THE ANALYSIS OF COOL-DOWN AND SNAP-THROUGH OF CROSS-PLY LAMINATES USED AS MULTISTABLE STRUCTURES

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ABSTRACT

This paper shows the modelling of the cool-down process of unsymmetric laminates with ABAQUS™. The attention is focused on the effects that thermal stresses have on the final shape of a composite structure. Unsymmetric laminates may possess more than a single equilibrium configuration, therefore during the cool-down the solution bifurcates at a given value of temperature and the numerical analysis must be coaxed to converge into one or the other branch of the solution. A methodology to overcome this problem is presented.

1. Introduction

Composite materials made of orthotropic layers can develop a residual stress field when subjected to a thermal field that varies with time. The thermal stresses are caused by the mismatch of coefficients of thermal expansion along fibres’ direction (e.g. $o(10^{-8})$ for the longitudinal direction and $o(10^{-6})$ for the transverse direction). Furthermore, if the material is not stacked symmetrically with respect to the mid-plane, bending and twisting moments are generated within the laminated structure, resulting in out-of-plane displacements. This happens during the curing process of a composite structure, when the material is heated up to ~170 °C and subsequently cooled-down to room temperature to achieve a desired degree of cure. The material displacements are usually difficult to predict accurately therefore unsymmetric stacking sequences are generally avoided during manufacturing. On the other hand, if the thermal behaviour of such material is thoroughly understood, its effects could provide an economic way to obtain structures with complex geometries or even with multiple equilibrium configurations. A few examples of complex structures obtainable from a flat tool are shown in Figure 1 and Figure 2.

Figure 1. Variable curvature shell
In addition to this, it has been observed experimentally that, if properly tailored\(^2\), unsymmetric laminates possess more than a single equilibrium configuration. This presents the great advantage to the designer such that one structure can achieve multiple geometric configurations without added mechanism. It is then possible to snap from one configuration to another by means of an actuation system\(^7\); the actuator is only required to provide energy during the snap-through process and not to maintain a configuration since both are stable.

Typical values for the material properties used throughout the paper are shown in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>(E_{11}) [GPa]</th>
<th>(E_{22}) [GPa]</th>
<th>(G_{12}) [GPa]</th>
<th>(\nu_{12})</th>
<th>(\alpha_1 [1/^\circ C])</th>
<th>(\alpha_2 [1/^\circ C])</th>
<th>(t [\text{mm}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>T300/914</td>
<td>130e9</td>
<td>10e9</td>
<td>4.4e9</td>
<td>0.33</td>
<td>-0.18e-6</td>
<td>30e-6</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table 1. Materials properties of carbon fibres laminates used

2. **Modelling the cool-down**

In order to be able to predict the equilibrium shapes and to describe the mechanical behaviour of unsymmetric composites, the attention is focused on the cool-down of the unsymmetric panels from the highest curing temperature to room temperature. It must be pointed out that moisture content can also have significant effects in developing the residual stresses but for the purpose of this analysis its effects will be neglected and the study will assume the temperature as the independent variable.

2.1. Stability consideration

The presence of multiple equilibrium states implies that during the cooling process, a singular point in the equilibrium path of the structure is encountered. Using the same notation as in Ref. 7, the incremental formulation of finite element approach to non-linear analysis is\(^9\)

\[
\dot{K} \Delta U = \dot{\tau} + \Delta \Gamma \dot{U} - F
\]

where \(\dot{K}\) is the tangent stiffness matrix of the finite element model at time \(t\) (function of stress and displacement in the structure), \(\Delta U = \dot{\tau} + \Delta \Gamma \dot{U}\) is the vector of nodal incremental displacements, \(\dot{\tau} + \Delta \Gamma \dot{U}\) is the vector of externally applied nodal loads at time \(t\) (in this particular case, it is a function of the temperature) and \(F\) is the vector of nodal forces corresponding to the internal stress at time \(t\).
If the structure is in a state of stable equilibrium, $'K$ is positive definite (i.e. all its eigenvalue will be real and positive) whereas it is non-positive definite if the structure is in a state of unstable equilibrium (i.e. there is at least one negative eigenvalue). During the numerical simulation, if a negative eigenvalue is found, it means that the solution jumped after a singular point and found an unstable equilibrium configuration; this does not necessarily affect the accuracy of the solution, it only means that between two successive increments there exists a configuration for which the tangent stiffness matrix is singular, a condition typical of limit point and bifurcation points. There are various strategies for dealing with this issue, in particular, for snap-through problems an explicit dynamic scheme (e.g. Abaqus/Explicit) is the most reliable because it accounts for the inertia effects but it can be very expensive from the computational point of view. For this reason a different strategy, based on modified static simulation, has been employed to perform the analysis here presented. The next section will provide further details.

2.2. Solution strategies

A square plate with a stacking sequence of $[0_2/90_2]^T$, if cured flat at high temperature, shows after cool-down, a cylindrical shape that can be easily snapped into a second cylindrical one by the application of a force. This type of plate has been thoroughly analysed by Hyer et al.$^{1,6,7}$ To highlight the possible interaction with more complex systems, a different structure is now considered. It consists of a rectangular plate made of both symmetric and unsymmetric laminates; the stacking sequence is shown in Figure 4.

Figure 4. Stacking sequence for the bi-stable panel

Figure 5 shows the two stable shapes. The panel is modeled using 800 four nodes shell elements (S4R) as shown in Figure 6. The number of nodes employed is 861 and the total number of degrees of freedom is 5166. The plate is clamped in the geometric centre to suppress rigid body motions and to reproduce free-free boundary conditions. Equilibrium configurations are predicted with two separate analyses: a “*Static” step and a “*Static, stabilize” step. In both analysis an initial
temperature of 140 °C and a final one of 0° C are applied to all the nodes of the model. Using the “*Static*” analysis, the solution always converges to the first equilibrium shape shown in Figure 5 and “negative eigenvalue warnings” are issued by the solver. This confirms the presence of a singular point and that the algorithm followed one branch of the solution till convergence is achieved. To capture the second equilibrium shape, a separate analysis makes use of the "stabilize" option. The automatic stabilisation, implemented in ABAQUS/Standard, adds viscous forces to the global equilibrium equation:

\[
P - I - F_v = 0
\]

where \(P\) and \(I\) are respectively, the external and the internal force vectors. \(F_v\) represents the viscous forces and has the form of:

\[
F_v = cM^*v
\]

where \(M^*\) is the artificial mass matrix calculated with unit density, \(c\) is the damping factor chosen as a fraction of the dissipated energy and \(v\) is the vector of nodal velocities. To ensure accuracy it is important to choose \(c\) as the smallest value that suppresses the local instabilities. This is usually done by increasing its value until the “negative eigenvalue warnings” disappear. The contour plots in Figure 7, show the distribution of the viscous dissipation energy and the total strain energy of the model. Figure 8 shows a comparison of the two energies for an element on the top left corner of the plate, where the viscous forces reach their maximum value. It can be observed that this value does not exceed 4% of the total strain energy therefore it can be stated that the artificial damping is not affecting the accuracy of the solution. An alternative way to obtain the second equilibrium shape is loading one of the two computed equilibrium configurations with a concentrated force. The structure will first deform elastically, then if the force is large enough, it will bifurcate. At this point, if the load is removed the panel will rest in the other equilibrium position. This procedure requires attention in the choice of the load and the boundary conditions, but provides also the full load-displacement diagram that characterise the extreme non linear behaviour of the structure. The next section will present a selection of the numerical results obtained.
3. **Prediction of equilibrium configurations**

The predicted equilibrium configurations at room temperature are shown in Figure 10 and Figure 11. A comparison with Figure 5 shows a good agreement with the experimental shapes. In the deformed configurations, the unsymmetric portion of the panel exhibits a cylindrical deformation with generators either parallel to the x-axis (second shape) or to the y-axis (first shape). As explained previously, this is due to the unsymmetric stacking sequence that causes the structure to buckle under the thermal load. During the first incremental iterations of the analysis, when the temperature change is still small (approximately 10 °C) and the deformations are within the linear range, the shell always deforms into a saddle configuration first (displacements are shown in Figure 9), then the equilibrium path bifurcates and the two solution schemes converge to the different geometric shapes. This point is further clarified in the next section.
3.1. Curvature – Temperature diagram

The temperature gradient generates out-of-plane displacements in the plate that can be quantified in terms of principal curvatures \( k_x \) and \( k_y \). In the next diagrams, the principal curvatures of the unsymmetric portion of the panel are shown. Figure 13 and Figure 12 show the curvature-temperature diagram for the 180 x 360 mm panel, with different number of layers (4 and 8 respectively). At 140 °C (right hand side of the diagram), the plate is flat; as soon as the temperature decreases, the deformations (i.e. the curvatures) start to develop; the final state is obtained when the temperature reaches 0° C (left hand side of the diagram). The stacking sequence, for the different number of layers tested, is shown in Table 2. The magnitude of the curvature depends greatly on the thickness of the plate and it decreases as the number of layers increases. The values shown in the diagrams are average values; however they are sufficient to describe the geometry in the two different states of equilibrium. Figure 14 shows \( k_x \) and \( k_y \) versus temperature for the only configuration of the 12-layered plate, it is in fact too thick to have two equilibrium states. There is a critical value for the non-dimensional thickness \( t^* \), (defined as the ratio between the thickness and the width of the plate) above which the shell does not have a bi-stable behaviour. The 8-layered plate is bi-stable and has a value for \( t^* \) of \( \frac{1}{180} \). The 12-layered one has \( t^* \) equal to \( \frac{1}{120} \) and it is not bi-stable, therefore \( t_{cr}^* \) is an intermediate value between \( \frac{1}{120} \) and \( \frac{1}{180} \).
Another important parameter, that can be obtained from the curvature–temperature diagrams, is the critical temperature $T_{cr}$: for $T > T_{cr}$, $k$ is nearly equal to $-k$, therefore for each value of the temperature there will be only one possible configuration (i.e. the saddle configuration). For $T < T_{cr}$, there will be two solutions for each value of $T$, either $k_1 \gg k_2$ (first cylindrical shape) or $k_2 \gg k_1$ (second cylindrical shape). The value of $T_{cr}$ is most easily identified in Figure 15, where for convenience, $-1 \cdot k$ is plotted. For the 8-layered plate $T_{cr}$ it is 93°C whereas for the 4-layered one it is 130°C. For values of the temperature above $T_{cr}$, the principal curvatures are nearly equal and opposite as it is expected for a saddle shape configuration, below $T_{cr}$ the difference between $k_1$ and $k_2$ diverges until an almost perfect cylindrical shape is achieved.

Figure 12. Principal curvature for the first shape

Figure 13. Principal curvature for the second shape
Figure 14. Principal curvature for the 12-layered plate

Figure 15. Identification of $T_{cr}$

<table>
<thead>
<tr>
<th>Number of layers</th>
<th>Stacking sequence symm. part</th>
<th>Stacking sequence unsymm. part</th>
<th>Total laminate thickness [mm]</th>
<th>$t^* = \frac{\text{thickness}}{\text{width}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>[0/90]_s</td>
<td>[0_2/90_2]_T</td>
<td>0.5</td>
<td>1/360</td>
</tr>
<tr>
<td>8</td>
<td>[0_2/90_2]_s</td>
<td>[0_4/90_4]_T</td>
<td>1</td>
<td>1/180</td>
</tr>
<tr>
<td>12</td>
<td>[0_3/90_3]_s</td>
<td>[0_6/90_6]_T</td>
<td>1.5</td>
<td>1/120</td>
</tr>
</tbody>
</table>

Table 2. Stacking sequences for different number of layers
3.2. Load – Displacement diagram

In this section the results related to the snap-through analysis are presented. The aim of this analysis is to compute the maximum out-of-plane load that the panel can withstand before changing configuration. To obtain the load corresponding to the limit point, the cool-down of the plate is modelled first, and then in a second step, the following boundary and loading conditions are applied:

1. The four corner nodes of the shell are restrained from moving along the vertical axis.
2. The geometric centre of the unsymmetric part of the shell is constrained to move along the vertical axis.
3. A concentrated force, parallel again to the vertical axis, is applied to the centre of the unsymmetric part.

A “*Static, stabilize” step is then performed. From the experimental point of view, this technique is equivalent to a load-controlled test. Figure 16 shows the load-displacement plot for the same 180 x 360 mm panel with different number of layers. As expected the critical load increases considerably with the thickness. In Table 3 the value of the critical load is reported.

![Stabilised load-displacement diagram for different number of layers](image)

**Figure 16.** Stabilised load-displacement diagram for different number of layers

<table>
<thead>
<tr>
<th>Num. of layers</th>
<th>Load [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4.92</td>
</tr>
<tr>
<td>8</td>
<td>13.1</td>
</tr>
<tr>
<td>12</td>
<td>17.38</td>
</tr>
</tbody>
</table>

**Table 3.** Critical load for different number of layers

Figure 17 shows a comparison between the applied load and the sum of the reaction forces at the four corner nodes. The difference between the two curves can be regarded as a measure of the effect of the viscous damping during the analysis. For loads less than the limit load the behaviour of the two analyses is identical, then, once the limit load is exceeded, the structure becomes unstable and while the
red curve continues horizontally due to stabilisation, the green one follows the unstable equilibrium path. Eventually the curves meet again in the new stable portion of the diagram. A curve similar to that named "reaction force" is expected if performing a "*Static, Riks" analysis step. This type of analysis considers the load as an additional unknown that varies accordingly with the tangent stiffness matrix. For this particular type of structure, where the global behaviour is driven by local instabilities, the "proportional loading method" does not seem to be robust enough and it has been discarded since the pseudo-dynamic analysis performed better.

3.3. Explicit integration analysis

As mentioned in the previous section the explicit analysis is still the most robust tool to predict snap-through behaviour because it takes into account the effects of inertia. Both the explicit-dynamic analysis and the stabilised algorithm always converge to the same shape as expected since their objective is to reach quickly the next stable solution for a given load.

4. Concluding remarks

The possibility of tailoring the deformations caused by residual stress fields relies on the accurate prediction of the post-curing shape of unsymmetric laminates. The paper presented a methodology to provide a numerical estimate for the equilibrium configurations of unsymmetric laminates that show multiple equilibrium states. Three different approaches are described and they all show a good
agreement. Values for the critical temperature and the critical load to induce bifurcation are obtained. Monitoring the reaction forces is also possible to obtain the full equilibrium path that brings the structure from one stable configuration to the other one. Finally a comparison with the Explicit-dynamic analysis is introduced.

REFERENCES

10. ABAQUS Online Documentation: Version 6.5-1