DESIGN OF A COMPLIANT AEROFOIL USING TOPOLOGY OPTIMISATION

D. Baker¹, M. I. Friswell²

¹Department of Aerospace Engineering, University of Bristol, UK
david.baker@bristol.ac.uk
²Department of Aerospace Engineering, University of Bristol, UK
m.i.friswell@bristol.ac.uk

ABSTRACT

A method of distributing a limited amount of material in a frame type structure with the objective of creating a compliant morphing aerofoil is presented.

The approach applies the Method of Moving Asymptotes (MMA) optimisation algorithm to an initial ground structure. By tailoring the cross sectional properties of the frame members the surface displacement of the structure when subjected to applied actuator and aerodynamic loading is controlled.

A multi-criteria objective function is formulated that provides control of the surface form of the structure without instilling predefined nodal displacements together with addressing the conflicting issues of adequate stiffness and flexibility. Local failure conditions are incorporated into the algorithm in the form of maximum, minimum and buckling stress constraints.

Keywords: Topology, Optimisation, Compliant, Aerofoil.
INTRODUCTION

Aircraft structural morphing provides a possible solution to an air vehicle able to perform multiple tasks during a mission with improved performance when compared with conventional fixed structure aircraft. Variable camber systems such as that demonstrated by the Mission Adaptive Wing (MAW) programme enabled significant performance improvements over similar fixed wing systems, particularly if the aircraft has to fly using several different lift coefficients [1]. However in general the extra requirements of weight and system complexity have prevented such concepts from becoming a practical proposition.

A compliant mechanism is a single piece structure designed to transmit motion and force mechanically relying solely upon elastic deformation of their constituent elements. As such they may be subject to advantages including high displacement accuracy, zero backlash and wear and ease of manufacture without assembly [2]. The design of such mechanisms treads a balance between achieving adequate stiffness in order that external loads may be supported yet be simultaneously flexible enough that the required motion due to applied loads is realised.

Various strategies for the design of compliant mechanisms have been developed in past studies however two basic categories may be defined. A kinematics approach replaces flexure joints with conventional pivots and a torsional spring system. This tends to provide a solution with concentrated areas of compliance within the structure, so called lumped compliance. An alternative is to take a more structural view of the problem using topology optimisation methods. Ananthasuresh and Kota [3] developed the homogenisation approach to compliant structure design, where the properties of a “composite material” composed of solid and void sections are parameterised. A similar approach involves multiplying the material properties with a “density function” [4]. In addition to this is the so called “ground structure” approach of truss or frame elements, where the layout of an elemental structure is found by allowing a certain set of connections between nodal points to be set as potential structural or vanishing members [5].

The focus of this paper is the design of a compliant system that is able to provide structural control and motion to the trailing edge of a morphing aerofoil. An initial skeletal frame type ground structure is selected whereupon the member cross-section dimensions and applied actuator deflection are controlled in order to provide a predetermined surface deflection.

PROBLEM FORMULATION

System Configuration

The system under consideration comprises a compliant mechanism forming the trailing 40% chord region of a NACA 0024 aerofoil. It is envisaged that this adaptive section would be located at the tips of a straight, symmetrical wing with a span of 3000mm and an aspect ratio of 7 and provide control to the outermost 500mm sections of the wing only (Fig. 1).
Fig. 1: Semi-span illustration incorporating tip deflection

The initially straight camberline is deflected in the trailing edge region according to a quadratic function and the respective NACA thickness function is subsequently applied in order to form the target surface shape that helps define the optimisation objective.

Two ground structures are proposed for the purpose of the investigation. The first is a single structure comprising 209 members connecting a network of 78 nodes with the node located at 60% chord in the midpoint of section subject to a horizontal actuated displacement (Fig 2a). A second system is formed by the mating of two separate structures each comprising 259 members connecting individual networks of 96 nodes (Fig. 2b). For the purpose of the optimisation only a single piece of the structure (upper section) is modelled, the resultant lower section is assumed to be a mirror of the upper and is subject to an equivalent but opposite actuated displacement.

Fig. 2: Ground structure formations featuring restraints and applied actuator displacements. a) Single piece structure with single actuator and 2 fixed nodes. b) Split structure with twin actuators and 4 fixed nodes.

Aerodynamic Loading

The pressure distribution on the wing surface is calculated using PANAIR [6], a higher-order panel method. The resulting pressure coefficients ($C_p$) at each panel location can be transformed to an applied pressure load ($p$) using Eq. (1)

$$p = \frac{1}{2} \rho V_x^2 C_p$$  \hspace{1cm} (1)
where $\rho$ is air density and $V_\infty$ is the free stream air velocity. This pressure may in turn be transformed to an applied panel force vector by multiplication with the respective surface panel area.

In order to provide an accurate calculation of the surface pressure distribution a dense network of surface panels is required and by interpolating this mesh between the surface structural nodes the applied structural load vector ($f_{\text{aero}}$) can be determined.

**Optimisation Problem**

**Objective function:** The problem of achieving shape change is formulated as minimising the distance between the achieved deflection of the actuated trailing edge surface and a preconceived target surface. The precise location of the deflected structural surface nodes is only measured with respect to the required target curve; in this manner required nodal deflections are not explicitly given but formulated as the minimum distance between node and the objective surface. The shape change error ($E_{\text{act}}$) is given by Eq. (2)

$$E_{\text{act}} = \sum_{i=1}^{n} w_i h_i$$  \hspace{1cm} (2)

where $w$ is a vector of weighted importance, for instance those nodes that form the interior of the structure are considered irrelevant and so are removed from the error formulation by the inclusion of a zero term. $h$ is the vector of calculated distances from the deflected nodes to the prescribed target surface.

The second objective is the minimisation of structural deflection due to applied air loading. Deflection due solely to the applied actuation force is calculated by the direct stiffness method as given by Eq. (3)

$$u = K^{-1} f_{\text{act}}$$  \hspace{1cm} (3)

where $K$ is the structural stiffness matrix and $u$ the vector of displacements due to the applied actuation load vector ($f_{\text{act}}$). Similarly the deflection due to applied actuation loads and aerodynamic loading ($v$) may be calculated using Eq. (4)

$$v = K^{-1} (f_{\text{act}} + f_{\text{aero}})$$  \hspace{1cm} (4)

The aerodynamic displacement error ($E_{\text{aero}}$) is then formulated as Eq. (5)

$$E_{\text{aero}} = ((u - v)^T (u - v))^{1/2}$$  \hspace{1cm} (5)
The multi-criteria objective \((J)\) may then be calculated as a weighted sum of the two error functions.

\[
J = \lambda E_{aero} + (1 - \lambda) E_{act}
\]  

\(\text{(6)}\)

**Design variables:** By modelling the structure as a network of individual frame members the cross sectional dimensions of the elements may be controlled to affect the output displacement. The cross sections are taken to be constant and rectangular and as such each may be defined using two design variables, member cross section height \((b_j)\) and width \((d_j)\). An additional design variable is the actuator input, defined as the applied horizontal displacement to the single input node.

**Constraint functions:** Local constraints of maximum, minimum and critical Euler buckling stress are applied to each member together with a global constraint of maximum material volume.

\[
\sigma_j^{\text{min}} \leq \sigma_j \leq \sigma_j^{\text{max}}, \quad \text{if } a_j \geq 0
\]

\[
\sigma_j^{\text{cr}} \leq \sigma_j, \quad \text{if } a_j \geq 0
\]  

\(\text{(7)}\)

where (for the \(j\)th member) \(\sigma_j^{\text{max}}\) is the stress limit in tension, \(\sigma_j^{\text{min}}\) is the stress limit in compression, \(\sigma_j^{\text{cr}}\) is the critical Euler buckling stress and \(a_j\) is the member cross sectional area. Thus the problem concerning the introduction of local stress constraints is that a typical optimum solution may contain degenerate parts with zero measure and so the formulation described by Eq. (7) proves unsuitable, however by modifying this to force based constraints degenerate parts may be included as illustrated by Eq. (7)\([7]\) where \(f_j\) is the element force.

\[
a_j \sigma_j^{\text{min}} \leq f_j \leq a_j \sigma_j^{\text{max}}, \quad \text{if } a_j > 0
\]

\[
a_j \sigma_j^{\text{cr}} \leq f_j, \quad \text{if } a_j > 0
\]  

\(\text{(7)}\)

Volume constraints were applied as a fraction of the maximum material volume \((Vol_{\text{max}})\) given the upper bounds of the design variables as shown by Eq. (8), for example the constrained volume \((Vol)\) may be expressed as \(Vol=0.5Vol_{\text{max}}\).

\[
Vol_{\text{max}} = \sum_{j=1}^{M} l_j b_j^{\text{max}} d_j^{\text{max}}
\]  

\(\text{(8)}\)

where (for the \(j\)th member) \(l_j\) is the member length.

**Optimisation technique:** A flowchart detailing the optimisation method is presented in Fig. 3. The optimisation problem is solved using the Method of Moving Asymptotes (MMA) algorithm \([8]\), a gradient based method that has found application in topology optimisation problems \([9,10]\). The topology is updated iteratively from random starting variables until convergence is reached (change in objective function of less than 0.1\%) or a pre-stated
maximum number of iterations is achieved. Gradients of the objective and constraint functions are computed using the forward step finite difference method.

![Flowchart of topology optimisation process using MMA algorithm.](image)

**RESULTS**

Topology optimisation was initially performed using the MMA algorithm on the ground structure shown in Fig. 2a with data provided in Table 1. The camberline of the compliant trailing edge section was modified using a quadratic function resulting in a vertical tip deflection of -20.5mm. Using the equation for equivalent flap angle ($EFA$) [5]

$$EFA = \tan^{-1}\left( \frac{u_{tip}}{0.4c} \right)$$  \hspace{1cm} (9)

where $u_{tip}$ is the vertical tip deflection and $c$ is the chord length, provides an $EFA$ value of -6.8°. The $EFA$ provides a simplified but convenient measure of the performance of the structure with respect to the required degree of compliance.

Two sets of results are shown by their respective states of deformation; Fig. 4a illustrates the solution if the lower design bounds of all members are set to zero. Fig. 4b is the solution in which the lower design bounds of the surface facing members, that is those members that
are presented to the surrounding airstream, are increased to $b_{i\text{min}}=0.5\text{mm}$ and $d_{i\text{min}}=0.5\text{mm}$. In this latter case the achieved $EFA$ is reduced from $-5.6^\circ$ to $1.9^\circ$ for solutions a) and b) respectively. This revision of the constraints on the design variables was placed on the model in order that a surface skin structure may be supported effectively by the compliant substructure. However such measures provide a clear limit on the achievable deformation due to the formation of excessive stresses in these surface members. A comparison of the deflected profile achieved by solution a) with the target profile is shown in Fig. 5. In addition to this is the resultant deflection when subjected to both actuation and aerodynamic loads. Actuator force and displacement values are listed in Table 2.

In an attempt to provide improved deflection characteristics of the structure whilst incorporating the increased lower bounds of the surface member cross sections a two piece ground structure is proposed as illustrated in Fig. 2b. The resultant optimised topology is displayed in Fig. 6. A comparison of the achieved and target surface forms of the split section topology is illustrated by Fig. 7. Calculation of the $EFA$ for the split structure both with and without aerodynamic loading reveals values of $EFA=-6.8^\circ$ and $EFA=-5.9^\circ$ respectively.

Fig. 4: Topology optimisation results of NACA 0024 profile with a) unrestricted surface members and b) restricted $b_i$ and $d_i$ surface members

Fig. 5: Surface form of deformed NACA 0024 topology with and without aerodynamic loads compared with target profile (unrestricted surface elements)

Fig. 6: Topology optimisation results of split NACA 0024 ground structure (upper section only).
**Fig. 7**: Surface form of deformed NACA 0024 split topology with and without aerodynamic loads compared with target profile.

<table>
<thead>
<tr>
<th>Optimisation Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus</td>
<td>2500MPa</td>
</tr>
<tr>
<td>Maximum Allowable Stress ($\sigma_{j}^{\text{max}}$)</td>
<td>45MPa</td>
</tr>
<tr>
<td>Minimum Allowable Stress ($\sigma_{j}^{\text{min}}$)</td>
<td>-45MPa</td>
</tr>
<tr>
<td>Maximum Element Cross-Section Width ($b_{j}^{\text{max}}$)</td>
<td>3mm</td>
</tr>
<tr>
<td>Minimum Element Cross-Section Width ($b_{j}^{\text{min}}$)</td>
<td>0mm-0.5mm*</td>
</tr>
<tr>
<td>Maximum Element Cross-Section Height ($d_{j}^{\text{max}}$)</td>
<td>3mm</td>
</tr>
<tr>
<td>Minimum Element Cross-Section Height ($d_{j}^{\text{min}}$)</td>
<td>0mm-0.5mm*</td>
</tr>
<tr>
<td>Air Velocity</td>
<td>70m/s</td>
</tr>
<tr>
<td>Angle of Attack</td>
<td>3°</td>
</tr>
<tr>
<td>Air Density</td>
<td>1.23kg/m$^3$</td>
</tr>
</tbody>
</table>

* $b_{j}^{\text{min}}$ and $d_{j}^{\text{min}}$ values increased for surface elements only

**Table 1.** Optimisation and structural analysis variables.

<table>
<thead>
<tr>
<th>Analysis Description</th>
<th>Actuator Displacement (horizontal)</th>
<th>Required Actuation Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Piece Structure, unrestricted elements</td>
<td>3.86mm</td>
<td>21N</td>
</tr>
<tr>
<td>Single Piece Structure, restricted elements</td>
<td>1.49mm</td>
<td>142N</td>
</tr>
<tr>
<td>Split Section Structure</td>
<td>-4.03mm</td>
<td>-135N</td>
</tr>
</tbody>
</table>

**Table 2.** Actuator force and displacement requirements.

**CONCLUSIONS**

A method for the design of a conformable aerofoil has been presented using design optimisation techniques in order to distribute a limited amount of material in a frame type structure. The objective of the design method was to create a structure that was able to provide a significant deflection of the trailing edge when subjected to the input displacement of a limited number of actuators whilst also ensuring a minimal deflection in response to an applied aerodynamic load. This is achieved by optimising a multi-criteria objective function comprised of a weighted sum of two displacement functions, the displacement error of the structure with respect to a target geometry and the resultant displacement due to aerodynamic loading to the structure.
Using the topology optimisation a single piece compliant structure forming the trailing section of a NACA 0024 profile could be tailored to produce an EFA of -5.6°, reducing to -5° when also subjected to a simulated aerodynamic load. Observation of the deformed structure reveals compliance is not distributed evenly throughout the structure but localised around the point of applied actuation displacement. The remaining trailing portion of the structure is then comprised of a rigid network of elements. In order to provide a supportive sub-structure to a compliant skin the minimum element cross-sectional dimensions \((b_j^{\text{min}}, d_j^{\text{min}})\) of those elements presenting themselves to the surrounding airflow were increased to 0.5mm. This measure reduced the resulting EFA to -1.9°, a figure further reduced to -1.3° when also subjected to aerodynamic loads. This reduced deflection is as a result of the imposition of stress constraints on the upper and lower surface facing members, where previously these elements were able to reduce to zero measure in order to permit the required member extension/contraction without violation of imposed constraints. In order to provide a system more able to achieve the desired EFA of -6.8° whilst maintaining the presence of surface forming members a split ground structure is proposed. Such a system removes the requirement of large member elongations in order to achieve significant trailing edge deflections. The upper section of the structure was created using topology optimisation methods and the lower surface formed by mirroring the upper section and applying a secondary actuation displacement in the opposing direction. This method provided an EFA of -6.8°, a value that reduced to -5.9° when subject to aerodynamic loads.

REFERENCES