Understanding the Influence of Input Variables on the Stability of a Two Linked Robot

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Abstract

The paper develops a mathematical model of the passive and stable dynamic walking process of a biped robot on sloping surfaces. The robot is modelled with two links corresponding to the left and right legs; the upper body is modelled as a mass located on the hip joint. The motion during the walking process consists of two phases, namely the single support (swing) phase and the double support phase. These phases occur sequentially in a bipedal walking cycle and are modelled as follows:

1. A set of differential equations analyze the double pendulum motion and describe the dynamics during the single support phase.
2. The impact of the swing leg with the ground when both legs are in contact, and the initial conditions for the next swing phase, are obtained by the conservation of angular momentum from a transition analysis.

Limit cycles in the phase plane and bifurcation diagrams are used to investigate the periodic motion of a biped robot. For simplicity, the biped robot was assumed to have point feet. The mass and length ratios of the robot and the slope of the plane are the main effective parameters that characterize the gait properties such as step period, velocity, step length and initial conditions. Investigation of such passive walking motion can help to determine the optimum energy required for a stable walking process, to understand the physics of human motion and lead to the development of a controller for active walking machines that uses the least torque.

1. Biped walking model

The biped robot based on passive walking proposed in this project has two rigid links which are connected to the hip joint (see Fig. 1) and three masses which are hip mass, non-support link mass and support link mass respectively. The lengths of links are same (L) and gravity denoted as g. This walking model is studied in (Goswami et al., 1996). The motion of the robot is constrained to the sagittal plane. The scuffing problem of the swing-leg, which is inevitable in the case of a biped kneeless robot of which motion is constrained to the sagittal plane, is neglected during the swing phase (McGeer, 1990). The dynamics of the biped robot consists of two parts with different governing equations: the differential equations describing the dynamics of the robot during the swing phase that the robot act as a double pendulum (see Fig. 1.a), and an impulse model of the contact event (see Fig. 1.b).

The dynamic equations of the swing phase are similar to the well-known double pendulum equations. During the swing phase, the stance leg acts as a supported link of double pendulum and the swing leg acts as a non-supported link of double pendulum.

The governing equations of the swing phase model (Goldstein, Poole and Safko, 2002), are written as:

\[ M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + K(\theta) = 0 \]
Where  \( \theta = [\theta_1 \ \theta_2]^T \). The details of the matrices are:

\[
M(\theta) = \begin{bmatrix} m_1L^2 + m_H L^2 + m_2L^2 & m_2Lc \cos(\theta_1 - \theta_2) \\ m_2Lc \cos(\theta_1 - \theta_2) & m_2L^2 \end{bmatrix}
\]

\[
C(\theta, \dot{\theta}) = \begin{bmatrix} 0 & m_2Lc \dot{\theta}_2 \sin(\theta_1 - \theta_2) \\ -m_2Lc \dot{\theta}_1 \sin(\theta_1 - \theta_2) & 0 \end{bmatrix}
\]

\[
K(\theta) = \begin{bmatrix} (m_1Lg + m_H gL + m_2Lg) \sin(\theta_1) \\ m_2Lg \cos(\theta_2) \end{bmatrix}
\]

These two second order differential equations describe the motion of the double pendulum’s system. As there are four unknowns, the motion is described by the following four first-order differential equations. The equations are solved using MATLAB ODE45.

\[
\dot{z}_1 = \theta_1, \quad \dot{z}_2 = \theta_2, \quad \dot{z}_3 = \dot{\theta}_1, \quad \dot{z}_4 = \dot{\theta}_2
\]

An impact occurs when the swing leg touches the walking surface. In this research the impact is assumed to be inelastic and without slipping, and the stance-leg lifts from the ground without interaction. The angular momentum of both (i) the robot about the impacting foot and (ii) the pre-impact support leg about the hip is conserved.

### 2. Bifurcation Diagrams and Phase Plane Limit Cycles

As discussed before the objective of this analysis is to predict regions of stable and unstable walking process as defined by initial conditions such as initial angular velocity (\( \dot{\theta} \)), angular position (\( \theta \)), mass ratio, length ratio and the slope angle. In a complex walking situation, these variables may be required to change at every step. The proposed analysis allows the prediction of stable regions for each step so that the next step can be planned. A Bifurcation diagram is plotted by varying values of mass and length ratio for different slope values and solving Equation 2 for angular velocity and displacement for each case. An example of the Bifurcation diagram with varying slope angle values, length ratio equal to 1 and mass ratio equal to 2 is shown in Figure 2a. An unstable region is a region of input values where Robot cannot walk. The stable region...
has three areas: (i) Single Periodic Step Cycle (ii) Multiple Periodic Step Cycles (iii) No periodicity in step cycles (Chaotic Area). The relationship between angular velocity and angular displacement can be plotted at any cross section of the Bifurcation diagram. This diagram is referred to as Phase Plane Limit Cycle. Figure 2.6 shows the Phase Plane Limit Cycle diagram drawn at the slope value of 4.85° (Section AA’, Figure 2a). The two cycle curves shown in Figure 2b correspond to the two Periodic Step Cycles identified in the Bifurcation diagram. Two periodic cycle means four steps (a schematic of single step is shown in Figure 1) are repeated in the walking process. For the given values of slope angle, mass and length ratios, these curves represent a range of stable values for angular velocity and angular displacement. The values within the close tolerance of these curves are also acceptable.

Figure 2: Bifurcation diagram: \( \beta = 1 \), \( \mu = 2 \) and slope angle \( \varphi \) increased from 0.1° to 5.2° (Fig 2.a). Phase plane limit cycles of a 2-periodic gait for Length ratio = \( \beta = \frac{b}{a} = 1 \), Mass ratio \( \mu = \frac{m_H}{m} = 2 \) and slope angle \( \varphi = 4.85° \) (Fig. 2.b).

3. Characterising the Influence of Input Variables

The actuators and controllers are used to keep the robot in the stable region. During the walking process, change in the environment can change the slope angle or a robotic leg may need to react to sudden changes in the angular velocity values. Understanding the influence of input variables on the stability of the walking process is essential in order to provide a feedback to the actuators and controllers attached to the robot. For any given value of slope angle, mass ratio and length ratio the actuators and controllers must ensure that angular velocity and angular displacement values are within the tolerance range of the corresponding Phase Plane Limit Cycle curves. If necessary the controllers may also change the mass ratio and length ratio. The information on the Bifurcation diagrams and Phase Plane Limit Cycles need to be generated for all combinations, a priori, so that controllers can make a decision on providing a corrective action.

Effect of mass ratio: By analysing bifurcation diagrams for different mass ratio, it was found that increasing the mass ratio will make the robot unstable at lower slope angle values (Fig. 2.a & Fig. 3.b). In other words, the high mass ratio means that the mass of the hip is heavier than the mass of legs which is likely to make the robot less stable.

Effect of length ratio: If the length ratio is increased i.e. the position of the leg mass is lowered from the centre of the link then the robot is likely to become unstable early. For example, the robot state is stable (single periodic step cycle) at slope angle 4, mass ratio 2 and length ratio 1
Figure 3.a shows the effect of changing the length ratio. A length ratio of 1.47 would push the robot in the chaotic region and at a 1.51 value it becomes unstable. Lowering the length ratio or moving the mass of the link towards the hip mass would keep the robot stable.

**Effect of slope angle:** As shown in Figure 2a, increasing the slope value will make the robot less stable.

![Figure 3a](image)

![Figure 3b](image)

**Figure 3:** Bifurcation diagram: $\mu = 2, \varphi = 4$ and length ratio ($\beta$) increased until the gait becomes chaotic (Fig. 3.a). Bifurcation diagram: $\beta = 1, \mu = 1$ and slope angle $\varphi$ increased from 0.1° to 5.7° (Fig. 3.b).

### 4. Conclusions

The objective of this research is to develop a multi-linked exoskeleton model that can assist disabled people during their walking process. The first step of this research was to develop an understanding of the computational issues involved.

A model has been developed to relate the slope angle, mass ratio and length ratio of a simple two linked robot with the angular velocity and angular displacement values associated with each link. The motion of the swing phase of the link is described by a double pendulum model. A basic contact phase model that conserves angular momentum about the impacting foot as well as the hip has been used.

Stable regions with single/multiple periodic step cycles for identified for different values of input variables and shown using the Bifurcation and Phase Plane Limit Cycle diagrams.

### References