Multiobjective Optimization for the Multi-Phase Design of Active Polymorphing Wings

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Advanced studies have been undertaken using Multidisciplinary Design Optimization (MDO) on the retrofitting of an outboard morphing wing system to an existing conventionally designed commercial passenger jet. Initial studies focusing on the single objective of specific air range improvement for a number of flight phases revealed increases of approximately 4-5% over the baseline aircraft with wing fences across each case. This validated the advantage of re-optimizing the geometric schedules for off-design conditions in comparison with fixed winglets, for which negative effects were observed. Due to the high number of design sensitivities of the outboard wing geometry it has now become necessary to conduct refined studies to analyse the effects of the wing system on additional operational performance metrics, such as take-off, initial climb, approach-climb and landing performance parameters, in order to ascertain a truly holistic representation of the benefits of morphing wing technology. In addition, further effort has been expended to couple the effects of each phase within a multiobjective framework. Thus, refined studies have been performed, incorporating a number of multiobjective optimization methods into a high-end, low fidelity aero-structural-control analysis together with a full engine model and integrated operational performance algorithm. Furthermore, updated aeroelastic functionality and improved aero-structural wing sizing allows for investigation of C-wing configurations. Results reveal the potential for significant field length reductions and climb performance enhancements, while maintaining improvements in cruise performance throughout the entire flight envelope and across multiple stage lengths.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>b</td>
<td>partition span</td>
</tr>
<tr>
<td>h</td>
<td>altitude</td>
</tr>
<tr>
<td>IGW</td>
<td>instantaneous gross weight</td>
</tr>
<tr>
<td>LFL</td>
<td>landing field length</td>
</tr>
<tr>
<td>MC_{CRZ}</td>
<td>cruise Mach number</td>
</tr>
</tbody>
</table>

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**MTOW**  maximum take-off weight  
**SAR**  specific air range  
**$T_S$**  specific excess thrust  
**TOFL**  take-off field length  
**$V_{ref}$**  landing reference velocity  
**$W$**  weight  
**$\gamma$**  climb gradient  
**$\theta$**  twist angle (°)  
**$\Lambda$**  sweep angle (°)  
**$\Phi$**  cant angle (°)

**Subscript**  
**APP**  approach  
**i**  partition number  
**L**  landing  
**LE**  leading edge  
**min**  minimum cant angle deflection  
**pri**  primary  
**sec**  secondary  
**sf**  skin friction  
**SSC**  second segment climb  
**TO**  take-off

**Acronyms**  
**FCA**  final cruise altitude  
**FOM**  figure of merit  
**GA**  genetic algorithm  
**ICA**  initial cruise altitude  
**KS**  Kreisselmeier-Steinhauser function  
**MCA**  mid cruise altitude  
**MDO**  multidisciplinary design optimization  
**MPMR**  maximum passenger, maximum range  
**SR**  short range

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**I. Introduction**

The application of Multidisciplinary Design Optimization (MDO) within engineering has been a natural evolution from parametric trade studies and hypo-dimensional, hyper-constrained nomographs (carpet plots) in tandem with the advent of computing capability and has become a leading method of research in the field of aircraft conceptual design. MDO offers the designer the ability to achieve a global performance constrained optimum for a minimized function across a large number of technical subspaces of interacting and often competing disciplines. MDO has already been successfully applied to the design of such aircraft configurations as the Business Jet, the novel Blended Wing Body and to a variety of both commercial and military configurations. Raymer\(^1\) and Isikveren\(^2\) have demonstrated this via development and utilisation of specialist MDO software, such as RDS-Professional and QCARD respectively, with great success. Each instance served to outline the attractive premise of achieving an improved level of efficiency, in terms of such sustainability related figures of merit (FOM) as drag and noise minimization, and such operational performance criteria as range and climb gradient improvements, compared with that of individual optimization of each discipline and subsequent incorporation into a conceptual design.

One of the most attractive fields of research within aviation to which MDO may be applied is that of active variable-geometry or morphing aircraft. In the analysis of such concepts it is necessary to study the design sensitivities of a number of geometric parameters that constitute the morphing configuration upon the datum aircraft, and thus the use of MDO to investigate this problem is a logical step. Historically the majority of variable-geometry aircraft research has been centred on variable-sweep configurations, which date to as early as 1937\(^3\) and feature on the Bell X-5, F-111, F-14 and B-1 Bomber among others, and there
have also been ventures into variable-camber or Mission Adaptive Wings (MAW). The notion of variable cant angle winglets was first examined by Bourdin,\textsuperscript{4} and from these seeds the MORPHing wingLET, or MORPHLET, concept was born.

The proposed MORPHLET wing system, displayed in Figure 1, comprises of up to 4 partitions that are to be retrofitted to the outboard wing section of an existing commercial narrowbody jet. Preliminary studies\textsuperscript{5} of the concept were originally framed around the optimization via MDO of the specific air range (SAR) improvement achieved at a specific flight condition, via manipulation of partition span, taper, cant and twist angles. Camber variation was also initially investigated but did not produce a favourable trade-off. When separately optimized for three specific cruise flight phases the MORPHLET system was predicted to offer up to 8.8\% SAR benefits, with a vortex-induced drag reduction of up to 18.5\%, of a similar order to that predicted by Whitcomb.\textsuperscript{6} Fixed non-planar wing tip systems, however, historically have often been considered to yield poor off-design performance, thus more detailed analysis was subsequently performed\textsuperscript{7} enhancing predicted results with improved aero-structural-performance analysis. As opposed to separate optimizations the wing system maintained a degree of structural and geometric continuity across each flight phase analysed, namely fixed partition spans and co-linear leading and trailing edge sweep for all cases beyond the initial cruise altitude (ICA) sizing case, while maintaining variation in partition twist and cant angles. These studies refined the anticipated SAR improvements to a more consistent 4.5-6.0\% across each flight case. An initial attempt was made for a multiobjective function optimization combining the SAR gains in each flight phase but for a single wing configuration only. The latest studies improve upon this by incorporating a multiobjective function that combines SAR enhancements for which the MORPHLET twist and cant angles vary for each flight phase, in addition to utilising critical case wing structure sizing.

![Four partition MORPHLET wing system retrofit concept.](image)

Non-planar wings are also interlinked with a number of key parameters relating to aircraft operational performance beyond drag reduction, which have been overlooked within previous analyses. Several low speed performance figures of merit, such as take-off (TOFL) and landing field lengths (LFL) and 1G maximum lift coefficients ($C_{L_{\text{max}}}$), second segment and approach climb gradients ($\gamma$), and the landing reference speed ($V_{r_{\text{ref}}}$) also have a strong sensitivity to winglet definition. It has therefore now become necessary to perform a detailed investigation into consolidating each unique flight phase specific air range increment along with a number of derived low speed performance parameters into a composite multiobjective function to be optimized. This is aimed to serve as a comprehensive analysis of the entire scope of benefit of the MORPHLET wing system.

### A. Multiobjective Optimization

The first attempts to formulate a multiobjective function for genetic algorithms was undertaken by Schaffer\textsuperscript{8} with the Vector Evaluated Genetic Algorithm (VEGA) approach, whereby the population is divided into subpopulations, each optimized by a single objective function for gene selection, before being recombined for crossover and mutation. However this approach was found to tend towards solutions optimized for one particular objective function, and had difficulty in generating a compromise of solutions. Attempts at generating Pareto Frontiers for multiobjective optimal trade-offs were made by Goldberg,\textsuperscript{9} who devised the
Non-dominated Sorting Genetic Algorithm (NSGA). In this method each population member is assigned a 
ranking value dependent on the number of other population members superior in each objective function, 
and an additional sharing value depending on the number of members falling within a predefined radius of 
each other that guarantees good distribution of optimal solutions. Another popular method of multiobjective 
optimization is to employ the fundamentals of Game Theory,\textsuperscript{10} with objective functions representing individual players with unique strategies. Competing players alternate turns optimizing their objective, generating Nash Equilibrium points where no objective function is able to further improve unless to the detriment of others. Further co-operative forms of this model have also been presented by Rao\textsuperscript{11} among others, whereby bargaining strategies are utilised by each player so as to maximize the total gain as opposed to solely their associated objective function, thus achieving more balanced solutions.

II. Methodology

The MORPHLET studies outlined have been undertaken through the development of a Multidisciplinary Design Optimization Suite that has been created and developed as a high-end, low fidelity aero-structural-control analysis tool, coupled with a full engine model and integrated operational performance algorithm, for conceptual aircraft design optimization. The suite utilises a genetic algorithm (GA) coupled with a Nelder-Mead simplex\textsuperscript{12} to give a global plus local search cocktail of methods. Conversion from a constrained multiple objective function into an unconstrained composite single objective function for this means is achieved via application of the Kreisselmeier-Steinhauser (KS) function.\textsuperscript{13} A comprehensive description of the MDO suite is given in Smith et al.\textsuperscript{7} and a latest schematic diagram is given in Figure 2. Various additional improvements have been made to the suite for the latest analysis, including a revised aerelastic prediction module and updated span loading and bending moment calculation that allows for investigation beyond the previous limit of 0° cant angle (90° dihedral) such that the software is able to investigate C-wing\textsuperscript{14} configurations that potentially offer significant vortex-induced drag reductions.

![Current MDO suite schematic diagram.](image)

A. Aerodynamics

The aerodynamic predictions are computed via the Tornado Vortex Lattice Method (TVLM),\textsuperscript{15} a linearised aerodynamic prediction tool. TVLM is coupled with several expert aerodynamic modules for buffet onset prediction,\textsuperscript{16} auto-trimming and auto-synchronisation for obtaining zero lift drag coefficients, whereby the coefficients are dynamically generated according to a set of semi-empirical expressions defined a priori, giving a trimmed full polar aerodynamic analysis.
B. Structures and Control

The expert structural analysis and wing weight prediction module, named 'UC700', has been devised by Ajaj et al.,\textsuperscript{17} to whom the reader is referred for an in-depth report on this algorithm. The module consists of a high-end wing box weight estimation model based on linear beam theory that incorporates Wing Equivalent In-plane Representation (WEIR) for wing systems and secondary structural weight predictions. In addition, a newly developed aeroelastic code has been installed as part of the structural sizing loop. Following on from the results of Smith et al.,\textsuperscript{7} in order to perform multiobjective analysis over a number of states it has been necessary to develop the wing sizing algorithm such that a wing weight estimate is produced for each flight state selected. From these, a critical wing weight, and thus, Maximum Take-Off Weight (MTOW) can be determined. The suite then proceeds to analyse each flight phase trimmed in accordance with this newly inferred gross weight. An alternate method of producing a wing structure comprising individual critical node sizes from each state was considered but would significantly increase the computational time of the software.

C. Performance

The operational performance predictions within the MDO suite are calculated via the use of Isikveren's fractional change theory.\textsuperscript{18} Normalised deltas of the basic geometric and flight condition parameters are incorporated into fundamental equations for performance predictions to give estimated fractional changes in such parameters as block fuel burn, maximum lift coefficients, reference landing approach and stall speeds, field lengths and climb gradients. Further enhancements of the operational performance predictions include the facility to re-optimize cruise altitudes for maximum range based upon the revised aircraft Instantaneous Gross Weight (IGW) and aerodynamic efficiency. The datum aircraft engine is not scaled by the suite, however engine performance is modified such that a vehicular efficiency prediction is obtained.

D. Optimization Procedure

The MDO suite utilises a genetic algorithm, developed by Chipperfield,\textsuperscript{19} to act as a global optimum search. Aircraft configurations are input into the objective function call as a chromosome of 'genes' that mimic the variables specific to that configuration. The structure of each chromosome is required to contain data pertinent to the MORPHLET outer partition local spans, as well as the inner and outer partition twist and cant angle schedules for each specific flight case to be investigated. In this manner the entire gamut of schedules can be examined within a single objective function call. This method is utilised as opposed to serial optimizations of each flight case examined, across which there was an inconsistency in wing weight and structure. For a two partition MORPHLET problem analysing four flight phases the number of design variables would thus be on the threshold of being a hyper-dimensional problem. Despite the need for increased population numbers and numbers of generations to find an optimum, as a consequence of an increased number of variables, this method serves as a significantly more time-efficient procedure to undertake in comparison with several single stage optimizations.

E. Multiobjective Formulation

For each flight phase the SAR improvement is calculated based upon the sizing case weight prediction and the flight case lift-to-drag ratio. Together with selected additional performance parameters, these FOM are cumulated via the KS method\textsuperscript{13} to give an unconstrained equivalent single objective function to be optimized by the genetic algorithm. The optimal solution determined by the suite is based upon this KS value. The genetic algorithm selection and reproduction processes, however, are directed by individual unconstrained KS values that are generated for each objective function. These values are then incorporated into the new multiobjective methods, each of which offers a number of alternate solutions to that of the KS value optimum. The multiobjective problem statement is given in Equation 1, where $i$ is a given partition (or panel) in the morphing system.
\[ \min F(X, \alpha) \]

\[ \text{where } F = [-L/D]_{\text{climb}} - \text{SAR}_{ICA} - \text{SAR}_{MCA} - \text{SAR}_{FCA}]^T \]

\[ X = [b_{P_i}, \Phi_{P_i, \text{climb}} \theta_{P_i, \text{climb}} \Phi_{P_i, \text{ICA}} \theta_{P_i, \text{ICA}} \Phi_{P_i, \text{MCA}} \theta_{P_i, \text{MCA}} \Phi_{P_i, \text{FCA}} \theta_{P_i, \text{FCA}}]^T \]

\[ \alpha = [M_{CRZ} \ h_{\text{climb}} \ W_{fuel}]^T \]

\[ \text{s.t. } g_j(X, \alpha) \leq 0 \quad j = 1, 2, \ldots, n \]

\[ \text{where } g = [\Delta TOFL \ \Delta LFL - \Delta \gamma_{SSC} - \Delta \gamma_{APP}]^T \]

\[ \Sigma b_{\min \Phi} \leq 36m \ (\text{ICAO Code C}) \]

\[ c_{\text{tip}} \geq 610mm \]

\[ -3^\circ \leq \theta_i - \theta_{i-1} \leq 3^\circ \]

\[ 0^\circ \leq \Phi_i - \Phi_{i-1} \leq 90^\circ \]

\[ 0^\circ \leq \Phi_i \leq \Phi_{i-1} \]

Each flight phase has an associated Mach number and fuel weight, based on stage length and fuel burn, that is constant for each population member investigated, and in addition the climb flight level is exempt from re-optimization. The optimization problem statement includes a number of practical engineering constraints in place to limit excessive spans, sweep and twist angles, and manufacturing tolerances (e.g. minimum tip chords). It was decided that TOFLs and LFLs would be employed as constraints as opposed to objective functions, as similarly with ICAO span constraints they do not give a continuous return with regard to airport compatibility charges, and thus, are more naturally suited to constraints.

Two additional multiobjective methods have been installed into the suite for the latest results. Firstly, Pareto Frontiers can be developed for a number of objective functions. A ranking and sharing method is used, as suggested by Goldberg,\textsuperscript{9} whereby each GA population member is evaluated in terms of how many other members of the population dominate it, i.e. are superior in every KS objective function. The rank of a candidate is thus 1 plus the number of other members that dominate that candidate, such that all non-dominated solutions are ranked 1. Each member is then in turn given a sharing value in accordance with the number of other members that fall within a set distance for all objective function values. This sharing radius, \( \sigma_{\text{share}} \), is calculated using Equation 2 for each objective function \( i \), where \( f_{\text{max}} \) and \( f_{\text{min}} \) denote the maximum and minimum values respectively so far encountered for each objective function and \( n_{\text{pop}} \) the population size.

\[ \sigma_{\text{share}} = 0.9 \frac{|f_{\text{max}} - f_{\text{min}}|}{n_{\text{pop}} - 1} \quad (2) \]

Sharing is applied in order to guarantee diversity along the Pareto Frontier. The sharing value is then calculated as 1 plus the number of other population members that fall within this radius of the candidate for every objective function. The fitness values used for selection for each population member are then functionally related to the ranking and sharing values.

The second multiobjective method uses the principles of Game Theory.\textsuperscript{10} Each population is divided into subpopulation groups that are optimized for each KS objective function, with the superior candidates then transferred to the adjacent subgroup for the next iteration in order to resemble the Game Theory players alternating turns to maximize their profit. In addition, a co-operative form of this method is utilised to allow players to form bargaining strategies that potentially improve the global gain for each iteration for a sacrifice to a player’s individual objective function. This method, as outlined by Rao,\textsuperscript{11} involves normalizing each objective function value in accordance with best and worst observed values for a given population member, and creating a fitness value for that member that is the product of these values, so as to direct the optimization procedure away from the worst objective function values.

III. Results

Results have been mainly focused on the maximum passenger, maximum range (MPMR) performance, whereby the climb specific excess power (equivalently lift-to-drag ratio) and initial, mid and final cruise
altitude SARs were concurrently maximized. The Pareto Frontier multiobjective results and the optimal flight schedules for this mission are given in Table 1 and plotted in Figure 3.

A. Maximum Passenger, Maximum Range Pareto Analysis

<table>
<thead>
<tr>
<th>Flight Phase</th>
<th>$\Phi_1 (^\circ)$</th>
<th>$\Phi_2 (^\circ)$</th>
<th>$\theta_1 (^\circ)$</th>
<th>$\theta_2 (^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climb MPMR</td>
<td>69.17</td>
<td>57.80</td>
<td>3.67</td>
<td>3.60</td>
</tr>
<tr>
<td>Initial Crz MPMR</td>
<td>59.61</td>
<td>56.74</td>
<td>2.76</td>
<td>0.17</td>
</tr>
<tr>
<td>Mid Crz MPMR</td>
<td>77.02</td>
<td>62.38</td>
<td>3.85</td>
<td>2.68</td>
</tr>
<tr>
<td>Final Crz MPMR</td>
<td>82.53</td>
<td>80.67</td>
<td>2.11</td>
<td>3.04</td>
</tr>
</tbody>
</table>

Table 1. Maximum Range optimal geometric schedules for each flight condition.

![Figure 3. Optimal geometric schedules for each flight case.](image)

The percentage improvements in comparison with the datum aircraft for various flight performance parameters are given in Table 2. In addition, results are given for a planar wing of equal span to the optimized configuration, as well as a fixed planar wing that is restricted in span by the ICAO compatibility constraint. Finally, results are given for a Sharklet wing tip, as featured on the new A320 Neo aircraft, to serve as a comparison between active and fixed devices. All results are stated with respect to the datum aircraft equipped with wing fences, as with the original A320. Please note that the individual phase wing weight increments have been quoted, though the maximum value observed was used for each specific air range calculation as described in the methodology.

The immediate conclusion from the results is that despite incorporating the critical wing weight penalty for each flight case there remains a consistent SAR improvement across each phase of over 6% in comparison with the wing fence baseline, surpassing the predictions from previous analyses. It can be observed that the critical sizing cases for the wing are the initial and mid cruise segments, whereby a compromise is sought between improving aerodynamic efficiency while minimizing wing weight. The wing outer partition therefore utilises moderate cant angles of around 60° (30° dihedral) for this effect, giving a compromised wing structural and systems weight increase of approximately 27%. The wing also maximizes the outer partition span up to the tip chord constraint imposed, indicating that with regard to span the increased efficiency counters the added wing weight. In final cruise it can be seen that the MORPHLET is not weight
Table 2. Maximum Range deltas in key performance parameters using Pareto Frontiers.

<table>
<thead>
<tr>
<th>Flight Phase</th>
<th>(\Delta C_{L})</th>
<th>(\Delta L/D)</th>
<th>(\Delta W_{\text{wing}})</th>
<th>(\Delta \text{MTOW})</th>
<th>(\Delta \text{SAR})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climb (\text{MPM}R)</td>
<td>-22.65</td>
<td>4.56</td>
<td>-</td>
<td>-</td>
<td>1.31</td>
</tr>
<tr>
<td>Initial Crz (\text{MPM}R)</td>
<td>-25.02</td>
<td>9.49</td>
<td>26.60</td>
<td>3.05</td>
<td>6.44</td>
</tr>
<tr>
<td>Mid Crz (\text{MPM}R)</td>
<td>-29.11</td>
<td>10.68</td>
<td>26.70</td>
<td>3.06</td>
<td>6.59</td>
</tr>
<tr>
<td>Final Crz (\text{MPM}R)</td>
<td>-29.27</td>
<td>10.66</td>
<td>19.27</td>
<td>2.21</td>
<td>6.42</td>
</tr>
<tr>
<td>Climb (\text{MPM}R) planar</td>
<td>-26.08</td>
<td>4.99</td>
<td>-</td>
<td>-</td>
<td>0.26</td>
</tr>
<tr>
<td>Initial Crz (\text{MPM}R) planar</td>
<td>-29.12</td>
<td>10.29</td>
<td>31.81</td>
<td>3.65</td>
<td>5.69</td>
</tr>
<tr>
<td>Mid Crz (\text{MPM}R) planar</td>
<td>-30.30</td>
<td>10.84</td>
<td>30.44</td>
<td>3.49</td>
<td>6.00</td>
</tr>
<tr>
<td>Final Crz (\text{MPM}R) planar</td>
<td>-29.36</td>
<td>10.69</td>
<td>19.27</td>
<td>2.21</td>
<td>5.67</td>
</tr>
<tr>
<td>Climb (\text{MPM}R) fixed</td>
<td>-5.52</td>
<td>0.93</td>
<td>-</td>
<td>-</td>
<td>0.03</td>
</tr>
<tr>
<td>Initial Crz (\text{MPM}R) fixed</td>
<td>-6.62</td>
<td>2.14</td>
<td>7.45</td>
<td>0.86</td>
<td>1.22</td>
</tr>
<tr>
<td>Mid Crz (\text{MPM}R) fixed</td>
<td>-4.32</td>
<td>1.25</td>
<td>0.52</td>
<td>0.06</td>
<td>0.18</td>
</tr>
<tr>
<td>Final Crz (\text{MPM}R) fixed</td>
<td>-5.12</td>
<td>1.73</td>
<td>-5.14</td>
<td>-0.59</td>
<td>0.61</td>
</tr>
<tr>
<td>Climb (\text{MPM}R) Sharklet</td>
<td>-8.28</td>
<td>1.41</td>
<td>-1.0</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>Initial Crz (\text{MPM}R) Sharklet</td>
<td>-8.63</td>
<td>3.00</td>
<td>5.32</td>
<td>0.61</td>
<td>2.34</td>
</tr>
<tr>
<td>Mid Crz (\text{MPM}R) Sharklet</td>
<td>-9.03</td>
<td>3.12</td>
<td>-4.58</td>
<td>-0.53</td>
<td>2.35</td>
</tr>
<tr>
<td>Final Crz (\text{MPM}R) Sharklet</td>
<td>-8.85</td>
<td>3.11</td>
<td>-6.80</td>
<td>-0.78</td>
<td>2.29</td>
</tr>
</tbody>
</table>

Table 3. Geometric deltas with performance and mid cruise SAR improvement breakdowns.

Table 3 gives the geometric changes and performance improvement breakdown over the datum aircraft of the American Institute of Aeronautics and Astronautics.
for the mid cruise mission segment. The reference span has been increased by 28% with a corresponding reference wing area increase of 7.6%. Due to the span extension the wing weight increases by 26.7%, the product of a 35% primary wing structural weight increase and a 7.25% secondary wing weight increment. The maximum take-off lift coefficient has substantially risen, by over 13%, as an additional consequence of the increased leading edge length, and as a result the take-off and landing field lengths have both been reduced by over 6.0% and 7.0% respectively. There are also significant reductions in the required cruise thrust and the landing approach velocity. As seen in Table 2 the vortex-induced drag reduction for a given cruise lift coefficient is reduced considerably by over 29% due to the increased span, although the enlarged wing area has driven up the skin friction drag and thus also the zero-lift drag coefficient. These contribute to an overall drag coefficient reduction of over 9.0%, and thus a L/D gain of 10.7%. This delta then coupled with the IGW increment of 3.83% gives a SAR enhancement of 6.59% for mid cruise.

<table>
<thead>
<tr>
<th></th>
<th>⊿L/D_climb(%)</th>
<th>⊿SAR_ICPA(%)</th>
<th>⊿SAR_MC(%)</th>
<th>⊿SAR_FCA(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.82</td>
<td>5.98</td>
<td>-10.8</td>
<td>5.09</td>
<td></td>
</tr>
<tr>
<td>1.65</td>
<td>6.45</td>
<td>6.59</td>
<td>-3.76</td>
<td></td>
</tr>
<tr>
<td>4.48</td>
<td>5.83</td>
<td>6.63</td>
<td>-13.8</td>
<td></td>
</tr>
<tr>
<td>2.85</td>
<td>0.65</td>
<td>5.77</td>
<td>8.22</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Alternate Pareto Frontier solutions outlining individual objective function optima.

A number of potential solutions were generated along the Pareto Frontier, each tending toward certain objectives. Solutions are given in Table 4 that represent the individual maxima observed for each objective function. A configuration optimized for final cruise can be seen to offer an additional 1.8% SAR gain, though to the detriment of climb and initial cruise performance due to the increased wing weight. Other solutions show only limited gains of less than half a percent. This suggests that the Pareto Frontier method has proved very successful at generating a compromise solution that remains close to each individual optimum.

B. Short Range Pareto Analysis

<table>
<thead>
<tr>
<th>Flight Phase</th>
<th>⊿C_D(%)</th>
<th>⊿L/D(%)</th>
<th>⊿W_wing(%)</th>
<th>⊿MTOW(%)</th>
<th>⊿SAR(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climb SR</td>
<td>-29.4</td>
<td>3.40</td>
<td>-</td>
<td>-</td>
<td>-1.98</td>
</tr>
<tr>
<td>Initial Crz SR</td>
<td>-32.4</td>
<td>11.9</td>
<td>36.2</td>
<td>3.87</td>
<td>6.11</td>
</tr>
<tr>
<td>Mid Crz SR</td>
<td>-32.7</td>
<td>12.5</td>
<td>33.9</td>
<td>3.62</td>
<td>6.49</td>
</tr>
<tr>
<td>Final Crz SR</td>
<td>-32.7</td>
<td>12.5</td>
<td>33.9</td>
<td>3.62</td>
<td>6.49</td>
</tr>
<tr>
<td>Climb SR Sharklet</td>
<td>-0.47</td>
<td>0.59</td>
<td>-</td>
<td>-</td>
<td>-0.04</td>
</tr>
<tr>
<td>Initial Crz SR Sharklet</td>
<td>-3.11</td>
<td>3.16</td>
<td>3.19</td>
<td>0.34</td>
<td>2.52</td>
</tr>
<tr>
<td>Mid Crz SR Sharklet</td>
<td>-3.03</td>
<td>3.13</td>
<td>0.63</td>
<td>0.07</td>
<td>2.46</td>
</tr>
</tbody>
</table>

Table 5. Short Range deltas in key performance parameters.

Analysis has also been conducted on a Short Range (SR) mission, with results presented in Table 5. The shorter range mission profile represents an approximate 17% reduction in take-off gross weight, due to the reduction in required fuel. As a result the climb and initial cruise lift coefficients are reduced, and thus the potential for vortex-induced drag reduction, a function of the square of the lift coefficient, and thus SAR improvement, is not as dramatic. Nevertheless, moderate improvements are achieved for both. The fuel weight for mid and final cruise segments remains the same as for the longer range mission, however, and thus the SAR gains are comparable for both stage lengths. Figure 4 gives the geometric schedules, with a notable decrease in climb cant angle (increased dihedral) in comparison with the MPMR mission. Additionally, improvements are given for the Sharklet wing tip, with initial cruise SAR gains reduced from 2.3% for the MPMR mission to just 1.6% over the baseline, half that of the MORPHLET, with even further detriment to the climb performance.
C. Game Theory Analysis

The MPMR mission profile was repeated using Game Theory as a multiobjective solution method incorporated into the genetic algorithm. The optimization was undertaken using an equal number of generations as for the Pareto Frontier solutions, with results displayed in Table 6. It can be observed that using this method the performance improvements are not of the same magnitude as for the Pareto Frontier results, with both climb and initial cruise performance failing to prevail through the procedure, and both mid and final cruise efficiencies falling short of previous results. This was a feature commonly observed in results generated in comparison with Pareto solutions, and therefore, this particular method was not deemed to be as successful in generating a comprehensively strong configuration for each objective.

<table>
<thead>
<tr>
<th>Flight Phase</th>
<th>$\Delta C_D$(%)</th>
<th>$\Delta L/D$(%)</th>
<th>$\Delta W_{wing}$(%)</th>
<th>$\Delta MTOW$(%)</th>
<th>$\Delta SAR$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climb MPMR</td>
<td>-9.24</td>
<td>1.60</td>
<td>-</td>
<td>-0.06</td>
<td>0.76</td>
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<tr>
<td>Initial Crz MPMR</td>
<td>-0.06</td>
<td>0.37</td>
<td>-0.48</td>
<td>-0.06</td>
<td>0.44</td>
</tr>
<tr>
<td>Mid Crz MPMR</td>
<td>-17.9</td>
<td>5.84</td>
<td>5.48</td>
<td>0.63</td>
<td>4.80</td>
</tr>
<tr>
<td>Final Crz MPMR</td>
<td>-19.7</td>
<td>6.84</td>
<td>6.91</td>
<td>0.79</td>
<td>5.75</td>
</tr>
</tbody>
</table>

Table 6. Maximum Range deltas in key performance parameters using Co-operative Game Theory.

IV. Conclusion

Multiobjective optimization has been performed to produce a MORPHLET design that incorporates multiple objective functions defining multiple stages of a flight envelope, using both Ranking and Sorting Pareto Frontier and Game Theory methods. Concurrent optimization of geometric twist and cant angles for each flight phase has allowed for improved wing weight sizing accuracy and robustness, and thus trim analysis, while minimizing computational time. In addition the enhancements to various low speed performance parameters have also been ascertained. It has been determined that a consistent 6.0% specific air range gain can be accomplished across all cruise phases for a maximum range mission profile, in addition to achieving a 4.5% lift to drag ratio improvement in climb. These represent greater values than those achieved by a planar wing of equivalent span, and vastly superior results to those of a span-limited planar wing. Furthermore, they represent consistently greater improvements across all flight phases than Sharklet wing tips.

For a shorter range mission the benefits are not as great, but still offer a considerable improvement over the datum aircraft and Sharklets. A number of performance parameters have been enhanced, including take-off and landing field length reductions of 6.5% and 7.5% respectively, a consequence of maximum lift
coefficient increases for take-off and landing of over 13%. In addition, approach speeds, the second segment climb gradient and cruise thrust requirements have all been significantly reduced. Solutions achieved via the application of Pareto Frontiers were found to give very encouraging results in comparison with each objective function individual optimum, and served as a superior method for this analysis in comparison with Game Theory.

Acknowledgments

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References