Efficient frequency analysis for wireless structural health monitoring systems

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Abstract
Structural health monitoring systems are becoming increasingly popular to detect and evaluate faults in structural systems. Many of these systems are applied to large structures and wireless sensor systems (often incorporating energy harvesting devices) avoid the significant problems of long cable runs in these cases. In wireless systems energy management is very important, and data transmission and computation are significant drains on the available energy. Often it is sufficient to transmit frequency domain data rather than time domain, and this reduces the bandwidth of the communication and hence reduces power demand. Thus, this paper considers the efficient calculation of the frequency response of the structure.

Introduction

Structural health monitoring is becoming popular in a wide range of applications from highway bridges to aerospace structures. The identification of the location and severity of cracks, loose bolts and other types of damage in structures using vibration data has received considerable attention [1]. Many of the approaches use modal data of a structure before damage occurs as baseline data, and compare all subsequent tests to this baseline [2]. Any deviation in the modal properties from this baseline data is used to estimate model parameters related to the damage severity and location. An alternative approach is to use frequency domain measurements directly, for example comparing the impedance of a joint with a baseline measurement [3].

In many structural health monitoring systems, the sensors are connected to the data acquisition and analysis system by wires, and hence there are no constraints on the power consumption of the monitoring algorithms. In this case the requirement is to determine the key features in the data that detect and diagnose any damage present and any signal processing will retain full accuracy. It is clear that the quantity of information is reduced from the time series data, through to the minimum data required to make a robust diagnosis and decision on the system. The key features might be changes in natural frequency, a change in vibration amplitude, an impedance measurement, acoustic emission measurements, and so on. Furthermore some systems are active, where the structure is forced with a known signal.

With larger structures the use of wires becomes cumbersome, and in the case of aircraft too heavy. Thus there is a move towards wireless SHM systems where the data transmission bandwidth is constrained. This is further complicated when the sensor is required to operate remotely from a power source. The options for power are limited to battery systems, possibly with the addition of energy harvesting. In this case, the power management of such a sensor becomes critical. In particular, the data acquisition, the signal
processing and the data transmission all require significant power resources. Zhou et al. [4] highlighted that the wireless transmission consumes the largest power, but lasts only a short time. Hence there is a trade-off between computation within the smart sensor and data transmission, assuming that the computation leads to a reduction in the transmission requirements. Transforming from the time domain to the frequency domain is a common approach and is very successful in extracting information from measured data, reducing the effect of any noise present and hence reducing the data transmission requirements.

The Fourier transform (FT) is fundamental to the frequency analysis of vibration data. Since the early nineteen seventies the FT has been implemented digitally using the fast and efficient FFT algorithm to determine the discrete Fourier transform (DFT). Although the FFT is fast, approximations have been developed. One approximation is given by the mono-bit FFT in which the DFT kernel is quantized [5-11]. This paper examines the errors caused by these approximations, and the possible applications in vibration data analysis.

**Quantization of the DFT kernel**

Consider a sequence of sampled data \( x(r) \) where \( r = 0, 1, \ldots, N \). The discrete Fourier transform (DFT) is given by

\[
X(k) = \sum_{r=0}^{N-1} x(r) e^{-j2\pi kr/N} \quad \text{where} \quad k = 0, 1, \ldots, N-1,
\]

and \( X(k) \) are the Fourier components. The hard quantization of the DFT kernel, i.e. the set of complex exponentials \( e^{-j2\pi kr/N} \), approximates the complex exponential by functions that are easier and faster to calculate. For example, the mono-bit DFT is obtained, for an angle \( \phi \) such that \(-\frac{1}{4}\pi \leq \phi < \frac{7}{4}\pi\), by

\[
e^{j\phi} = f(\phi) =
\begin{cases}
1 & -\frac{1}{4}\pi \leq \phi \leq \frac{1}{4}\pi \\
-j & \frac{1}{4}\pi < \phi \leq \frac{3}{4}\pi \\
-1 & \frac{3}{4}\pi < \phi < \frac{5}{4}\pi \\
j & \frac{5}{4}\pi \leq \phi < \frac{7}{4}\pi 
\end{cases}
\]

Note that there is some choice at the boundaries of the angle \( \phi \) in the definition of the quantization given in Equation (2), that is the choice of interval in which to place \( \phi = \frac{2i-1}{4}\pi \), for integer \( i \). The definition of intervals in Equation (2) has the great advantage of retaining the symmetry about the Nyquist frequency in the DFT, where \( X(k) = \bar{X}(N-k) \) for \( k = 1, \ldots, N/2-1 \), and the overbar denotes the complex conjugate. The definition given by Gómez-García and Burgos-García [9], for example, does not have this property. As \( N \) becomes large the difference in the mono-bit DFTs resulting from different boundary definitions in Equation (2) become small, as the proportion of values of \( \phi \) on the boundaries decreases with increasing \( N \).

In practice the mono-bit DFT is computed using a variant of the FFT algorithm. With the standard DFT algorithm the original data is recovered by using the inverse DFT; for the quantized kernels this is not the case and the original data may only be recovered approximately. Determining the savings in computation and memory, and hence power, between the mono-bit and classical DFTs is difficult, as the results depend significantly on the implementation and the application involved. The purpose of this paper is to highlight
that there are alternative methods that may be used to advantage, and for any application where the power management is critical requires a careful analysis of the accuracy requirements of the output DFT. What is clear is that the look-up table required for the kernel is much smaller for the mono-bit DFT. Furthermore the mono-bit DFT can be implemented using additions rather than multiplication, potentially saving significant computation.

Quantitative analysis of the effect of quantization of the FFT kernel

The question that arises when the FFT kernel is quantized is the effect on the amplitudes of the resulting FFT at different frequencies. To consider these issues we consider the Fourier transform (or more accurately series) of a sinusoidal signal over a finite time interval, so that we can avoid issues related to the sampling. Consider the signal, \( x(t) \), with frequency \( \Omega \), so that

\[
x(t) = x_0 \cos \Omega t.
\]  

(3)

The signal is measured over a time period, \( T \), where we assume that \( T = m(2\pi/\Omega) \) for some integer \( m \). This means that the \( m \) cycles of the signal are measured and there is no leakage when the signal is transformed to the frequency domain. Since a finite time period is consider we calculate the Fourier series of the signal as

\[
x(t) = \sum_{r=-\infty}^{\infty} c_r e^{j r \Omega_0 t}.
\]  

(4)

where \( \Omega_0 = T/2\pi \) is the fundamental frequency based on the measurement period, and for complex constants \( c_r \) given by

\[
c_r = \frac{1}{T} \int_0^T x(t) e^{-j r \Omega_0 t} dt.
\]  

(5)

Note that the \( c_r \) terms represent the values of the Discrete Fourier Transform (DFT) at frequency \( r \Omega_0 \). With no errors in the DFT kernel the only non-zero coefficient would be \( c_m \). For the standard definition of the DFT, the exponential is defined in the usual way as \( e^{j \Omega t} = \cos \Omega t + j \sin \Omega t \). When the DFT kernel is quantized the exponential is approximated by a function that is periodic, but contains higher frequency components. Thus the approximate function may be written as a Fourier series so that

\[
e^{j \Omega t} \approx f(t) = \sum_{n=-\infty}^{\infty} a_n e^{j n \Omega t}.
\]  

(6)

Thus, combining Equations (5) and (6) and since \( \Omega = m \Omega_0 \), the approximate value of the DFT coefficients is then

\[
c_r = \frac{1}{T} \int_0^T \left( \cos m \Omega_0 t \right) \left( \sum_{n=-\infty}^{\infty} a_n e^{j n \Omega_0 t} \right) dt = \frac{1}{T} \sum_{n=-\infty}^{\infty} \left( a_n \int_0^T \left( \cos m \Omega_0 t \right) e^{j n \Omega_0 t} dt \right).
\]  

(7)

Note that the integral is only non-zero if \( m = \pm rm \), and hence the sum is simplified to give

\[
c_r = \begin{cases} 
\frac{1}{2} (a_{m/r} + a_{-m/r}) & \text{if } m/r \text{ is an integer} \\
0 & \text{otherwise}
\end{cases}
\]  

(8)
Since $c_r$ can only be non-zero if $|r| \leq m$, that is for sub-harmonics of the signal. The situation for the sampled data is more complicated, particularly when the frequency lines are not present at the sub-harmonic frequencies. In this case these frequency components will appear at higher frequencies; the values of these frequencies will also depend on the sample rate. This will be demonstrated in the numerical examples.

For the mono-bit FFT, given by the approximation in Equation (2), the coefficients $a_n$ are

$$a_n = \begin{cases} 2\sqrt{2}/n\pi & \text{if rem}(n,8) = 1 \text{ and } n > 0 \\ 2\sqrt{2}/n\pi & \text{if rem}(n,8) = 3 \text{ and } n < 0 \\ -2\sqrt{2}/n\pi & \text{if rem}(n,8) = 5 \text{ and } n > 0 \\ -2\sqrt{2}/n\pi & \text{if rem}(n,8) = 7 \text{ and } n < 0 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

where rem$(n,8)$ denotes the remainder after division of $n$ by 8. Figure 1 in the paper by Nelson [7] gives the Fourier transform based on the quantized kernel and highlights the presence of these sub-harmonics for the mono-bit FFT.

**Examples**

A number of simple examples will be given to demonstrate the properties of the DFT kernel quantization using the mono-bit DFT given in Equation (2). The performance of the mono-bit DFT is compared with the classic FFT. Let $N = 2^{11} = 2048$, $T = 6.4s$, so that $df = 0.15625Hz$.

**Case 1. Sinusoidal signal with no leakage and available sub-harmonic frequency lines**

Let $x(t) = \cos(2\pi f t)$ where $f = 16.40625$ Hz, so that $m = 3 \times 5 \times 7 = 105$. Clear the excitation frequency has been chosen so that the 3rd, 5th and 7th sub-harmonic frequency lines are present. Figure 1 shows the results from the classic and mono-bit DFT. Obviously the classic DFT gives the exact result. The mono-bit DFT shows a clear frequency line at the excitation frequency and its amplitude is given from Equation (9) by $a_1 = 0.9003$. There are clearly distinct frequency lines at the 3rd, 5th and 7th sub-harmonic frequencies, given by 5.46875, 3.28125 and 2.34375 Hz respectively, with amplitudes $a_3 = a_1 / 3 = 0.3001$, $a_5 = a_1 / 5 = 0.1801$ and $a_7 = a_1 / 7 = 0.1286$. Since frequency lines are not available for the lower frequency sub-harmonic, these components appear at higher frequencies in the spectrum.
Case 2. Sinusoidal signal with no leakage

The frequency of the sinusoidal signal is changed to $f = 10\text{Hz}$, so that $m = 64$ and in this case no frequency lines exist for the sub-harmonics. Figure 2 shows the comparison between the classic DFT and the mono-bit DFT. The mono-bit FFT gives a similar spectrum to the classic DFT, and the amplitude at the excitation frequency is again given by $a_1 = 0.9003$. The sub-harmonics cannot appear in the low frequency spectrum, but their contribution is spread amongst higher frequency components.
Case 3. Sinusoidal signal with leakage

The frequency of the sinusoidal signal is now changed from 10 Hz to 10.078125 Hz. This frequency is equal to the original frequency shifted by $df/2$. This will give the worst case of leakage, as shown in Figure 3. In normal signal processing practice the input data is multiplied by a window function to minimize the leakage. However, this is not done here because the windowing requires further computation that will increase the power requirements.

![Figure 3: The classic and mono-bit DFT for a sinusoidal signal with leakage](image)

Case 4. Sinusoidal signal with leakage and noise

Signal noise is inevitable in any measurement. To simulate this situation random noise is added to the input signal thus: $x(t) = \cos(2\pi ft) + 0.3\text{randn}(t)$, where the MATLAB randn function provides Gaussian random noise with a standard deviation of 1. Again $f = 10.078125$ Hz so that leakage is present. Figure 4 shows the resulting spectra. As expected the noise produces components in the spectrum, but the level of noise is essentially the same in the classic and the mono-bit DFT analysis.
Case 5. Transient response of a single degree of freedom damped system

Suppose the signal is a damped sinusoid given by the response of a single degree of freedom (sdof) system with natural frequency 10 Hz and a damping ratio of 1%. Figure 5 shows the classical single degree of freedom response, and also the result of the mono-bit DFT, which again shows the additional discrete frequency components.

Quantization of the signal

Signal quantization is desirable because, potentially, there will be a substantial saving in power required if the ADC has lower resolution, or ultimately is a 1 bit device.
Case 6. Sinusoidal signal with signal quantization

The input signal is now quantized to 1 or −1, depending whether the input is positive or negative. Figure 6 shows the results for a sinusoidal signal with frequency \( f = 10.078125 \text{Hz} \). The classic DFT shows that leakage is present and that the signal quantization has introduced harmonics of the signal frequency. Remarkably, the harmonic signal frequency component is still clearly detected in the spectra and there is little difference between the classic and mono-bit DFTs for low frequencies.

![Figure 6: The classic and mono-bit DFT for a quantized sinusoidal signal](image)

Results with measured data

The mono-bit DFT is now tested on real measured data. The structure was a three storey bookshelf structure, that was built and tested at the Los Alamos National Laboratory [12]. The data consists of time responses of 8192 samples measured with sampling rate of 1600Hz. The data used was the undamaged case, with an input voltage to the shaker of 2V, and the measurement point was 3DP [12]. Figure 7 shows the complete time signal, and Figure 8 shows a zoomed portion that gives more detail of the response. The signal was transformed to the frequency domain using the classical DFT and also the mono-bit DFT. Figure 9 shows the results and highlights that the quantization of the kernel for the mono-bit DFT has very little influence in the frequency domain, particularly near the peaks.
Figure 7: The complete time signal for the measured signal from the bookshelf structure

Figure 8: A portion of the time signal for the measured signal from the bookshelf structure
Figure 9: The classic and mono-bit DFT for the measured data from the bookshelf structure

Conclusions

This paper has investigated the performance of the mono-bit DFT on vibration data, with a view to reducing the computation and data transmission requirements for wireless sensor systems. The paper has concentrated on the performance aspects, rather than detailed power consumption estimations, and has demonstrated that significant approximations in the calculation of the DFT can yield surprisingly good results in the frequency domain. Certainly dominant frequencies in the response signal may be identified, and this may be sufficient to detect the onset of damage. Much more work needs to be performed on the detailed implementation of these methods, and the estimation of the savings in power obtained.

References


