Modelling the Dynamic Response of a Morphing Wing with Active Winglets

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This paper investigates the capabilities of the SimMechanics Simulink multibody modelling tool in analysing the dynamic and shape changing response of morphing aircraft. The research is focused on morphing aircraft that undergo large discrete shape variation in which the “morphing” is due to a set of rigid bodies moving relative to each other. The aerodynamic loads contribution is calculated via a quasi-steady linear vortex-lattice code that has been implemented within the Matlab-Simulink environment in order to be directly linked to the SimMechanics block. The methodology is applied to an actual morphing aircraft demonstrator that is a flying wing based on an “active winglets” concept which means that manoeuvres are controlled by a two degrees of freedom tilting-wing-tip system. Particular attention is given to the effects of the variation in the shape on the inertia properites, the centre of gravity location and modal parameters. Finally a response simulation with respect to a time change in shape input is discussed.

Nomenclature

cg = Centre of gravity
b = Wing span
cr = Root chord
ct = Tip chord
\bar{c} = Mean aerodynamic chord
S = Wing surface
\Lambda_{LE} = Leading edge sweep angle
AR = Aspect ratio
\lambda = Taper ratio
vb = [U V W] Velocity vector in body axes (xb yb zb)
\omega_b = [P Q R] Angular velocity vector in body axes
\theta_b = [\phi \theta \psi] Euler angles vector
pb = [Xb Yb Zb] Position vector of body axes origin with respect to inertial frame
R = Matrix of direction cosines between body and inertial frame
Eb = Matrix relating Eulerian velocities to angular quasi-velocities

I. Introduction

Conventional aircraft are typically shaped in a way to accomplish a specific type of mission or task: a military aircraft like the Lockheed Martin F-16 realises supersonic flight and high-speed-maneuuvrability thanks to a thin and short highly sweep wing while a civil aircraft like the Canadair CL-215 amphibious firefighter needs a long and thick wing in order to realize flight at low-speed. The strong dependency between geometry and mission capabilities arises because the shape governs the interaction with the atmosphere. Unlike birds and nature, the ability to accomplish different and contradictory tasks for heavier-than-air
flying machines represents a challenge for designers who have to provide aircraft with the ability to “morph”; i.e. capable of changing shape and geometrical features. The shape variation could be continuous, smooth and seamless as with variable camber, variable twist and self adapting wings involving the development and research of new smart materials and actuation systems. Alternatively the shape variation could be more drastic involving the relative motion of big portions of the aircraft, such as telescoping or variable sweep wings. Examples in the past are the B-1 bomber in which aft variable-sweep-wing settings are used in high subsonic and supersonic flight in order to enhance performance, or the XB-70 Valkyrie where wing tips were tilted down in supersonic flight in order to take advantage of the “compression lift” effect.

When the morphing process involves large geometrical variations, centre of gravity location and inertia properties can vary significantly so that the typical rigid flight mechanics equations of motion do not describe the dynamics and the actual transient of the system during morphing. The conventional nonlinear equations are based on the assumption that the inertia properties of the aircraft stay constant with respect to a body reference system. Moreover the body reference system is located at the centre of gravity of the whole aircraft which means that static moment contributions, coupling longitudinal and rotational inertia forces, disappear. However, when relative motions occur between bodies extra terms arise, such as the rate of change of inertia components, that drastically affect the dynamics of the system.

In order to study the dynamics and response of such systems many researchers have reformulated the flight mechanics equations of motion assuming the morphing aircraft to be a multibody system comprising rigid bodies. Others have built a set of tools based on the coupling of commercial multibody and finite element codes in order to study the response of a folding wing aircraft with a full multibody-aeroelastic dynamic model. From a control point of view, shape reconfiguration of a biologically inspired micro-air vehicle has been investigated using an $H_\infty$ model.

The investigation of the transient dynamics becomes important when the morphing aircraft changes shape in order to provide direct control authority for manoeuvring. The task of the present work is to investigate the capabilities of SimMechanics in describing the dynamic and shape-changing transients of morphing aircraft that comprise of a set of rigid bodies moving relatively to each other and in which the elasticity of each body can be neglected. Working within the SimMechanics environment gives the advantage of performing control directly with Simulink. The aerodynamic contributions are provided, as a first step in the current analysis, by a potential-quasi-steady vortex lattice code coupled with the multibody model built with SimMechanics. This approach is applied to investigate the dynamics of an actual morphing aircraft by Bourdin et al. The concept is based on a flying wing with independently tilted wing tips, where the “active winglets” allow basic manoeuvres to be performed: aerodynamics, static stability and the ability to perform a steady-turn with this novel concept have been investigated. The present work focuses on building tools for future work in control and response analysis.

The paper is organised as follows: Section II provides a geometrical description of the flying wing and an investigation of how the two winglet rotational degrees of freedom affect centre of gravity location and inertia properties. Section III firstly provides a general overview of SimMechanics capabilities in Modelling morphing aircraft and secondly describes the aerodynamic and multibody dynamic Modelling of the flying wing. Section IV shows results and discussion concerning the longitudinal trim, the effect of winglets position on system poles, and finally an example of the response with respect to an winglet-impulse input. Conclusions are presented in Section V.

## II. Description of the Model

### II.A. Geometry

The airframe under investigation is a prototype (see figure 1) which has been obtained by modifying a commercial remotely controlled flying wing made in EPP foam. The original configuration consisted of a tapered-planar wing with no twist and 30 deg LE sweep angle and 1.8 m span. As there is no negative sweep angle at the wing tip, the longitudinal static stability is provided (i.e. given a positive static margin) by a “reflexed” 12% thick aerofoil section that provides a small amount of down lift near to the trailing edge. Longitudinal and lateral control were provided through a pair of elevons.

For morphing purposes, the model has been modified to introduce two hinges parallel to the wing plane of symmetry at a distance, calculated along the wingspan, of 0.6 m from the root chord. Two separate actuators allow the winglets to rotate relative to the baseline thus modifying the dihedral angles from that point on. In this configuration each winglet is 0.3 m long representing 50% of the baseline semispan; a second
Figure 1. Flying wing with active winglets scheme and dimensions (black spheres on the left pictures represent masses distribution on the wing).

Table 1. Main geometrical features for the baseline (no winglets), and short and long winglet configurations

<table>
<thead>
<tr>
<th></th>
<th>baseline</th>
<th>short</th>
<th>long</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ [m]</td>
<td>1.2</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>$S$ [m$^2$]</td>
<td>0.311</td>
<td>0.361</td>
<td>0.399</td>
</tr>
<tr>
<td>$AR$ [-]</td>
<td>4.631</td>
<td>6.234</td>
<td>8.105</td>
</tr>
<tr>
<td>$\bar{c}$ [m]</td>
<td>0.266</td>
<td>0.252</td>
<td>0.241</td>
</tr>
<tr>
<td>$c_t$ [m]</td>
<td>0.185</td>
<td>0.148</td>
<td>0.111</td>
</tr>
<tr>
<td>$c_r$ [m]</td>
<td>0.333</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$ [-]</td>
<td></td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_{LE}$ [deg]</td>
<td></td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

configuration will also be investigated where each winglet is 0.15 m long (25% of baseline semispan). The two configurations are referred to as the “long” and “short” winglet configurations respectively.

Pure longitudinal control is provided by converting the original elevons into a pair of symmetrically moving elevators. The elevators represent about 12% of the inner aerofoil section chord and 30% of the baseline tip chord. The main geometrical features are summarized in Table 1.

II.B. Mass and Inertia Properties

An investigation of how shape variation due to the winglet folding action affects the location of the cg, the inertia and inertia-rate properties is presented in this section.

The payload of the flying wing consists mainly of the motor, a receiver and two batteries with a total weight of 0.5546 kg. They are located along the root chord section and it is possible to tailor the longitudinal position of the cg by moving on the receiver or battery positions. A pair of actuators each weighing 24 g are located near the right and left baseline wing tips in order to actuate the winglets, while another pair of actuators weighing 41 g are positioned near the root section to control the elevator surfaces.

In order to define the inertia properties of each body (baseline, right and left winglets) a body-fixed coordinate system with origin at the body cg has been defined for each body. Axis orientations are defined with longitudinal axis $x^i_b$ positive forward and vertical axis $z^i_b$ positive downward. The mass and inertia properties of baseline and right winglets (in short and long configuration) are presented in Table 2. The weight of the baseline reported in the table includes all of the devices previously described. Thus the flying wing has a total weight of 0.995 kg for the long winglets configuration, and 0.963 kg with the short winglets.

The left winglet products and moments of inertia can be obtained from those of the right winglet simply by imposing $I^L_{xy} = -I^R_{xy}$, since the two winglets are mirrored with respect to the $xz$ plane. It could be observed that the reference system chosen for the main body represents a principal axes as the products of inertia are zero: it means that $xz$ and $zy$ are both planes of symmetry for the baseline and that the asymmetric shape
Table 2. Mass and Inertia properties of baseline, short and long right winglet

<table>
<thead>
<tr>
<th></th>
<th>baseline</th>
<th>short winglet</th>
<th>long winglet</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass [g]</td>
<td>931</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>(I_{xx}) [kg·m(^2)]</td>
<td>4090E-5</td>
<td>2.98E-5</td>
<td>23.49E-5</td>
</tr>
<tr>
<td>(I_{yy}) [kg·m(^2)]</td>
<td>1291E-5</td>
<td>4.80E-5</td>
<td>11.77E-5</td>
</tr>
<tr>
<td>(I_{zz}) [kg·m(^2)]</td>
<td>5377E-5</td>
<td>7.79E-5</td>
<td>35.26E-5</td>
</tr>
<tr>
<td>(I_{xy}) [kg·m(^2)]</td>
<td>0</td>
<td>1.35E-5</td>
<td>10.66E-5</td>
</tr>
<tr>
<td>(I_{xz}) [kg·m(^2)]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(I_{yz}) [kg·m(^2)]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

of the reflexed aerofoil section has been neglected in the computation of the products of inertia.

For this configuration the shape variation depends on \(\delta_r(t)\) and \(\delta_l(t)\), respectively the right and left dihedral angles; generally they vary as a function of time accordingly to the command imposed by the pilot to control manoeuvres. Therefore even the centre of gravity location and the inertia tensor are functions of time\(^a\):

\[
x_{cg} = x_{cg} [\delta_r(t), \delta_l(t)]
\]

\[
\mathbf{I} = \mathbf{I} [\delta_r(t), \delta_l(t)]
\]

where the symmetric inertia tensor is defined as

\[
\mathbf{I} = \begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{xy} & I_{yy} & -I_{yz} \\
-I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}
\]

In order to calculate the inertia tensor of the whole configuration a generic multi-body system composed of a set of \(N\) rigid bodies is considered; it is possible to define for the \(i\)-th body of mass \(m_i\) \((i = 1 \ldots N)\), a local coordinate system \(\mathbf{e}^i\), with origin at the body cg, and with respect to which the local inertia tensor \(\mathbf{I}^i\) is defined. The global inertia tensor of the system is defined with respect to a global coordinate system \(\mathbf{e}^g\). The vector \(\mathbf{r}_i\) locates the position of the origin of the \(i\)-th local system with respect to the global one as a function of time. An orthonormal transformation between the axes \(\mathbf{e}^g\) and \(\mathbf{e}^i\) is performed through \(\mathbf{R}^{ge_i}\), a tensor of first rank collecting the direction cosines between the two axes so that

\[
\mathbf{e}^g = \mathbf{R}^{ge_i} \mathbf{e}^i
\]

Finally the global inertia tensor can be calculated as

\[
\mathbf{I}^g = \sum_i \mathbf{I}^{gi}
\]

where \(\mathbf{I}^{gi}\) is the inertia tensor of the \(i\)-th body calculated with respect to the global coordinate system and can be obtained as

\[
\mathbf{I}^{gi} = \mathbf{R}^{ge_i} \mathbf{I}^i \mathbf{R}^{ge_i \top}^{-1} + m_i \left( \mathbf{1} \mathbf{r}_i^T \mathbf{r}_i - \mathbf{r}_i \mathbf{r}_i^T \right) i = 1 \ldots N
\]

where \(\mathbf{1}\) is the identity matrix. The first term of Eq. (6) simply rotates the inertia tensor from the local to the global axes, while the second term arises because the two origins are not coincident.

\(^a\)Vectors will be defined using a bold lower case notation while a bold upper case notation will be used for matrix
In order to estimate the magnitude of the variation of the inertia properties for the specific case of the flying wing, two geometrical variations are presented for both short and long winglets configurations. The first configuration is a symmetrical deflection of both winglets from $-90$ deg to $+90$ deg (both dihedral angles are measured respect to the planar configuration and are positive when they are folded up). The second is a variation of just the right winglet, leaving the left winglet planar. In both cases a sinusoidal variation in time for the dihedral angles is imposed as shown in figure 2. The time needed for the winglets to fold up from the lower position to the upper is about 0.66 s and has been calculated considering the maximum angular velocity of the actuator, which is $\dot{\delta}_{\text{max}} = 428$ deg/s. In the same figure the dihedral angular velocity is presented as a function of time.

In order to evaluate variation of the cg, the cg of the planar configuration ($\delta_R = \delta_L = 0$) is taken to be the reference configuration. In that symmetric situation the cg is located along the root chord at 0.66 $c_r$ for the long winglets case and at 0.63 $c_r$ for the short winglets. Figures 3 and 6 show the variation of the cg position with respect to the reference configuration for the symmetric and asymmetric variation respectively\(^b\). In the symmetric case only the vertical component moves while for asymmetric case there is also a lateral movement. In both cases there is no variation in the longitudinal component; thus such a geometrical change will affect the static margin only through the wing aerodynamic centre that is strongly affected by the winglets’ location. There would be a variation of the longitudinal component only if the winglet hinge lines were not parallel to the $xz$ plane.

In order to calculate the inertia tensor through Eqs. (5) and (6) a global reference system with origin at the cg has been chosen, and thus the global system is moving with the winglets. The results of that computation are shown in figures 4 and 5 when both winglets are folded up, and 7 and 8, when only the right winglet is folded. It can be seen from the previous figures that the maximum absolute value of inertia moment is about z axis for the planar winglets configuration (as the winglet masses are in the furthermost position with respect to the z axis). When the winglets are not in a planar configuration the global reference axes are not principal axes anymore thus a non-zero $I_{xz}$ component appears for the symmetric case, while all the products of inertia are non-zero for the asymmetric case. Therefore an inertia coupling involving all the axes will arise when the winglets are folded asymmetrically. Unlike the case of typical rigid-bodies, inertia time derivatives affect the motion of non-rigid body. They have been calculated numerically using a Simulink derivative block, and are reported in the same figures.

The main results of the above analysis are summarized in Table 3. It can be seen that a symmetrical deflection of the winglets causes larger variation in terms of inertia and cg than the asymmetrical case. Moreover the long winglets configuration represents the worst case as it exhibits a maximum inertia variation of about 16% relative to the initial inertia value ($I_{zz}(t = 0)$), and a maximum variation of cg position of 5% relative to the root chord. The same values for the short winglets case are respectively 5.2 and 1.4%.

Therefore a multi-body approach is needed in order to analyse the dynamics of the large winglets configuration while variation for the short winglets configuration seems to be negligible.

\(^b\)Note that the graphs are plotted with respect to time, rather than to dihedral angle, for consistency with the inertia-rate plots.
Figure 3. Centre of gravity variations as functions of time when both winglets are folded from $-90$ to $+90$ deg - long and short configuration compared.

Figure 4. Inertia and rate of inertia variation as functions of time when both winglets are folded from $-90$ to $+90$ deg - long winglets case.

Figure 5. Inertia and rate of inertia variation as functions of time when both winglets are folded from $-90$ to $+90$ deg - short winglets case.
Figure 6. Centre of gravity variations as functions of time when right winglet is folded from $-90$ to $+90$ deg, left winglet is planar - long and short configuration compared.

Figure 7. Inertia and rate of inertia variation as functions of time when right winglet is folded from $-90$ to $+90$ deg, left winglet is planar - long winglets case.

Figure 8. Inertia and rate of inertia variation as functions of time when right winglet is folded from $-90$ to $+90$ deg, left winglet is planar - short winglets case.
Table 3. Main results from the analysis of the effect of the shape change on the inertia properties and centre of gravity location.

<table>
<thead>
<tr>
<th></th>
<th>long</th>
<th>short</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symmetric</td>
<td>Asymmetric</td>
</tr>
<tr>
<td>$I_{zz}^{max}$ [kg·m²]</td>
<td>9411E-5</td>
<td>9411E-5</td>
</tr>
<tr>
<td>$\dot{I}_{zz}^{max}$ [kg·m²/s]</td>
<td>6074E-5</td>
<td>3044E-5</td>
</tr>
<tr>
<td>$(\Delta I_{zz}/I_{zz})^{max}$ [%]</td>
<td>16.2</td>
<td>7.5</td>
</tr>
<tr>
<td>$(\Delta z_{cg}/c_r)^{max}$ [%]</td>
<td>5.3</td>
<td>2.6</td>
</tr>
<tr>
<td>$(\Delta y_{cg}/c_r)^{max}$ [%]</td>
<td>0</td>
<td>2.6</td>
</tr>
</tbody>
</table>

III. Modelling Morphing Aircraft Using SimMechanics

SimMechanics is a block diagram Modelling environment that allows the simulation of rigid body machines and their motions, using the standard Newtonian dynamics of forces and torques. It is possible to model and simulate mechanical systems with a set of tools to specify bodies, mass properties, kinematic constraints and coordinate systems, and to initiate and measure body motions. SimMechanics defines a fixed or “absolute” inertial coordinate system (CS) called World. The World CS has an origin (0,0,0) and a triad of right-handed, orthogonal coordinate axes.

A machine is represented by a chain of body and joint blocks. Each rigid body comprising the machine is represented by a “body block” in which it is possible to specify the mass and the inertia tensor about the body cg. A set of coordinate frames fixed to the body can be specified in order to position and orient the body relative to the World coordinate system or to other frames. Moreover coordinate frames can locate points where external loads act on the body or points at which motion can be measured.

Bodies are connected together by a “Joint block” representing one or more mechanical degrees of freedom between two bodies:

- prismatic primitive (1 translational DOF)
- revolute primitive (1 rotational DOF)
- weld primitive (no DOF)
- bushing primitive (6 DOF)

The machine motion is linked relative to the inertial frame through a “Ground Block” that represents an immobile ground point at rest in the absolute inertial World reference frame. The vertical direction or up-and-down is determined by the gravity vector direction (acceleration $g$) relative to the World axes that is by default directed along the y positive direction. It is possible to set the z axis as the vertical direction (positive down as in flight mechanics convention) using the “Machine Environment” block in which the gravity vector can be changed.

By assuming a morphing aircraft to be a multi-body system composed of a set of rigid bodies moving relative to a “main body”, a point on the main body can be chosen to be the origin of a body coordinate frame ($F_B$), of axes $x_b$ $y_b$ $z_b$ and the motion of the entire aircraft can be described by the six rigid degrees of freedom, three translational and three rotational, of the main body. Locating the origin of $F_B$ on the cg results in a floating frame that would allow translational motion to be decoupled from rotational motions. That frame is known as Koenig’s frame. It should be noted that setting a coordinate system on bodies in SimMechanics is a background property of a model that is possible to reset before starting a simulation, but does not dynamically change during a simulation. Therefore it is not possible to refer the motion to a floating point as in Koenig’s frame, thus the origin of $F_B$ will be chosen to be the main body cg fixed during the simulation (unlike the entire aircraft, each rigid body has constant mass and inertia properties).

The main body frame can be linked to the ground using a “Bushing joint” that represent three translational (P1 P2 P3) and three rotational (R1 R2 R3) degrees of freedom. A moving frame $\xi_i$ ($i = 1 \ldots 3$) is obtained from an inertial frame $x_i$ by means of three successive rotations, say R1 R2 R3, that in the bushing joint block are set by default about axes $x_1$ $y_2$ and $z_3$, resulting in the systems $y_i$, $z_i$, $\xi_i$, respectively. The
The "Body sensor" block allows the measurement of the velocity and angular velocity vectors of the origin of the main body frame with respect to the body axes, \( \mathbf{v}_b \) and \( \mathbf{\omega}_b \) respectively. Typically these variables are called quasi-coordinates because their integration is meaningless. Once the components \( U, V, W \) of the velocity in body axes have been obtained, it is possible to evaluate absolute velocity \( V_T \), angle of attack \( \alpha \) and sideslip \( \beta \) as:

\[
V_T = \sqrt{U^2 + V^2 + W^2} \\
\alpha = \tan^{-1}(W/U) \\
\beta = \sin^{-1}(V/V_T)
\]  

(7)

Finally the Body sensor block provides position \( \mathbf{p}_b \) of the origin of the body frame with respect to the inertial frame, and the nine components of the body-to-inertial rotation matrix \( \mathbf{R}(\theta_b) \) that are a function of the Euler angles collected into the vector \( \theta_b \). Thus Euler angles can be obtained from the components \( r_{ij} \) of the rotation matrix as:

\[
\phi = \tan^{-1}(r_{32}/r_{33}) \\
\theta = -\sin^{-1}(r_{31}) \\
\psi = \tan^{-1}(r_{21}/r_{11})
\]  

(8)

Therefore at each time step of the simulation it is possible to evaluate with SimMechanics the quasi-coordinates vector \( \mathbf{x} \):

\[
\mathbf{x} = [\mathbf{p}_b \ \theta_b \ \mathbf{v}_b \ \mathbf{\omega}_b]
\]  

(9)

where \( \mathbf{v}_b = [V_T \ \alpha \ \beta] \). Initial conditions are set using an “Initial Condition” block but it should be noted that they have to be provided to the system using a different set of coordinates because the SimMechanics internal set of Lagrangian coordinates \( \mathbf{y} \) collect inertial position, Euler angles and their first order derivatives:

\[
\mathbf{y} = [\mathbf{p}_b \ \theta_b \ \dot{\mathbf{p}}_b \ \dot{\theta}_b]
\]  

(10)

The relation between the two coordinates can be found using:

\[
\dot{\mathbf{p}}_b = \mathbf{R}(\theta_b)\mathbf{v}_b = f(\theta_b, \mathbf{v}_b) \\
\dot{\theta}_b = \mathbf{E}_b(\theta_b)\mathbf{\omega}_b
\]  

(11)

where \( \mathbf{E}_b(\theta_b) \) is a matrix relating Eulerian velocities to angular quasi-velocities. It should be noticed that in Eq. (11), components of velocity in the inertial frame are a linear function of the component of the velocity in body frame \( (U \ V \ W) \) but, because of Eq. (9), are non linear functions with respect to the \( \mathbf{v}_b \) vector components \( (V_T \ \alpha \ \beta) \).

External loads acting on the machine can be fed to the system using “Body actuator” blocks, which allow external loads (such as aerodynamic forces and moments) to be transferred to the origin of body coordinate frame. In the next sections the specific case of the flying wing with active winglets is considered.

### III.A. Flying wing: Aerodynamic and Thrust Loads Evaluation

The aerodynamic loads of a morphing aircraft could be a complex function of the kinematic state, control and geometrical shape. Consider as an example the case in which the flying wing undergoes an asymmetric
shape variation. If the right winglet is folded up leaving the left unfolded, the primary effect will be a loss in lift on the right wing causing a positive roll moment. The winglet in a non-planar position will provide a side force that causes a positive yawing moment about the cg and a lateral translation. Finally the aerodynamic centre will shift forward causing a pitching up moment. Thus in order to guarantee positive static margin the aerodynamic centre must not overtake the centre of gravity in its foremost position (both winglet completely folded up or down by ±90 deg). It is clear that there could be, as consequence of the shape variation, a non-negligible lateral-longitudinal coupling. Moreover if the rate of shape variation is high enough there could even appear a significant unsteady effect.

Because of the complex relation between aerodynamic loads and the kinematic-control variables, we have chosen, as a first step in the dynamic analysis, to use an aerodynamic code that computes those contributions at each time step. This “black box” approach is opposite to the “look-up tables” approach, where forces and moments are interpolated between multi-dimensional tables that relate explicitly the loads dependency on kinematic and controls. Those tables could in principle be previously generated numerically by mean of an aerodynamic code or filled by experimental data. The use of an aerodynamic code within the simulation may slow down the computation time and because of this the numerical method implemented has to have a good compromise between accuracy and computation time. The code used here for generating aerodynamic loads is AVL, an inviscid quasi-steady vortex lattice method that allows the study of non-planar geometric configurations. Basically each lifting surface is discretized by a certain number of unknown horseshoe vortices. These vortices can be found as a solution of a linear system of equations built up when a flow-tangency boundary condition is applied. It should be pointed out that the AVL is a code based on a potential flow hypothesis and thus is best suited for thin lifting surfaces at small angles of attack and sideslip. Moreover angular rates (yaw, pitch, roll) and those involving shape variation (such as winglet dihedral rates) have to be small enough in order to guarantee the quasi-steady flow. Even if there is an uncertainty regarding the unsteadiness of the flow, AVL gives the possibility to correct the inviscid hypothesis by adding a parabolic lift-drag polar model to each aerofoil section.

In order to run AVL in the Matlab-Simulink environment, a Simulink block based on an “avl2matlab.m” function has been built (see figure 9). The block needs two groups of input: the first one is the instantaneous values of control variables i.e. left dihedral, right dihedral and elevator angles, δℓ, δr and δe respectively; the second group are the kinematic variables v_b and ω_b with respect to the origin of the body frame (AVL allows the point at which the aerodynamic torques are calculate to be specified). The output is a six component vector comprising three forces in body axes (X, Y and Z) and three moments (L, M and N) with respect to the origin of the body axes.

The flying wing thrust is provided by a propeller with 0.17 m diameter, powered by an electrical motor and positioned at the trailing edge of the root chord. A simple zero-order-lag model for the thrust has been implemented in the model and thus the throttle command δt is a fraction of the maximum trust $T_{max}$:

$$T = \delta_t T_{max}$$

(13)

The thrust action line is coincident with the longitudinal body axis direction $x_b$; therefore the model does take in account the moment about the cg due to the thrust. Another simplification is that the density is assumed to be fixed at its the sea level value, which means that in the current analysis altitude does not affect the flight performance.

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cThe static margin is related to the pitch stiffness as $K_N = (x_N - x_{cg})/\bar{c} = -dC_m/dC_L$ where $x_N$ is the neutral point of the whole aircraft; for a flying the wing neutral point and wing aerodynamic centre are coincident; $K_N$ has to be positive in order to have longitudinal static stability. If the neutral point is moving forward, the $C_{m,\alpha}$ slope will be less negative and this will cause at the same angle of attack a positive pitching-up moment.
III.B. Flying Wing: Dynamic Model with SimMechanics

The flying wing comprises three bodies, essentially two winglets and the baseline. The baseline is considered to be the main body and a right-handed body fixed frame ($x_b$ forward, $z_b$ downward) is attached at its cg located at 0.60 $c_r$.

The SimMechanics model of the wing is a box (see figure 10), where two groups of input are fed: the first group is the set of the zeroth, first and second order derivatives of the left and right dihedral angles, while the second group is the aerodynamic force and moment vector provided by AVL. The dihedral angle derivatives are obtained numerically using a Simulink derivative block. At each simulation time step the output is the vector $x$ (see Eq. (9)) containing the twelve quasi-states, and six of these states ($v_b$ and $\omega_b$) are fed back to the AVL block in order to give the loads. Specifically the morphing airframe is modeled by:

- 3 Body blocks for baseline, right and left winglets
- 2 Joint blocks representing 1 rotational DOF (or “Revolute joint”), linking each winglet to the baseline
- 1 Bushing Joint representing a 6 DOF linking the baseline to the ground

The zeroth, first and second order derivatives of the dihedral angles are directly feed into two “Joint actuator” blocks that actuate the winglets independently. A useful tool of SimMechanics is the “equivalent ellipsoid” representation of a machine as shown in figure 12. The equivalent ellipsoid is a homogeneous ellipsoidal-shape solid, centred at the body centre of gravity, with the same principal moments of inertia and principal axes.

Finally the input control vector $u$ for the model is defined as:

$$ u = [\delta_r \ \delta_{\ell} \ \delta_e \ \delta_t]$$  \hspace{1cm} (14)

which collects the right and left dihedral angle, elevator and throttle inputs into a single vector.
Figure 13. Short winglets case: elevator angle for longitudinal trim as a function of dihedral angle and velocity.

Figure 14. Long winglets case: elevator angle for longitudinal trim as a function of dihedral angle and velocity.

Figure 15. Short winglets case: angle of attack for longitudinal trim as a function of dihedral angle and velocity.

Figure 16. Long winglets case: angle of attack for longitudinal trim as a function of dihedral angle and velocity.

Figure 17. Short winglets case: throttle for longitudinal trim as a function of dihedral angle and velocity.

Figure 18. Long winglets case: throttle for longitudinal trim as a function of dihedral angle and velocity.
IV. Results

IV.A. Longitudinal Trim

Trim analysis represents a starting point to define operational points on which to perform linearization, stability and response analyses. In this section the longitudinal trim for the flying wing with active winglets is considered. The routine implemented is based on collecting the $\dot{y}$ vector at each iteration and finding values of $\alpha$, $\delta_e$ and $\delta_t$ that minimize the cost function:

$$\text{cost} = \ddot{X}_b^2 + \ddot{Y}_b^2 + \ddot{Z}_b^2 + \ddot{\psi}^2 + \ddot{\theta}^2 + \ddot{\phi}^2 + \dot{\psi}^2 + \dot{\theta}^2 + \dot{\phi}^2$$

(15)

using the following constraints:

$$V_T = V_T$$
$$\delta_e = \delta_r$$
$$\delta_e = \bar{\delta}$$
$$\dot{\psi} = \dot{\psi}$$
$$\dot{\theta} = \dot{\theta}$$
$$\theta = \alpha$$

Computation has been carried out considering a range in absolute velocity between 10 and 25 m/s and a range of dihedral angles (symmetrically varied) of $\pm 90$ deg. Results are plotted in figures 13, 15 and 17 for the short winglets case, and figures 14, 16 and 18 for the long winglets case.

It can be seen that the longitudinal trimmed condition of flight for a flying wing, at a given velocity, requires negative angles of elevator (elevator up) which means that a small amount of down lift is created near to the trailing edge in order to balance (for positive static margin) a negative pitching moment about the cg. Therefore the wing is flying with a negative-like camber section which means a positive $C_m$ and, as a consequence, positive longitudinal static stability. At a fixed velocity the elevator shows the maximum negative deflection when the winglets are in a planar configuration, since the negative pitching moment for balance increases as the static margin increases (the lift is the same but the arm with respect to the cg will increase). At fixed dihedral angle the elevator angle decreases with the velocity because lower values of angle of attack are needed to trim at higher velocities. The latter can be seen in figures 15 and 16 where trim angle of attack is plotted as a function of dihedral angle and velocity. At fixed velocity the trim angle of attack reaches the maximum for the planar configuration. This is despite the fact that as the lifting surface area increases, less angle of attack would be required in order to generate the same lift. The reason for such different behaviour is because, as previously shown, the trim elevator angle increases in its negative value as long as the winglets approach the planar configuration: this means that a higher angle of attack would be required to compensate for the loss in lift due to the elevator (in other words as long as the sections become more cambered the zero lifting line will change in a way that the effective angle of attack becomes smaller). Finally in figures 17 and 18 the trim throttle command is plotted as a function of dihedral angle and velocity. Although changes in thrust command with dihedral angles are negligible, as long as the velocity increases the thrust has to increase in order to balance the higher drag.

IV.B. Linearisation and stability analysis

Given the non-linear equation of motion for the morphing aircraft:

$$\dot{y} = f(y, u)$$

(20)

where $y$ and $u$ are defined in Eqs. (10) and (14) respectively, it is possible to linearise the previously described SimMechanics model about an equilibrium point defined by the equilibrium state $\bar{y}^e$ and equilibrium

\[\begin{align*}
\dot{\bar{y}} &= \bar{f}
\end{align*}\]

\[\begin{align*}
\bar{f} &= f(\bar{y}, 0)
\end{align*}\]

The ellipsoid of inertia is a solid with the same rotational dynamics behaviour of the body. The rotational behaviour of a body depends only on its ellipsoid of inertia and not on its shape, because an infinite number of bodies with different shapes could correspond to the same ellipsoid of inertia.

It is possible to realise positive longitudinal static stability for an aircraft only if $C_m > 0$ and $C_{m\alpha} < 0$. The first condition is verified for an aircraft with positive cambered wing, only introducing the tail, while for flying wings, introducing negative sweep angle or a negative camber. The latter option would be in fact the less efficient configuration.
control vector $u^e$, in order to obtain the linear system:

$$\dot{\hat{y}} = A_y\hat{y} + B_y\dot{\hat{u}} \quad (21)$$

where $\hat{y}$ and $\dot{\hat{u}}$ are perturbation values about the equilibrium point. In order to give a physical interpretation to the results it is worth working in quasi-coordinate space $x$ (as defined in Eq. (9)). Thus linearising Eqs. (11) and (12) about the equilibrium point it is possible to find the constant matrix $\hat{T}$ relating the perturbed quasi-coordinate vector to the perturbed coordinate state vector:

$$\hat{y} = \hat{T} \hat{x} \quad (22)$$

This relationships allows the state-space matrix to be rotated into the quasi-coordinates state:

$$A_x = \hat{T}^{-1} A_y \hat{T}$$

$$B_x = \hat{T}^{-1} B_y \quad (24)$$

It should be noted that the transformation does not affect the eigenvalues, but only the eigenvectors. In order to investigate the effect of the winglets’ positions on the dynamic stability, eigenvalues and eigenvectors were obtained by evaluating the state-space matrices corresponding to linearisations at several conditions of longitudinal trimmed steady-flight. The velocity is fixed at 15 m/s and the dihedral angles are varied symmetrically (see Section IV.A) from 0 to 90 deg. It should be noted that even if a different location of the whole aircraft cg exists at each configuration, the non-linear equations of motions were linearised about the same fixed point, namely the origin of the body frame as defined in Section III.B. Therefore in the modal analysis, longitudinal and lateral stability should not be decoupled as the linearisation is about a point different from the cg. Nevertheless the analysis of the state space matrices shows that the coupling effect is weak and thus it should be possible to recognise pole distributions and modal shapes of a typical uncoupled analysis about the cg. Anyway in order to maintain a more general approach for morphing aircraft the analysis considers all the eight variables obtained from the quasi-coordinate vector $x$, excluding the inertial positions $p_b$ and the attitude $\psi$. 

### Table 4. Short winglets case: modal properties of oscillatory and non-oscillatory modes for three different dihedral angles.

<table>
<thead>
<tr>
<th>$\delta$ [deg]</th>
<th>$T_n$ [s]</th>
<th>$\zeta_n$</th>
<th>$T_n$ [s]</th>
<th>$\zeta_n$</th>
<th>$T_n$ [s]</th>
<th>$\zeta_n$</th>
<th>$\tau$ [s]</th>
<th>$\tau$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.5541</td>
<td>-0.0751</td>
<td>0.2758</td>
<td>-0.5672</td>
<td>2.293</td>
<td>-0.0683</td>
<td>-0.0358</td>
<td>41.5076</td>
</tr>
<tr>
<td>45</td>
<td>7.8341</td>
<td>0.1113</td>
<td>0.3056</td>
<td>-0.5631</td>
<td>1.3394</td>
<td>-0.1043</td>
<td>-0.0413</td>
<td>-44.802</td>
</tr>
<tr>
<td>90</td>
<td>8.1144</td>
<td>-0.181</td>
<td>0.4518</td>
<td>-0.6177</td>
<td>0.939</td>
<td>-0.1443</td>
<td>-0.056</td>
<td>42.0963</td>
</tr>
</tbody>
</table>

### Table 5. Long winglets case: modal properties of oscillatory and non-oscillatory modes for three different dihedral angles.

<table>
<thead>
<tr>
<th>$\delta$ [deg]</th>
<th>$T_n$ [s]</th>
<th>$\zeta_n$</th>
<th>$T_n$ [s]</th>
<th>$\zeta_n$</th>
<th>$T_n$ [s]</th>
<th>$\zeta_n$</th>
<th>$\tau$ [s]</th>
<th>$\tau$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.6642</td>
<td>-0.1022</td>
<td>0.2282</td>
<td>-0.5881</td>
<td>2.3502</td>
<td>-0.1035</td>
<td>-0.00286</td>
<td>21.7611</td>
</tr>
<tr>
<td>45</td>
<td>7.7189</td>
<td>-0.1828</td>
<td>0.278</td>
<td>-0.5566</td>
<td>1.1561</td>
<td>-0.1788</td>
<td>-0.0356</td>
<td>-18.461</td>
</tr>
<tr>
<td>90</td>
<td>9.2073</td>
<td>-0.3813</td>
<td>0.7067</td>
<td>-0.7186</td>
<td>0.6689</td>
<td>-0.2512</td>
<td>-0.0614</td>
<td>35.8752</td>
</tr>
</tbody>
</table>
The root loci with respect to the winglet dihedral angle parameters are shown in figures 19 and 20 for the short and long winglet cases respectively. As expected, for each configuration the system exhibits three oscillatory modes and two non-oscillatory modes: due to the negligible coupling behaviour previously observed, it is possible to identify the oscillatory poles to be the phugoid, short period and dutch-roll modes, while the non-oscillatory poles are the spiral and rolling modes. Modal parameters for three different winglet positions (0, 45 and 90 deg) are compared in Tables 4 and 5 for the short and long winglet cases respectively, while pole real and imaginary parts are plotted with respect to the dihedral angles respectively in figures 21 and 23 for the short winglets, figures 22 and 24 for long winglets.

It can be seen that the modal behaviour between the long and short cases are similar. Thus, unless stated otherwise, the following analysis will be referred to the long winglets configuration. The spiral mode is the only one that changes stability: starting from a small positive value for the planar configuration, the real part of the pole becomes negative when the dihedral angle is about 6 deg, reaches the maximum negative value for 27 deg and then turns toward the imaginary axis and become unstable again when the angle exceeds 72 deg. Therefore it is possible to identify a range of dihedral angles between 6 and 72 deg for which the spiral mode is stable. Nevertheless the unstable spiral for the planar configuration exhibits a time constant \( \tau \) of about 31 s so that a pilot should be able to correct the divergent trajectory.

The short period mode is highly affected by the position of the winglets. Specifically the period of oscillation \( T_s \) increases by about 200% when the dihedral angles are varied from 0 to 90 deg (the behaviour would be the same for winglets folded down) while the damping \( \zeta_s \) increases by about 22%. An approximate approach\(^\text{12}\) for the analytical calculation of the short period modal parameters may show that the square of the short period frequency \( \omega_s \) is directly proportional to the pitch stiffness and inversely proportional to the inertia moment about the \( y \) axis; folding the winglets up (or down) from the planar configuration means reducing the static margin and consequently decreasing the frequency. The same approximate approach would show that the product \( \omega_s \zeta_s \) is affected mainly by the pitch damping coefficient. Therefore even if a decrease in static margin means a decrease in pitch damping, the decrease in frequency results in a global augmentation of the damping.

For what concerns the phugoid mode, the linearisation and eigenvalue evaluation process is influenced by poor numerical conditioning in a way that, instead of producing smooth and continuous variations, the long period poles vary irregularly about a small region near the imaginary axis (sometimes with small positive real part) thus it is not possible to identify a specific behaviour.

The out-of-plane position of the winglets strongly affects the lateral dutch-roll mode as it causes a decrease in the period of oscillation of about 71% and an increase in damping of 141%. An approximate approach\(^\text{11}\) for the analytical calculation of dutch-roll modal parameters shows that the square of frequency is directly proportional to the yaw stiffness coefficient and inversely proportional to the inertia momentum about the \( z \) axis. The behaviour of these poles with respect to the dihedral angles can be explained by two effects: firstly the inertia moment \( I_{zz} \) decreases when the winglets are folded up (see Section II.B); secondly as the winglets approach a vertical position they behave similar to pair of fins thus increasing the yaw stiffness. The damping is strongly dependent on the yaw damping which increases (negatively) as the winglets folds up, again because in the non-planar configuration they behave as vertical tails.

The rolling real pole moves towards the imaginary axis when the winglets are folded out of the planar configuration so that it remains stable but with a higher time constant. Thus the mode will take more time to decay. The reason is that the real eigenvalue is proportional to the roll damping coefficient (typically negative) which decreases because the lifting surface area decreases (in a typical aircraft the contribution of the wing to the roll damping is bigger than that due to the vertical tail).

Finally modal shapes for oscillatory and non-oscillatory modes are presented in figures 25 and 26 respectively, for the planar winglets configuration. In order to compare the contributions of each state to the modal shape, the eigenvectors have been non-dimensionalised using the velocity of equilibrium \( V_T \) as reference velocity, and \( b/2V_T^2 \) as the reference time for yaw and roll rate, and \( c/2V_T^2 \) for pitch rate. The diagrams confirm that the coupling effect is negligible. For example the short period mode involves only longitudinal states as pitch rate and angle of attack, while the dutch-roll modal shapes involve mainly sideslip, bank angle, roll and yaw rates.

IV.C. Response

The results of a response analysis are discussed in this section. Specifically, the analysis will focus on how inertia rate terms affect the transient of the morphing wing. The flying wing with long winglets is set in a
Figure 19. Short winglets case: root locus with respect to dihedral angle (varying from 0 to 90 deg).

Figure 20. Long winglets case: root locus with respect to dihedral angle (varying from 0 to 90 deg).

Figure 21. Short winglets case: Pole real part with respect to dihedral angle (varying from 0 to 90 deg).

Figure 22. Long winglets case: Pole real part with respect to dihedral angle (varying from 0 to 90 deg).

Figure 23. Short winglets case: Pole imaginary part with respect to dihedral angle (varying from 0 to 90 deg).

Figure 24. Long winglets case: Pole imaginary part with respect to dihedral angle (varying from 0 to 90 deg).
steady longitudinal condition of flight where equilibrium states and controls are the following:

\[
\begin{align*}
V_T &= 15 \text{ m/s} \\
h &= 30 \text{ m} \\
\alpha &= 4.34 \text{ deg} \\
P, Q, R, \beta, \phi, \psi &= 0 \\
\theta &= \alpha \\
\delta_L, \delta_R &= 0 \text{ deg} \\
\delta_e &= -7.7 \text{ deg} \\
\delta_t &= 0.22 [0-1]
\end{align*}
\]  

(25)

In order to simulate the winglet actuators, a simple first order lag transfer function with time constant 0.025 s is included between the commanded signal block of each winglet and the multi-body equation of motion block. Thus the commanded input signal is filtered by the actuator transfer function and then the zeroth, first and second order derivative are fed into the SimMechanic joint block. It should be noticed that if zero values of the angular velocity and acceleration were fed into the block then the effects of the inertia rate terms on the response can be disabled.

For the current analysis, responses due to two step inputs are computed and compared. After one second of simulation a step input is imposed causing the left and right winglets to fold from the planar configuration up to 45 deg for the first case (see figure 27), and down to -45 deg for the second case (see figure 28), which gives a variation of 10% in wing span for both cases. The resulting angle of attack, pitch and pitch rate are shown in figures 29, 31 and 33 respectively for the up-folded winglet step, and figures 30, 32 and 34 for the down-folded case. The results are directly compared with those obtained by the simplified model where the inertia effects are neglected. In this case the only effect which influences the dynamics is the pitching-up moment produced by the static margin variation, which means that a rise in the angle of attack, pitch angle and pitch rate is produced when folding the winglet up or down. In contrast, when inertia effects are taken in account an additional effect caused by the moving masses is superimposed on the nose-up moment due to the static margin variation. The folded-up winglet causes a nose-up moment that excites the short period
Figure 27. Step input 0 to +45 deg.

Figure 28. Step input 0 to -45 deg.

Figure 29. Angle of attack response for a +45 deg step input.

Figure 30. Angle of attack response for a -45 deg step input.

Figure 31. Pitch angle response for a +45 deg step input.

Figure 32. Pitch angle response for a -45 deg step input.

Figure 33. Pitch rate response for a +45 deg step input.

Figure 34. Pitch rate response for a -45 deg step input.
mode, leading to a sharp increase in the response magnitude. In contrast, for a folded-down winglet a nose-
down moment counteracts the nose-up due to the static margin variation leading to an opposite response
magnitude. In both cases, after about 0.5 s the behaviour of the responses follows that of the simplified
model.

V. Concluding Remarks

In this paper the dynamic response of a morphing flying wing with a 2 DOF tilting-wing-tip system has
been investigated. The effect of the winglet positions on the inertia properties and cg location has been
analysed for short and long winglets configurations showing that the effect is not negligible, especially
for the long winglets case. A dynamic simulation model of the flying wing has been built using SimMechanics
Simulink tool and aerodynamic loads were provided by AVL. From the root locus analysis, for different wing
shape variations, a typical aircraft modal distribution has been recognised and sensitivities of the modal
parameters with respect to the winglets’ position has been discussed. Finally the response to two different
step variations in shape has been shown and the results compared to those of a simplified model where the
inertia effects are neglected. Results have shown a strong and non-negligible dependency of the dynamic
transient behaviour on the shape variation.

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