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The optimisation of hierarchical structures with applications to morphing aircraft

M.I. Friswell*, J.E. Herencia, D. Baker, P.M. Weaver

Department of Aerospace Engineering, University of Bristol
Queen's Building, University Walk, Bristol BS8 1TR, UK
m.i.friswell@bristol.ac.uk

Abstract

Many structures, both man made and in nature, are hierarchical in the sense that there are structures on more than one length scale and the performance is enhanced by the optimization of such a system. The length scales involved can go down to the micro and nano scales, for example considering the crystal structure of solids. However the emphasis in this paper is the macro length scale and the use of hierarchical designs to morph aircraft wings. Two examples will be given, namely the optimization of the composite lay-up to enhance a structure's anisotropic properties and the optimization of truss and skin elements in a compliant mechanism approach to morphing aircraft.

1 Introduction

Although significant efforts have been expended in research into adaptive structures and morphing aircraft, examples of practical solutions are still very few and far between. In essence the past examples have relied on applying significant actuation force to enable a shape change. Biological inspiration shows an alternative approach, increasing the role of information to reduce the energy requirements. From the structural perspective the objective is to produce fully integrated, hierarchical structures with compliance control. The requirements of the structure are conflicting: the structure must be stiff to ensure the external loads cause only small deformations, but must be flexible to enable shape changes. The only solution to this conflict is to carefully design the structure to decouple the two actions. The skin of a morphing aerofoil is a good example; the skin must be stiff to withstand the aerodynamic loading, but flexible to allow the aerofoil section to deform.

If one observes living structures they are constructed from a hierarchy of structure from the cell level to the macro level. Engineering has far fewer levels, for example composite structures have stiff fibres in a flexible matrix and in essence only have two levels. Chiral structures use two levels to produce interesting properties such as a negative Poisson's ratio. Introducing hierarchy into adaptive structures has huge potential, but one of the major challenges is the design of the structure across the length scales. This requires equivalent models of the smaller scale structures in order to optimise the higher levels; the use of lamination parameters for composite structures is an example of this. Of particular importance will be the interfaces between different components, for example trusses and skins. The possibility to vary compliance and integrate actuation is also important.

This paper will demonstrate two morphing aircraft structures that are hierarchical, namely an aeroelastically tailored composite structure of conventional design and a compliant trailing edge constructed of truss elements and a morphing skin. These examples will show the huge potential of such an approach to design optimal structures, although there is still significant research to produce robust and efficient means to design and construct such systems.

2 Morphing aircraft

The design of conventional fixed wing aircraft is constrained by the conflicting requirements of multiple objectives. Mechanisms such as deployable flaps provide the current standard of adaptive aerofoil geometry, although this solution places limitations on maneuverability and efficiency, and produces a design that is non-optimal in many flight regimes. The development of new smart materials together with the always present need for better UAV performance is increasingly prompting designers towards the concept of morphing aircraft (Wlezien et al. [21], Jha and Kudva [12], Sanders et al. [17], Bae et al. [3], Friswell and Inman [7], Campanile [4]). These aircraft possess the ability to adapt and optimize their shape to achieve dissimilar, multi-objective mission roles efficiently and effectively. One motivation for such uninhabited aircraft are birds that morph between cruise and attack missions by changing their wing configuration accordingly. Birds also use camber and twist for flight control. The Wright Brothers used wing warping as a seamless flight control in their first flying machine. Morphing wings for flight control bring new challenges to the design of control laws for flight. Because configuration changes move the aerodynamic centre, control of the aircraft during planform morphing requires attention. Hence both morphing mechanisms and control systems must be considered.

The structural technologies available to achieve shape changes in a morphing aircraft fall into two major categories, namely planform changes using *rigid* mechanisms, and compliance (for example wing twist or compliant mechanisms). Methods using compliance are of interest in this paper, and in particular two approaches will be considered. The first is aeroelastic tailoring, where the stiffness distribution of a conventional wing layout is optimized based on weight or aerodynamic objectives. Composite materials present high specific strength and stiffness ratios, and primary flight structures, such as wings or fuselages, are mainly designed using stiffened panels. Structures made of composite materials can be stiffness tailored, and this is a significant advantage over their metallic counterparts. The second approach is the use of compliant mechanisms to deform the camber of the airfoil section. Such structures may be designed using topology optimization or direct optimization of the geometry of truss structures. However satisfactory designs often require truss element properties that can only be realized using an hierarchical structure approach.

3 Aeroelastic tailoring by optimizing the composite lay-up

Composite materials have the potential to be stiffness tailored, which fits in well with the purpose of aeroelastic tailoring (Shirk et al. [18], Weisshaar and Ryan [20]). Composites can show anisotropic properties or elastic coupling terms. Two common types of anisotropy are membrane and flexural, and these are mainly related to unbalanced or off axis laminates. Flexural anisotropy is influenced by the laminate stacking sequence, whereas membrane anisotropy is affected by the laminate volume fractions or ply percentages content. For instance, if flexural anisotropy is neglected in buckling analysis, buckling load factors can be unconservative. Designing for elastic tailoring is characterized by the use of the composite anisotropy to improve composite structural performance (Herencia et al. [11], Fukunaga et al. [9], Fukunaga and Sekine [8]). For example, a composite plate with flexural anisotropy (that is bend-twist coupling terms) under bending loads will twist. Rehfield et al. [16] and Lemanski and Weaver [13] provided a set of guidelines to design composite wings or flaps with bend-twist coupling. However, due to practical, manufacturing considerations, composites have been restricted to symmetric or mid-plane symmetric laminates with 0, 90, 45 and -45 degree ply angles.

Over the years, local and global optimization techniques have been developed for composite design. Local optimizations have concentrated on the specific components or part subassemblies of an aircraft structure made mainly of laminated composites. Gradient, Genetic Algorithm (GA) or Gradient-GA based techniques have been developed. Composite materials may be modeled and characterized by their stacking sequences or alternatively by lamination parameters. On the other hand, global optimizations have addressed the wing as a whole accounting for the interaction between components. Several methods have been proposed from single to multilevel approach, accounting for aero-structure interaction having high fidelity models, characterizing the wing as a beam and approximating the aerodynamics, and so on. However, existing studies have limited use of aero-structure coupling, practical design rules, anisotropy in the laminated composites and structural sizing constraints. Herencia et al. [10] reviewed the literature on the optimization of composite lay-ups and the application to morphing aircraft structures.

The aim is to provide a method to design an aircraft wing of laminated composites that possesses morphing capabilities employing aeroelastic tailoring. Morphing is achieved passively so that the aircraft wing will adapt itself to improve its performance during the designed flight conditions. The approach consists of an aeroelastic steady-state scheme with aero-structure coupling embedded within a global optimization. The global optimization is divided into two levels. At the first level, Mathematical Programming (MP) is used to optimize the wing under structural and aerodynamic constraints. The wing-box panels (skins and spars) are modeled using lamination parameters accounting for their anisotropy. The panels are assumed to be symmetric or mid-plane symmetric laminates with 0, 90, 45 or -45 degree ply angles. Each of the wing-box panels is subjected to a combined in-plane loading under strength, buckling and practical design constraints. At the second level, the actual lay-ups of the wing-box panels are obtained using a GA, accounting for manufacture and design practices. Thus the aeroelastic steady-state simulation with aero-structure coupling is embedded within a two level global optimization, which accounts for composite material anisotropy, lift and induced drag variations as well as internal load redistribution under structural and aerodynamic constraints.

3.1 Example wing geometry, structure and loading

A swept back wing configuration as shown in Figure 1 is considered. The wing structure is divided into the Wing-Box (WB), the Leading and Trailing Edge (LE and TE) and the ribs. The wing-box structure comprises the skin (top and bottom) and the spars (front and rear). Each of these substructures consists of several components or unstiffened panels. This study concentrates on the wing-box structure which has a predetermined layout. The wing-box is assumed to be subdivided into five rib bays, with the skin and spar panels having different properties in each bay. The skin and spar panels, due to aerodynamic forces, experience a combined loading; however it is assumed that this loading will be mainly membrane in nature. For analysis purposes and simplicity, it is also assumed that the skin and spar panels are flat and can be represented as rectangular plates with simply supported conditions along the interface with the adjacent panels.

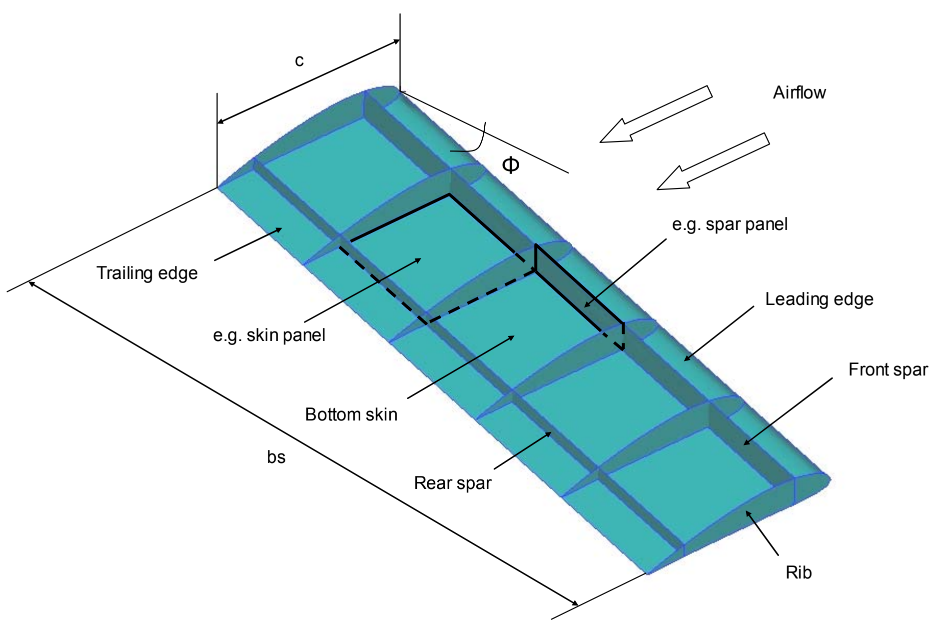

Figure 1: Wing structure layout for aeroelastic tailoring

An aeroelastic steady-state response technique is employed to calculate the aerodynamic loads. From initial flight conditions and wing geometry an aerodynamic mesh is produced. PANAIR is then used to compute the aerodynamic pressure loads. The aerodynamic pressure loads are subsequently converted into nodal loads to be applied to a finite element (FE) model. The structural and aerodynamic meshes share the same characteristics. The material properties together with the structural mesh and nodal loads are used to generate the FE model of the wing. FE analysis is then carried out using MD NASTRAN to obtain the structural deformation. The scheme is iterative and at each iteration the geometric displacement of the grid positions is evaluated. The process terminates when the difference of the geometric distance between iterations is less than a specified tolerance. A relaxation factor is used to achieve numerical stability.

3.2 Global optimization strategy

The global optimization strategy is shown as a flow chart in Figure 2 and is divided into two stages similar to that given by Herencia et al. [11]. At the first stage, the wing-box panels are optimized using lamination parameters and gradient based techniques under structural and aerodynamic constraints. The optimum thicknesses and values of the lamination parameters for each of the panels are identified. At the second stage, a GA is used to target the optimum lamination parameters to obtain the actual lay-ups for each of the skin and spar panels.

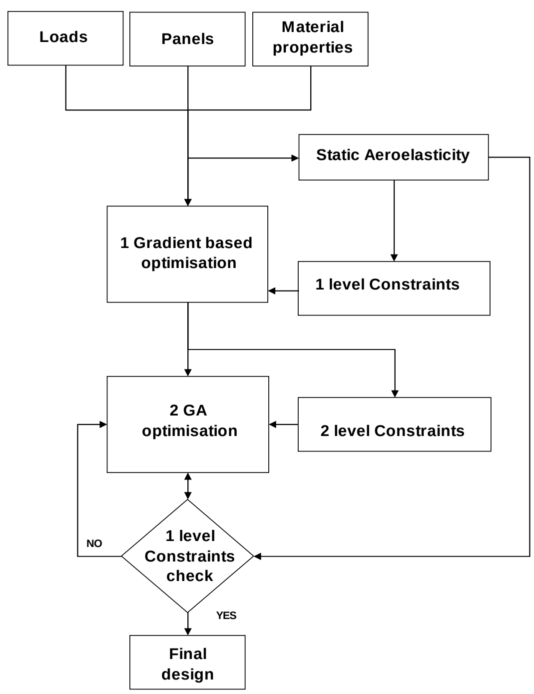

Figure 2: Global optimisation flow chart

The objective function at the first stage is the mass of the wing-box panels (skins and spars), and the inequality constraints consist of structural constraints such as strength, buckling or practical design rules, and aerodynamic constraints (lift and induced drag). The design variables are the thicknesses and the membrane and bending lamination parameters of the skin and spar panels. Note that for every optimization cycle at this level, an aeroelastic-steady state scheme is performed.

Classical Laminate Theory (CLT) is applied to the wing-box panels (skins and spars) assuming laminates are symmetric or mid-plane symmetric, and provides the link between the discrete lay-up and the continuous model used for the optimization. However, not all laminates defined by a particular set of lamination parameters can be manufactured, and constraints must be included by defining a feasible region in the space of lamination parameters, as given by Miki and Sugiyama [15] and Diaconu and Sekine [5]. Furthermore practical design constraints are applied, for example that at least 10% of each ply angle (0, 90, 45, -45) should be included.

At the second stage, a standard GA is employed at this level to solve the discrete lay-up optimization problem. The lamination parameters from the first optimization level are targeted to obtain the actual lay-ups for the skin and spar panels. Note that at this level the GA is applied separately to each of the wing-box panels.

3.3 Numerical example

A typical UAV aircraft with a swept back wing configuration was used to test the proposed two level hierarchical approach. For the flight case, which constitutes the up-bending case, it was assumed that the aircraft is flying at 5000 m, at a speed of 100 m/s, and an angle of attack of 8°. The load factor was assumed to be 2.5 representing a maneuver case. For sizing purposes an up-bending case and a down-bending (70% of up-bending) case were considered. The airfoil used was NACA 4412. The wing had a swept back angle of 15°. The wing chord and semi-span were 400 and 1200 mm, respectively. The front and rear spars were located at 20% and 80% of the wing chord, respectively. The wing-box was made of P100/AS3501 with a thickness of 0.125 mm and the following properties: $E_{11} = 369000$ N/mm^2, $E_{22} = 5030$ N/mm^2, $\nu_{12} = 0.31$, $G_{12} = 5240$ N/mm^2, and $\rho = 1.6 \cdot 10^{-6}$ kg/mm^3. The leading edge, the trailing edge and the ribs were made of an aluminum alloy with the following properties: $E = 72000$ N/mm^2, $G = 26900$ N/mm^2, $\nu = 0.3$ and $\rho = 2.7 \cdot 10^{-6}$ kg/mm^3. The thicknesses of the leading and trailing edges for the 5 rib bays starting at the wing root were: 2, 1.75, 1.5, 1 and 0.875 mm, respectively. Each of the ribs had a thickness of 0.25 mm.

The two level approach was applied to carry out an optimization with both structural and aerodynamic constraints. The structural constraints were: strength, buckling, practical design and ply contiguity constraints. At the first level, a minimum thickness for the skin and spar panels was set as 0.875 mm. The minimum percentage of each ply angle was limited to 10%. Strength constraints were applied to limit the strains in the *x*, *y* and *xy* directions in both tension and compression to 3600με, 3600με, and 7200με, respectively. At the second level, a GA code was used with a population of 40, 200 generations, a 0.7 probability of crossover, a 0.05 probability of mutation and assuming that all weighting factors for the lamination parameters were equal to 1. The aerodynamic constraints imposed were to have at least 99% of the lift and to have an induced drag less than the optimum wing design without aerodynamic constraints (Herencia et al. [10]).

Firstly, the two level optimization approach was performed with structural constraints and without aerodynamic constraints (i.e. lift and induced drag were not restricted). Figure 3 shows the optimum design in this case. Figure 4 shows the optimum design when the aerodynamic constraints are included and where a reduction of 1.4% in the drag was achieved. Figure 5 compares the weight of optimal solution in both cases, for the continuous (first level) and discrete (second level) optimizations. Clearly there is an increase in weight when the discrete lay-up is realized, and there is a weight penalty for the improvement in aerodynamic performance. Finally Figure 6 shows the deformation in both cases, and highlights that there is a bend-twist coupling that improves the performance when the aerodynamic constraints are included.

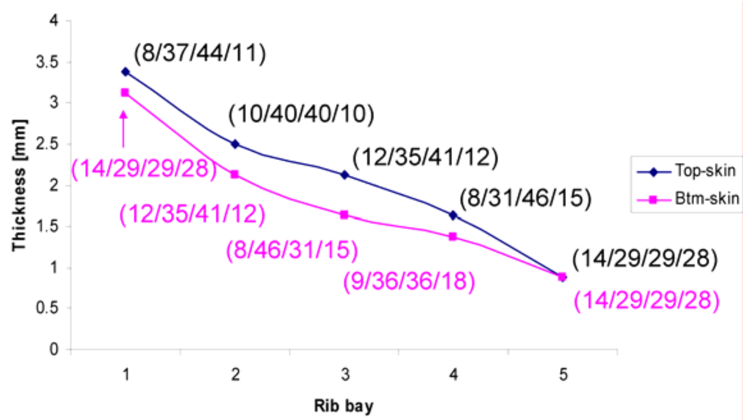

Figure 3: The optimum wing design with structural constraints

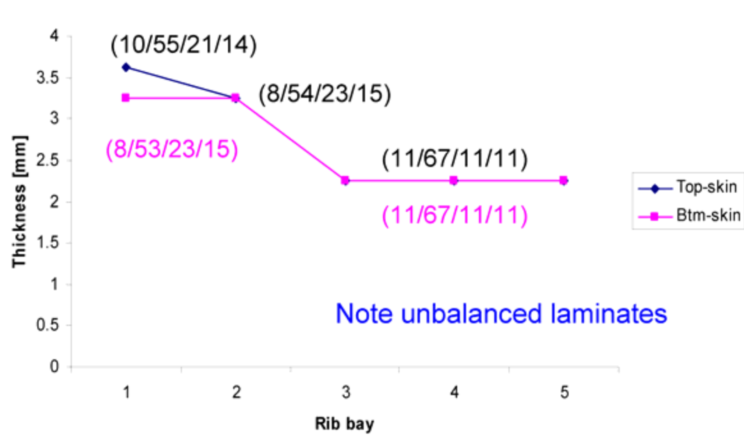

Figure 4: The optimum wing design with structural and aerodynamic constraints

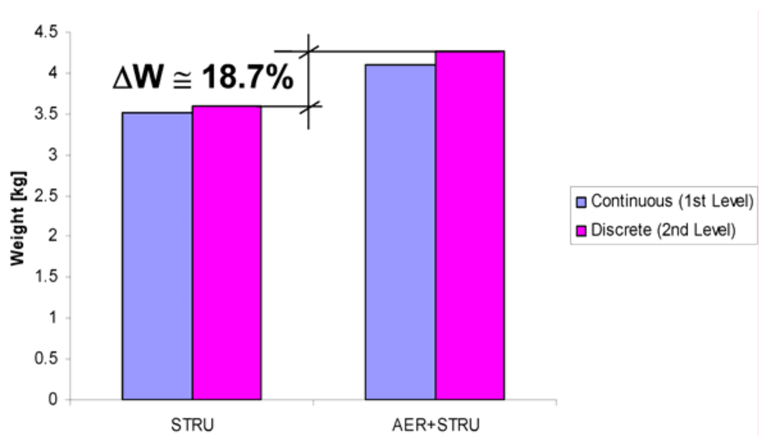

Figure 5: The weights of the optimum wing designs

Figure 6: The deformation for the optimum wing designs, without (left) and with (right) aerodynamic constraints

4. Compliant mechanisms

A compliant mechanism is a single piece structure designed to transmit motion and force mechanically relying solely upon elastic deformation of their constituent elements. As such they may be subject to advantages including high displacement accuracy, zero backlash and wear and ease of manufacture without assembly (Xu and Ananthasuresh [22]). The design of such mechanisms treads a balance between achieving adequate stiffness in order that external loads may be supported yet simultaneously be flexible enough that the required motion due to applied loads is realized. Various strategies for the design of compliant mechanisms have been developed in past studies however two basic categories may be defined. A kinematics approach replaces flexure joints with conventional pivots and a torsional spring system. This tends to provide a solution with concentrated areas of compliance within the structure, so called lumped compliance. An alternative is to take a more structural view of the problem using topology optimization methods. This continuous optimization problem is formulated by defining a structural element only by the loads it is to carry, its volume (cost) and design requirements such as stress and strain limitations. The physical size, shape and connectivity of the structure are unknown. Ananthasuresh and Kota [1] developed the homogenization approach to compliant structure design, where the properties of a composite material composed of solid and void sections are parameterized. A similar approach involves multiplying the material properties with a density function (Zhou and Rozvany [24]). A different approach to topology optimization is the so called ground structure approach of truss or frame elements, where the layout of an elemental structure is found by allowing a certain set of connections between nodal points to be set as potential structural or vanishing members (Anusonti-Inthra et al. [2]). Here a certain set of connections between constituent members are permitted and the optimal dimensions of the members are the design variables. By permitting the reduction of the variables to zero then the inclusion or not of the said member is decided. Such a formulation is not without problems, for example the stiffness matrix for the ground structure with certain members of zero cross sectional area may become singular. In some cases this connection set is prescribed to include all possible connections between nodal points (Frecker et al. [6]). However many practical applications place restrictions on overlaying connections and maximum member length resulting in allowable sets of nodal connection.

The focus of this paper is the design of a compliant system that is able to provide structural control and motion to the trailing edge of a morphing aerofoil. An initial skeletal frame type ground structure is selected where upon the member cross-section dimensions and applied actuator deflection are controlled in order to provide a predetermined surface deflection. The objective is defined as the sum of squares distance of the surface nodes to the required surface. The actuator locations are chosen using a forward subset selection procedure, which has been shown to be very efficient compared to an exhaustive search or methods such as a GA.

4.1 Example trailing edge compliant mechanism

The selection procedure will now be demonstrated on a complex 2D truss structure. In order to develop a 2D truss pattern that fulfills the requirements for static and kinematic determinacy a set of fixed nodes and free nodes are defined. The connectivity of the nodes is then decided by the iterative connection of each free node to two fixed nodes by two truss elements. Upon connection to the truss system a previously free node is considered fixed and so may be employed to connect further free nodes at the following iteration. Figure 7 shows the base structure which comprises of 1752 truss elements arranged in a regular rhombic pattern without patch elements at the aerofoil surface. Examples with patch elements forming triangular shaped cells at the aerofoil surface have been considered, although the highest strains are often required at the aerodynamic surface and the extra stiffness of these elements causes large differences to the target shape.

Consider first the case when the truss elements have fixed dimensions but the actuator locations may be chosen. Here we assume that there are 88 actuators and the maximum strain in the active truss element is ±10%. Figure 8 shows the resulting deflection and the actuator locations. It is immediately apparent that if the ends of the skins are rigidly fixed then the upper skin is in tension and the lower skin is in compression. This skin must also be sufficiently stiff to withstand the aerodynamic loads. Furthermore this skin must be supported between the surface nodes and this requires a truss element with a very small axial stiffness and a high bending stiffness. Although such a truss element could possibly be designed by varying the cross-section of the beam (for example a very thin I beam), the required characteristics are much more likely to be obtained by the design of an internal structure to the beam. Figure 9 shows a potential solution.

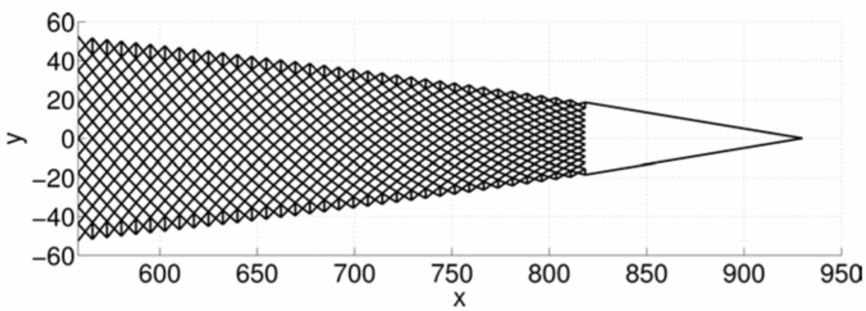

Figure 7: 1752 element determinate truss structure that forms the basis for the actuator selection problem and described as a rhombus-triangle lattice

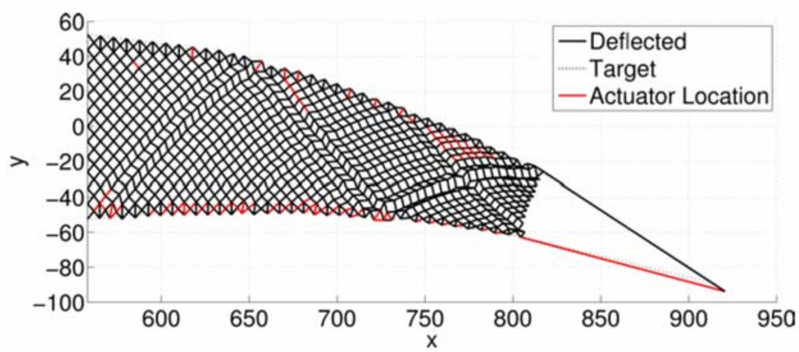

Figure 8: Rhombus-triangle cell structure with actuator and target deflection curve. Actuator locations selected using the forward method with a form finding objective

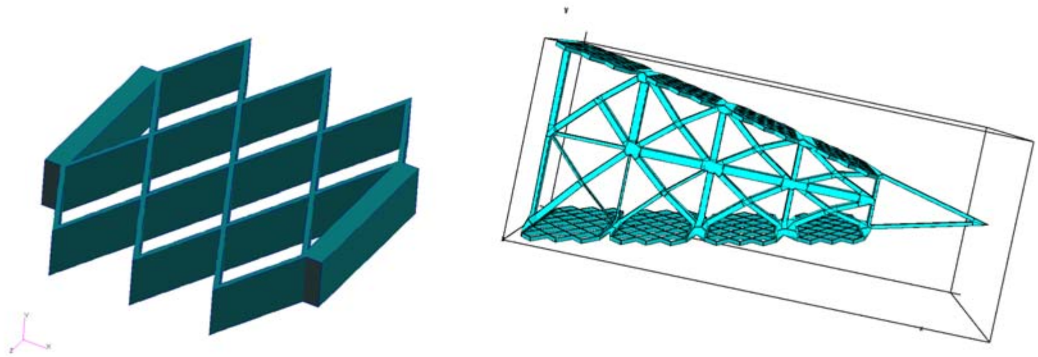

Figure 9: A possible solution for the surface truss elements using a cellular structure

4.2 Morphing skins

It is clear from the design of compliant mechanisms using truss structures that elements with unusual properties are required. In this section we consider the skin (Thill et al. [19]). Suppose that the surface truss elements are designed to support the skin at the mechanism. This means that the skin must be very flexible in-plane in the direction of the airflow, although bending stiffness in this direction is not required. However the skin must form an aerodynamic surface between the ribs and this must be able to withstand the aerodynamic loads. Hence the bending stiffness in the span-wise direction must be high. It is impossible for any isotropic material to fulfill these requirements and the skin must be designed with internal structure. One possible solution is a corrugated skin (Yokozeki et al. [23]), shown in Figure 10. Figure 11 shows the simple model that is used to estimate the equivalent stiffness properties, both in plane and bending, for the skin. These properties may be used in the model to design the compliant mechanism, and requires far fewer degrees of freedom than a detailed model of the corrugations.

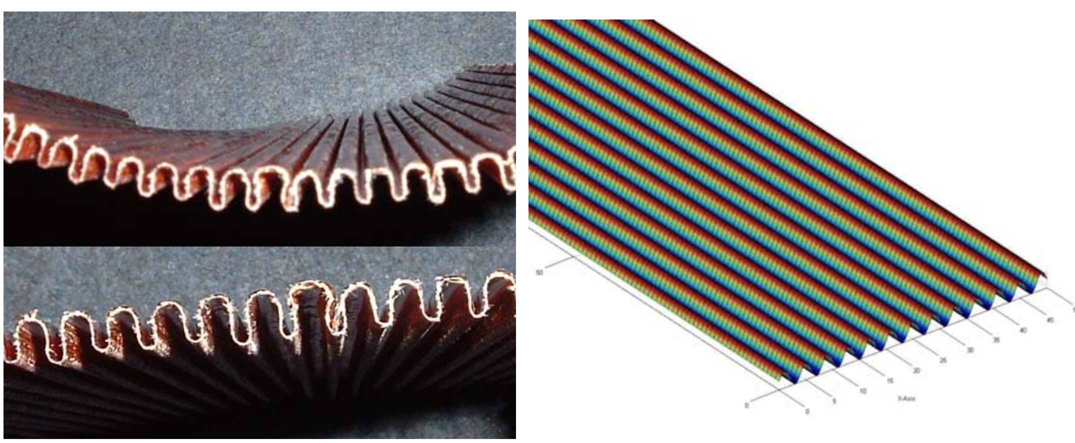

Figure 10: The corrugated skin and detailed FE model

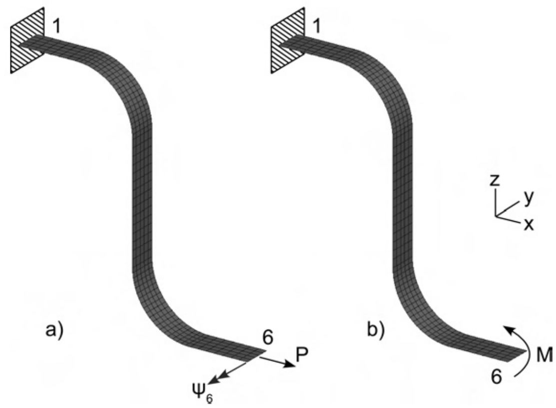

Figure 11: The simplified model to obtain the equivalent stiffnesses

5 Conclusions

This paper has considered morphing aircraft examples where the ability to design panels and substructures with highly anisotropic properties has the potential to significantly improve the performance. The realization of such anisotropic *materials* requires the optimization of a hierarchy of structures. The interface between the different levels in the hierarchy requires equivalent models of the low level structures to perform the high level optimization, but also requires that any restrictions of the feasible properties are included. In this way the high computational cost of an optimization over multiple levels can be reduced by using two or more steps.

6 Acknowledgements

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