
<tr>
<td>Tecoflex 100A</td>
<td>16.6</td>
<td>0.50</td>
<td>5.5333</td>
<td>20.8</td>
</tr>
<tr>
<td>Arnitel 640A</td>
<td>23.5</td>
<td>0.33</td>
<td>8.8346</td>
<td>23.7</td>
</tr>
<tr>
<td>Riteflex 640A</td>
<td>35.8</td>
<td>0.60</td>
<td>11.1875</td>
<td>22.2</td>
</tr>
<tr>
<td>RP6444 urethane</td>
<td>182</td>
<td>0.50</td>
<td>60.6</td>
<td>-</td>
</tr>
<tr>
<td>Typical epoxy</td>
<td>2096</td>
<td>0.30</td>
<td>806</td>
<td>3-4 (failure)</td>
</tr>
</tbody>
</table>

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HOMOGENISATION-BASED MULTI-SCALE CONSTITUTIVE MODEL

The main assumption in the homogenisation-based multiscale constitutive theory of heterogenous solids is that the macroscopic or homogenised strain tensor $\epsilon$ at any arbitrary point $x$ of the macroscopic continuum is the volume average of the microscopic strain tensor field $\epsilon_\mu$ defined over a local representative volume element (RVE). The RVE is such that its domain $\Omega_\mu$ has a characteristic length much smaller than that of the macroscopic continuum and, at the same time, is sufficiently large to represent the mechanical behaviour of the heterogeneous medium in the averaged sense.

At any instant $t$, the macroscopic or homogenised strain tensor $\epsilon$ at a point $x$ can be expressed as

$$\epsilon(x, t) = \frac{1}{V_\mu} \int_{\Omega_\mu} \epsilon_\mu(y, t) \, dV,$$  \hspace{1cm} (2)

where $V_\mu$ is the volume of the RVE associated to point $x$, $y$ denotes the local RVE coordinates and $\epsilon_\mu = \nabla^\text{sym} u_\mu$, with $\nabla^\text{sym}$ denoting the symmetric gradient operator and $u_\mu$ the RVE (or microscopic) displacement field.

Further, it is possible to decompose the displacement field $u_\mu$ as a sum of a linear displacement $\epsilon(x, t) y$, which represents a homogeneous strain, and a displacement fluctuation field $\tilde{u}_\mu$.
i.e.,
\[ u_\mu(y,t) = \varepsilon(x,t)y + \tilde{u}_\mu(y,t). \] (3)

The displacement fluctuations field represents local variations about the linear displacement \( \varepsilon(x,t)y \) and do not contribute to the macroscopic scale strain. The field \( \tilde{u}_\mu \) depends on the presence of heterogeneities within the RVE.

In general, the present multi-scale constitutive theory requires the prescription of kinematical constraints upon the selected RVE. In what follows, the choice of this set of kinematical constraints will coincide with the widely used Periodic boundary displacement fluctuations model. This is typically associated with the modelling of periodic media. The fundamental kinematical assumption in this class of constitutive models consists in prescribing identical displacement fluctuation vectors for each pair of opposite points \( \{y_+, y_-\} \) on the RVE boundary \( \partial \Omega_\mu \), such that:
\[ \tilde{u}_\mu(y_+,t) = \tilde{u}_\mu(y_-,t) \quad \forall \{y_+, y_-\} \in \partial \Omega_\mu. \] (4)

The adopted RVE in the present work is shown in Figure 4. We consider a periodic linear elastic medium with a circular reinforcing fibre located in the centre of the RVE, embedded in a softer matrix. The volume fraction of reinforcement considered here are: 20, 30, 40, 50 and 60 %. The finite element implementation of the homogenisation scheme was carried out in ANSYS [12]. For further details about computer implementation of this type of models, we refer, for instance, to [13, 14]. Three-dimensional solid structural elements (Solid45) were used in all the RVE finite element meshes. The lamina mechanical properties for volume fractions of fibers between 20% and 60% are calculated with the graphite fibre and epoxy matrix materials given in Table 1 and 2.

OUT-OF-PLANE DEFLECTION

The morphing wing skin panels can be modeled with different aspect ratios (ARs) and with different possible boundary conditions. Also, the deflection behavior of orthotropic skin panels modeled as plates is different from the isotropic plates. Therefore, before performing the optimization, the effects of aspect ratio on the out-of-plane deflection of skin panels modeled as plates are studied in this section.

An aluminium plate and a composite plate with the same structural dimensions are considered. The composite plate is made of graphite fibers and epoxy matrix given in Tables 1 and 2. Two possible boundary conditions representative of the morphing skin are considered. In the first case, the boundary conditions of the plate are simply supported at two sides and fixed at other two sides (SS-FF). In the second case, the boundary condition of the plate are fixed at all four sides (FF-FF). In the first case, the fibers of the composite plate are parallel to the fixed edges. This represents a morphing skin which is allowed to morph in the spanwise direction.

The maximum deflection due to uniform pressure loading is evaluated with the ANSYS for aspect ratios varying from 0.5 to 2.0. The percentage increase in the maximum deflection for different aspect ratios compared to the maximum deflection at an aspect ratio of 1.0 is calculated and are shown in Figs. 5 and 6. The results show that the deflection of an isotropic plate increases almost 100% with an increase in the aspect ratio from 1 to 2 in the FF-FF boundary condition. Further, the increase in deflection converges around the aspect ratio of 2.0. However, for the orthotropic plate, the deflection remains constant with the increase in aspect ratio (L/B ratio), whereas it increases dramatically with the L/B ratio less than 1. For example, the deflection increases by 600% for the L/B ratio of 0.5 as shown in Fig. 5.

For the SS-FF boundary condition case, the deflection of orthotropic plate increases to almost 30% when the aspect ratio reaches 1.5 and remains relatively constant after that, as shown in Fig. 6. As the composite fibers are in the direction perpendicular to the simply supported boundary conditions, the change from a square plate to a rectangular plate has little effect on the composite plates for a L/B ratio greater than 1. However, when the aspect ratio is less than 1, the deflection of the composite plate increases dramatically compared to the isotropic plate. This is due to the fact that \( E_2 \), which is much less than \( E_1 \) for the composite plate, plays a major role in the out-of-plane bending for aspect ratios less than 1. Note, the thickness of the plate is kept constant for all the aspect ratios. These results show that the optimal design of plates can vary with the aspect ratio of plates. Therefore, the optimization has to be performed for different aspect ratios. In the following sections, the optimization is performed to find the optimal distribution of composite fibers of the plate with the aspect ratios of 1.0, 1.3, 2.0 and 4.0.

OPTIMIZATION FORMULATION

In this section, an optimization is performed to find the optimal fiber distributions of composite plates representative of morphing skins. The objective is to maximize the ratio of in-plane...
stretching of the laminate to the bending stiffness by varying the volume fraction of fibers for the plies. The optimization is performed for two cases. In the first case, the maximum out-of-plane deflection for a given uniform pressure loading is minimized by varying the fiber volume distribution. In the second case, the ratio of in-plane displacement to the maximum out-of-plane displacement is maximized. That is the laminate is optimized to be flexible along the in-plane direction and at the same time stiffened enough to minimize the out-of-plane deflection due to uniform pressure loading.

The boundary conditions of a laminate representative of a morphing skin is considered in this study and shown in Fig. 7. The plate is divided into 9 sections along the length ‘L’, as shown in Fig. 8. The term aspect ratio or L/B ratio is interchangeable in the following discussions. A symmetric balanced laminate with four plies is considered in this study. The design variables $X_1$ to $X_{10}$, shown in Fig. 8, represent the volume fraction of fibers. The upper and lower limits of the volume fraction of fibers in a laminate depends on the fiber packing and failure modes [10, 15].

In this study, the design variables are allowed to vary between 20 to 60% volume fraction of fibers with a 5% increment. However, the average volume fraction of the laminate is kept at 40%. That is the fibers of a plate with the uniform volume fraction of 40% is redistributed to improve its structural efficiency. A uniform pressure of 50 Pa is applied to the surface and a uniform in-plane loading of 1 N/mm is applied along the edge of the plate. The maximum out-of-plane displacement and in-plane displacement for the given loadings are calculated with ANSYS. The laminate is modeled with shell99A elements for which the number of ply layers, ply angles and material properties of each layer are given as input.

The objective function to minimize the maximum out-of-plane displacement of the plate and constraints for the optimiza-
dispacement for optimization case II can be written as

\[ \text{Minimize, } \left[ \frac{\Delta(X)^{\text{max}}}{\Delta_{40}^{\text{max}}} \right] \]  \hspace{1cm} (5)\]

The constraint is

\[ g(X) = \frac{\sum_{i=1}^{n} (X_i)}{n} = 40\% \]  \hspace{1cm} (6)

where \( \Delta(X)^{\text{max}} \) is the maximum out-of-plane displacement of plate and the \( X_i \) are the design variables, that is the volume fraction of fibers (%). The displacement \( \Delta_{40}^{\text{max}} \) denotes the value corresponding to the plate with uniform fiber volume fraction of 40%. Thus the uniform fiber distribution with 40% volume fraction is considered as the baseline design.

Similarly, the objective function to minimize the maximum out-of-plane displacement of the plate and maximize the in-plane displacement for optimization case II can be written as

\[ \text{Maximize, } \left[ w_1 \left( \frac{\delta(X)}{\Delta_{40}} \right) + w_2 \left( \frac{\Delta_{40}^{\text{max}}}{\Delta(X)^{\text{max}}} \right) \right] \]  \hspace{1cm} (7)

where \( \delta(X) \) is the in-plane displacement of the plate, and \( w_1 \) and \( w_2 \) are the weights of the functions. Here, the two objective functions are combined to form a single aggregate objective function using the weighted linear sum of the objectives method. In this study, equal weights are assigned to both the objective functions, i.e., \( w_1 = 0.5 \) and \( w_2 = 0.5 \). The optimization case II has the same constraint as given in Eqn. (6). The ply angle values are assigned to be zero while optimizing the fiber distribution. It is likely that the optimal ply angle values will be zero for the boundary conditions and objective function of this problem.

The optimization is performed with a real coded genetic algorithm (GA) coupled with the ANSYS for objective function evaluations [16]. The optimization was performed with a population size of 40 and 200 generations was used as the stopping criteria based on the computational time. The main issue, in terms of computational time, in this optimization study was the implementation of constraints given in Eqn. (6). The constraint requires the average value each potential design, i.e., the average of volume fraction of the fibers, to be equal to 40%. However, the GA creates the new designs which may not satisfy the constraints, thereby increasing the number of generations to find the optimal solution.

**OPTIMIZATION CASE I.** The optimization is performed for aspect ratios of 1.0, 1.3, 2.0 and 4.0. The results of the optimization case I are shown in Fig. 9 and Table 3. The optimal fiber distribution for the AR of 1.0 shows that the top ply has almost 60% volume fraction (VF) and the second ply has 20% VF. For the AR of 1.3, the optimal result shows a parabolic type distribution for both the ply layers with almost 30% VF at the edges and reaching 60% at the middle of the plate length, \( L \). For the AR of 2.0, the optimal distribution of the top ply shows a parabolic type distribution whereas the second layer shows a dual peak type distribution. Similarly, for the AR of 4.0, the fiber volume distribution has multiple peaks for both of the ply layers.

The maximum out-of-plane deflections of optimal fiber distributions are compared with the maximum out-of-plane deflections of uniform fiber distributions and given in Table 3. In addition to the optimal case and uniform fiber distribution case, the fiber distribution of 60% VF at the top layer and 20% VF at the second layer, so that the averageVF is 40%, is also given in Table 3. For easy comparison, all deflections are normalized with the deflections of the uniform fiber distribution (\( \Delta_{40} \)). The results show that the optimal distributions corresponding to the AR 1.3 and 2.0 are better than the 60/20 VF case whereas for the AR of 1.0 and 4.0, the 60/20 VF case is slightly better than the optimal distributions. However, both the 60/20 VF case and optimization results show almost 20-30 percent reduction in the deflection compared to the uniform distribution of 40% VF. These results show that more fibers at the ply layers away from the neutral axis increase the bending strength of a laminate considerably.

**TABLE 3. MAXIMUM OUT-OF-PLANE DISPLACEMENT FOR VARIABLE FIBER DISTRIBUTIONS**

<table>
<thead>
<tr>
<th>Plate L/B</th>
<th>( \Delta_{40} )</th>
<th>( \Delta_{60/20} )</th>
<th>Optimal Case I</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(( / \Delta_{40} ))</td>
<td>(( / \Delta_{40} ))</td>
<td>(( / \Delta_{40} ))</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>0.7361</td>
<td>0.7384</td>
</tr>
<tr>
<td>1.3</td>
<td>1.0</td>
<td>0.7319</td>
<td>0.7156</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0</td>
<td>0.7320</td>
<td>0.7081</td>
</tr>
<tr>
<td>4.0</td>
<td>1.0</td>
<td>0.7323</td>
<td>0.7685</td>
</tr>
</tbody>
</table>

**OPTIMIZATION CASE II.** The optimization case II is performed with the objective function and constraints given in Eqns. (7) and (6), respectively. The results of the optimization are shown in Fig. 10. A flexibility ratio (FR) is defined as the percentage of in-plane displacement to the maximum out-of-plane displacement of the plate. This flexibility ratio, in terms of stiff-
ness, measures the in-plane flexibility for an unit out-of-plane bending stiffness. The FR ratio is given for the optimization results of case I and II, the 60/20 VF case and the uniform fiber distribution case in Table 4. Similarly, the percentage increase in the FR of optimal results and the 60/20 VF case compared to that of uniform fiber distribution case is given in Table 5.

The optimal fiber distribution of the first and second ply layers follows a parabolic type distribution for the AR of 1.0 and 1.3. For the AR of 2.0, the first ply is almost 60% VF and the second ply is 20% VF. However, the plies of AR 4.0 follow corrugated type distributions with 60% VF for the first ply and 20% VF for the second ply. Table 4 shows that the parabolic and corrugated type distributions have better flexibility and bending stiffness compared to the uniform fiber distribution. The FR ratio for the optimal results show an increase of 31 to 47% from the uniform fiber distribution as given in Table 5. Also, the FR for the optimal results of case I show an increase of 27 to 40% and the 60/20 VF case shows an increase of 29% from the uniform fiber distribution.

**TABLE 4. FLEXIBILITY RATIO (FR=δ/Δ×100) FOR VARIABLE FIBER DISTRIBUTIONS**

<table>
<thead>
<tr>
<th>Plate L/B ratio</th>
<th>(FR)_{40}</th>
<th>(FR)_{60/20}</th>
<th>Optimal case I</th>
<th>Optimal case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>9.1177</td>
<td>11.7807</td>
<td>11.8038</td>
<td>12.5530</td>
</tr>
<tr>
<td>1.3</td>
<td>10.0894</td>
<td>13.1089</td>
<td>14.2684</td>
<td>14.9080</td>
</tr>
<tr>
<td>2.0</td>
<td>15.4054</td>
<td>20.0147</td>
<td>21.6622</td>
<td>21.8947</td>
</tr>
<tr>
<td>4.0</td>
<td>30.9345</td>
<td>40.1732</td>
<td>39.5447</td>
<td>40.6432</td>
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</tbody>
</table>

**TABLE 5. PERCENT INCREASE IN FLEXIBILITY RATIO FOR VARIABLE FIBER DISTRIBUTIONS**

<table>
<thead>
<tr>
<th>Plate L/B ratio</th>
<th>(FR)_{60/20}</th>
<th>Optimal case I</th>
<th>Optimal case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>29.21</td>
<td>29.46</td>
<td>37.68</td>
</tr>
<tr>
<td>1.3</td>
<td>29.39</td>
<td>41.42</td>
<td>47.76</td>
</tr>
<tr>
<td>2.0</td>
<td>29.92</td>
<td>40.61</td>
<td>42.12</td>
</tr>
<tr>
<td>4.0</td>
<td>29.87</td>
<td>27.83</td>
<td>31.38</td>
</tr>
</tbody>
</table>

In addition to the above results, the FR of the laminate with a uniform distribution of VF of fibers varying from 20 to 60% is shown in Fig. 11. The results show that the FR increases with
the VF of fibers in a non-linear manner and converges for the VF above 60%. This can be understood as the fibers dominate the laminate for the VF above 60%. However, the FR of the laminate with a uniform VF of 60% is less than the FR of the optimal distributions whose average VF is 40%. These results clearly show that the variable fiber volume distributions enhance the characteristics desirable for morphing skin applications.

FIGURE 11. VARIATION OF FLEXIBILITY RATIO WITH THE VOLUME FRACTION OF FIBERS OF LAMINA

CONCLUSION

In this study, an optimization problem is formed to enhance the in-plane flexibility and bending stiffness of wing skins modeled as composite plates. Initially, the optimal fiber and elastomer materials for a highly flexible fiber reinforced elastomer laminate are studied with the materials available in the literature. In the next stage, the effects of boundary conditions and aspect ratio on the out-of-plane deflection of the laminate are studied. Finally, an optimization is performed to minimize the in-plane stiffness and maximize the bending stiffness by spatially varying the volume fraction of fibers of a laminate. The following conclusions are drawn from this study:

1. The longitudinal Young’s modulus and major Poisson’s ratio of FRE lamina show considerable variation for the various fiber and elastomer properties considered. The lateral and shear modulus are of the order $10^{-3}$ less than longitudinal modulus. The minor Poisson’s ratio is of the order of $10^{-3}$ for all of the fiber and elastomer combinations because of the highly orthotropic nature of the FRE lamina. The preliminary results show that most of the elastomers satisfies the stiffness and Poisson’s ratio requirements of the morphing skins. However, the non-linear stress-strain behavior, the elastic recovery and the thermal resistant property of
elastomers are the major parameters in designing the FRE lamina for morphing skins and have to be investigated.

2. The out-of-plane deflection of the plate varies considerably with the aspect ratio and boundary conditions as the mode of deflection varies. Therefore, the optimal distribution of volume fraction of fiber varies with the aspect ratio of the plate.

3. The optimization performed to minimize the out-of-plane deflection shows that the parabolic type fiber distributions increase the bending stiffness. The optimal distribution of fibers minimizes the out-of-plane deflection by 25 to 30% compared to the uniform fiber distribution.

4. The optimization performed to increase the flexibility ratio of the laminate has shown that the non-uniform distribution of fibers increases the flexibility ratio by almost 30-47% over that for a uniform distribution of fibers. The flexibility ratio of optimal fiber distributions with an average volume fraction of 40% is higher than the uniform fiber distribution with a volume fraction of 60%.

These results clearly show that the variable fiber distribution plays a major role in enhancing the desirable characteristics of morphing skins modeled as composite laminates.

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REFERENCES


