Stochastic model updating of a spot welded structure

C. Mares, J. E. Mottershead, M. I. Friswell†
University of Liverpool, Department of Engineering, Brownlow Hill, Liverpool, UK
†University of Bristol, Department of Aerospace Engineering,
Queen’s Building, University Walk, Bristol, UK
Email: mares@liverpool.ac.uk

Abstract
The usual model updating method may be considered to be deterministic since it uses measurements from a single test system to correct a nominal finite element model. Variability of the test structure and uncertainty related to the finite element modelling show that multiple sets of experiments should be taken into consideration. In this paper stochastic model updating is carried out on a spot-welded benchmark structure using a contact finite element model, which incorporates the uncertainty specific to the modeling of the spot welds. The analytical model is updated based on a Monte-Carlo inverse procedure.

1 Introduction
Model updating attempts to optimise a finite element model by minimizing the prediction errors with respect to test data from a single test system [1], [2]. An important aspect of the updating process remains the choice of updating parameters and the parameter set which can result in an improved prediction should always be justified physically. The uncertainty in the model should be located and parametrised sensitively to the predictions. The validation of the model should be made by assessing the model quality within the operational domain and the robustness to modifications of the loading conditions, design modifications, coupled structure analysis and changes of the boundary conditions.

But predictions based on a single calibration of the model parameters, cannot give a large confidence interval for numerical simulations to represent the actual structure and the effect of the most significant uncertainties and errors through the modeling and simulation process onto the predicted response quantities should be assessed [3,4]. As more complex models are being developed, it is conceivable that the new designs may involve hundreds of random variables and a large number of responses at many structural locations should be determined in a stochastic sense. The concept of the variability response function was introduced by [10] and was related to the assumption that the inhomogeneity and uncertainty of the system properties (e.g., mass density, cross section, bending rigidity) are determining upper bounds of the response variability and reliability (in terms of safety indices) of the structural system.

Multiple realizations of an experiment (numerical or physical) lead to the concept of the meta-model [9] and the possibility to express the distance between models and operate design modifications based on statistical concepts as opposed to the comparison between deterministic models based on nominal variables.

The Monte-Carlo simulation appears to be the only universal method which can provide accurate solutions for problems in stochastic mechanics, the meta-model being a source for a statistical problem description, confidence measures, correlation with experimental data, global dependencies and selection of dominant design variables, the only disadvantage being that of computational time consumed for the simulations [5], [6], [7], [11].

In this paper a stochastic Monte-Carlo correlation and inverse uncertainty propagation is carried out on a benchmark structure with spot-welds. When modeling spot welds, it is difficult to take into account many
local effects such as geometrical irregularities, residual stresses, material in-homogeneities and defects due
to the welding process [12,13]. Furthermore a detailed and complex local model of the joint characteristics
is not possible in the case of industrial applications where thousands of spot welds are used to manufacture
actual structures.

These conceptual modeling uncertainties (or lack of system knowledge) are incorporated in an interface
finite element [1] which connects the welded panels in the area of the spot-weld. The material properties
of the element determine the local stiffness in the tangential and normal directions on the surface, and
together with the elasticity of the curved regions of the folded plate, they become the random parameters
used in the Monte-Carlo simulation of the dynamic properties of the finite element model. An inverse
propagation is carried out using a linear regression model and with the purpose of updating a nominal
finite element model with spot welds based on a set of experimental measurements.

The structure of the paper is the following: the theoretical description of the model updating based on a
gradient and multiple multivariate regression is presented in Section 2. The finite element model of the
interface element is described in Section 3. The experimental and finite element correlation of one sample
is presented in Section 4 and the Monte-Carlo stochastic updating follows. A discussion of the results and
conclusions brings the paper to a close.

2 Theoretical aspects

2.1 Uncertainty propagation

A model of a mechanical system presents a set of physical parameters whose possible variations around a
nominal value create a model of uncertainty:

\[ x = x_0 + \Delta x \]  

(1)

These variables can be different structural parameters as material and geometrical properties or aspects of
modelling related to the boundary conditions etc. and their variation may be correlated or independent.

For a finite element model the global matrices are expressed as linear combinations of constant element or
substructure matrices multiplied by the variable parameters and affecting all the terms in the substructure
stiffness or mass matrix. More sensitive geometric parameters are studied for the case of complex joints
[15-17] or by eigenvalue decomposition of the stiffness matrix and modification of its eigenvalues and
eigenvectors [18,19]. For example a general parametrisation for the stiffness matrix may be written as,

\[ K(x) = \sum x_j K_j^x \]  

(2)

with similar decomposition for the the mass or damping matrices. The choice of updating parameters and
of variation bounds for them, which can be justified physically, remains the main aspect of any analysis.
The possibility to describe the variation of the updating parameters by a probability distribution involves
experimental data, which is not always accessible. In the case of equivalent models where the parameters
are model dependent (like in the case of generic element eigenvalues for example) a possible assumption,
which could be used, is that of a uniform probability distribution over an interval justified by design
consideration.

In the Monte-Carlo process a random parameter vector obtained from the parameter distribution can be
used to characterize the uncertainty in some output variables of interest. The random output variables
\( y \) can be physical quantities observed at a single location, a single physical quantity observed at \( p \) locations, or at \( p \) time instants, combinations of these etc.

This process produces a posterior predictive distribution and can be used for an inverse estimate when compared to the experimental data obtained from measurements on the actual system.

For the \( n \) random observations of each \( i \) th variable the mean value is obtained as:

\[
\bar{y}_i = \frac{1}{n} \sum_{k=1}^{n} y_{ki} = \frac{1}{n} \vec{y}_i e_n
\]

\( e_n = [1, 1, \ldots, 1] \); \( e_n^t e_n = n; \quad \vec{y}_i = [y_{1i}, y_{2i}, \ldots, y_{ni}]^t \)

and collectively in terms of the data vector:

\[
\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i = \frac{1}{n} y_1 \quad \tilde{y}_2 \quad \cdots \quad \tilde{y}_n e_n = \frac{1}{n} Y^t e_n
\]

The sample mean vector \( \bar{y} \) is an unbiased estimate to the population mean vector \( \mu \) (over all possible values) though never equal to it. For the observed \( p \) variables the covariance matrix comprising the sample variances and covariances is defined by:

\[
S = \begin{bmatrix}
    s_{11} & s_{12} & \cdots & s_{1p} \\
    s_{21} & s_{22} & \cdots & s_{2p} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{p1} & s_{p2} & \cdots & s_{pp}
\end{bmatrix}
\]

and can be computed as:

\[
S = \frac{1}{n-1} \left[ \bar{y}_1 \quad \bar{y}_2 \quad \cdots \quad \bar{y}_p \right] \left[ I - \frac{1}{n} e_n e_n^t \right] \left[ \bar{y}_1 \quad \bar{y}_2 \quad \cdots \quad \bar{y}_p \right] = \frac{1}{n-1} \left[ Y^t Y - n \bar{y} \bar{y}^t \right]
\]

The sample covariance matrix \( S \) is an unbiased estimator of the population covariance \( \Sigma \) but never equal to it. Considering the system described by the relationship between the random variables of the input and of the output:

\[ y = F(x) \]

the inverse propagation of the error can be achieved by comparing the means of the output vector with the desired output. From different stochastic optimisation techniques, the gradient-and-regression approach yields a linearised version of (5) by using a Taylor expansion about the mean vector of the sample \( \bar{x} \) at an iteration \( k \):

\[
y_k \equiv F(\bar{x}) + \frac{dF}{dx} (x_k - \bar{x}) \equiv \bar{y} + G(x_k - \bar{x}); \quad G = \frac{dF}{dx}
\]
where the second equality is valid for a linearised model when \( \bar{y} = F(\bar{x}) \). If the target mean vector of the output variable is \( \bar{y}_0 \) and at an iteration \( k \) the population mean \( \mu_{x,k} \) lead to a mean vector \( \bar{y} \neq \bar{y}_0 \), an improved estimate \( \mu_{x,k+1} \) of the population mean of the input can be obtained by using (6):

\[
\bar{y}(\mu_{x,k+1}) \equiv \bar{y}(\mu_{x,k}) + \frac{dF}{dx}_k (\mu_{x,k+1} - \mu_{x,k}) = \bar{y}_0
\]

\[
\mu_{x,k+1} = \mu_{x,k} + G_k^{-1}(\bar{y}_0 - \bar{y}(\mu_{x,k}))
\]

with convergence of the iterative process when \( \mu_{x,k+1} \equiv \mu_{x,k} \quad \text{and} \quad \bar{y}(\mu_{x,k}) \equiv \bar{y}_0 \). Usually the distance between the points in the output population and the target mean vector is the Euclidian distance or the Mahalanobis distance:

\[
D_E^2 = (y_k - y_0)'(y_k - y_0)
\]

\[
D_M^2 = (y_k - y_0)'S_y^{-1}(y_k - y_0)
\]

The Mahalanobis distance can be determined as an Euclidian norm after normalization by the variances and elimination of covariances and due to the scaling through the use of the covariance matrix is unitless expressing the difference in mean by a number of standard deviations relative to the pooled covariance matrices (to be defined in the following section).

### 2.2 Multivariate multiple regression

In order to obtain a linearized model, a multivariate multiple regression analysis is carried out at each iteration in order to estimate a linear relationship between the predictive variables and the criterion variables, or the input \( x = \{x_1, x_2 \ldots x_q\} \) and output \( y = \{y_1, y_2 \ldots y_p\} \) variables as (the iteration index dropped):

\[
Y = e_n a' + XB + E; \ a \in \mathbb{R}^{q \times p}; \ B \in \mathbb{R}^{p \times p}; \ x \in \mathbb{R}^{q \times 1}; \ Y \in \mathbb{R}^{p \times 1}.
\]

\[
a = \begin{bmatrix} a_1 & a_2 & \cdots & a_p \end{bmatrix}, \ B = \begin{bmatrix} b_1 & b_2 & \cdots & b_p \end{bmatrix}, \ E = \begin{bmatrix} e_1 & e_2 & \cdots & e_p \end{bmatrix}
\]

\[
e_n = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}.
\]

Each row in the model describes an observation vector in the output vector as a linear function of the corresponding observation in the input vector corrected by a random deviation. Several assumptions have to be made in order to obtain the coefficients:

\[
E(Y) = XB;
\]

\[
\text{cov}(y_i) = \Sigma;
\]

\[
\text{cov}(y_i, y_j) = 0
\]

where \( y_i \) is the ith row of \( Y \). With these assumptions one states that a) the linear model is correct and that no additional input observations \( x \)'s are needed; b) each of the \( n \) observation vectors (rows) in \( Y \) has the
same covariance matrix and c) that the observation vectors (rows) are uncorrelated with each other. Thus the observation vectors are independent and have the same covariance matrix. That is we assume that the \( y_i \)’s within an observation vector are correlated with each other but independent in the \( y_i \)’s in any other observation vector [20,21]. Application of a least squares technique for each variable individually \( y_j \), leads to the estimation of the parameters \( \hat{a}, \hat{B} \) of the linear model which have the following properties: a) the estimator \( \hat{B} \) is unbiased (sampling the same population one would get the same average value for \( B \); b) the least squares estimates \( \hat{\beta}_j \) in \( \hat{B} \) have minimum variance among all possible linear unbiased estimators without requiring normality of the \( y_i \)’s (the Gauss-Markow theorem) and c) all \( \beta_j \) are intercorrelated with each other, the relationship of the \( x_i \)’s with each other affecting the relationship of the \( \beta_j \)’s with each other and since the \( y_i \)’s are typically correlated the \( \beta_j \)’s in different columns are correlated. A model corrected for means is obtained [20,21] by:

\[
\hat{B} = \begin{bmatrix} \hat{b}_1 & \hat{b}_2 & \cdots & \hat{b}_p \end{bmatrix} = S_{xx}^{-1}S_{xy};
\]

\[
a' = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \cdots & \hat{a}_p \end{bmatrix} = \frac{1}{n} e_n' [Y - X\hat{B}];
\]

The final model resulting from the multivariate multiple regression reads:

\[
\hat{Y} = e_n a' + X\hat{B}; a \in \mathbb{R}^{n \times p}; B \in \mathbb{R}^{p \times p}; x \in \mathbb{R}^{n \times q}; Y \in \mathbb{R}^{n \times p}.
\]

### 3 Equivalent spot-weld finite element model

A generally applicable and simple joint/interface element for 3D analysis is presented in [14]. The element can model interfaces between solid and shell finite elements. The formulation is quite general, the element has zero thickness and the derived element is isoparametric. On the contact surfaces of the two shell elements, the displacements at any point are given by:

\[
\begin{align*}
\{u\} &= \sum N_i \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}_{TOP} - \sum N_i \frac{d_i}{2} \begin{bmatrix} -v_{2i} \\ v_i \end{bmatrix}_{TOP} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix}_{TOP} \\
\{u\} &= \sum N_i \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}_{BOT} + \sum N_i \frac{d_i}{2} \begin{bmatrix} -v_{2i} \\ v_i \end{bmatrix}_{BOT} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix}_{BOT}
\end{align*}
\]

where the \( N_i \) are the isoparametric shape functions, the \( v_{il}, v_{2i} \) are the unit vectors in the direction of the local tangent axes, \( \alpha_i, \beta_i \) are the rotation angles around these axes and \( d_i \) is the thickness of the shell at each node.

These equations can be rewritten as:
The displacements in the directions normal and tangential to the contact surface are obtained by:

\[
\begin{bmatrix} u \\ v \\ w \end{bmatrix}_{TOP,BOT} = N_{TOP,BOT} a_{TOP,BOT} ;
\]

\[
a_{TOP,BOT} = [a_1, a_2, \ldots, a_n]_{TOP,BOT} ; \quad a_i = \begin{bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{bmatrix}.
\]

\[
N_{TOP,BOT} = [N_1, N_2 \ldots N_n]_{TOP,BOT} ; \quad N_i = -N_i[I, \frac{d_i}{2} v_{ij}, -\frac{d_i}{2} v_{ii}] ;
\]

The displacements in the directions normal and tangential to the contact surface are obtained by:

\[
\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & n \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}.
\]

where \( s_1, s_2, n \) are the tangent vectors and the normal vector to the surface area. The relative displacements at the surface are the slip in the two tangent directions and the convergence/separation:

\[
\begin{bmatrix} \delta_{s1} \\ \delta_{s2} \\ \delta_n \end{bmatrix} = \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}_{TOP} - \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}_{BOT}.
\]

Between the tractions acting on the contact and relative displacements the following relationships are defined:

\[
\begin{bmatrix} t_{s1} \\ t_{s2} \\ t_n \end{bmatrix} = \begin{bmatrix} k_{s1} & 0 & 0 \\ 0 & k_{s2} & 0 \\ 0 & 0 & k_n \end{bmatrix} \begin{bmatrix} \delta_{s1} \\ \delta_{s2} \\ \delta_n \end{bmatrix} ; t = D\delta
\]

where \( t_{s1}, t_{s2}, t_n \) are the shear tractions and contact pressure and \( k_{s1}, k_{s2}, k_n \) are the shear and normal stifnesses.

\section{4 Updating results}

Through a process of validation, one aims at determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model. In order to perform the updating of the spot weld models, a benchmark has been built, consisting of hat section steel plate and a flat plate joined together by spot welds at the flanges as shown in Fig. 1. This structure is designed to
represent simplified models of the beams used in the construction of car bodies, for example the roof pillars. The correctness and accuracy of the conceptual model of the spot-weld connections represented by an interface finite-element are assessed by vibration testing in free-free conditions. Considering the experimental results as a faithful reflection of the reality, a validation process should identify the errors and uncertainty in the conceptual computational model by comparing the computational and the experimental results.

![Fig. 1. Benchmark structure.](image)

The variability of the experimental pool was given by 10 similar benchmark structures and a modal test was carried out for determining the first 5 normal modes. After cutting and bending, all 10 plates and hats were measured, numbered and returned for spot-welding as numbered pairs. In order to isolate the error in the spot weld models from those in the rest of the benchmark structures, the plate and hat belonging to one specimen were tested before being welded together and the plates were updated separately before updating the complete benchmark structures so that the error in the welded structures should arise mainly from the spot weld models. This corresponds with the usual situation when a deterministic model updating is performed based on a building-block approach.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Test</th>
<th>FE</th>
<th>Error (%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.74</td>
<td>23.92</td>
<td>-0.76</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>65.95</td>
<td>66.28</td>
<td>-0.50</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>76.78</td>
<td>74.48</td>
<td>3.00</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>130.23</td>
<td>130.73</td>
<td>-0.38</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>156.14</td>
<td>151.94</td>
<td>2.69</td>
<td>99</td>
</tr>
<tr>
<td>6</td>
<td>216.33</td>
<td>217.02</td>
<td>-0.32</td>
<td>99</td>
</tr>
</tbody>
</table>

Table 1 Measured natural frequencies and finite element predictions (Hz) for the plate of specimen #1

## 4.1 Plate correlation

A roving hammer test with thirty hammer points and two measurement points was carried out and the identified modes, the frequency errors and MAC values are presented in Table 1. The plate model was corrected against the actual plate of specimen #1. The thickness of the plate was measured at different points on the plate span and an average value of 1.448mm was used in the finite element model. With a
mass and Young’s modulus correction the model was found to be very accurate for the bending modes and less accurate in torsion.

4.2 Hat correlation

A roving hammer test with 40 hammer points and 4 measurement points was carried out and the hat model was corrected using the first tested specimen. Measurement of the thickness at different points on the hat span showed a variation both along the span of the model and on the cross-section, the main differences appearing in the curved areas. The initial (design) curvature radius of 5mm was found to be approximately 4mm and this change in the finite element model improved the correlation considerably. The hat was divided in 5 regions (Fig. 2 a) and the thickness of these zones was used for model updating by minimising the frequency errors for the target modes. Due to the uncertainty related to the thickness variation a Floating-Point Genetic Algorithm [22] with 30 individuals and 50 generations was set-up for model optimisation and the frequency errors and MAC values for the best solution are presented in Table 2.

In the case of updating the two components alone, there is an aleatory uncertainty related to the dimensions and material properties of the shells especially in the curved areas of the hat and in the updating process one faces an epistemic uncertainty (ignorance or lack of knowledge about the system due to limited experimental data). It is also possible that some extension/shear/bending coupling effects are reducing the performances of the shell element used and this might represent a conceptual modelling uncertainty. Choosing the best solution based on the frequency distance for a single sample and using these component models for all 10 samples means that in the next step, the modeling errors related to this sample and not an average error, are propagated through the updating loop. The modification of the initial measured average thickness of 1.468mm for all the zones is presented in Table 3.

<table>
<thead>
<tr>
<th>Upper Curved Zone</th>
<th>Lower Curved Zone</th>
<th>Vertical Walls</th>
<th>Upper Plate</th>
<th>Contact Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.400</td>
<td>1.400</td>
<td>1.593</td>
<td>1.512</td>
<td>1.600</td>
</tr>
</tbody>
</table>

Table 3. Thickness (mm) of the hat zones after updating

Fig. 2. a) Hat finite element model parametrisation for model updating; b) GA convergence after 50 iterations.
4.3 Experimental modes for the welded structure

A roving hammer test with 70 hammer points and 4 measurement points was carried out on the first specimen assembled from the tested parts. For the other 9 specimens a reduced test was performed with only 2 hammer points and 4 measurement points. The natural frequencies were matched based on a reduced mode shape wireframe with the objective that mode swapping and any experimental uncertainty related to the mode type would be eliminated based on further tests if needed.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Exp</th>
<th>INITIAL</th>
<th></th>
<th>UPDATED</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FE</td>
<td>Error (%)</td>
<td>MAC</td>
</tr>
<tr>
<td>1</td>
<td>71.631</td>
<td>67.994</td>
<td>5.08</td>
<td>95</td>
</tr>
<tr>
<td>2</td>
<td>256.83</td>
<td>257.80</td>
<td>-0.38</td>
<td>93</td>
</tr>
<tr>
<td>3</td>
<td>275.23</td>
<td>274.00</td>
<td>0.45</td>
<td>94</td>
</tr>
<tr>
<td>4</td>
<td>338.37</td>
<td>345.80</td>
<td>-2.19</td>
<td>95</td>
</tr>
<tr>
<td>5</td>
<td>403.65</td>
<td>407.87</td>
<td>-1.04</td>
<td>94</td>
</tr>
<tr>
<td>6</td>
<td>631.01</td>
<td>632.06</td>
<td>-0.17</td>
<td>86</td>
</tr>
<tr>
<td>7</td>
<td>644.23</td>
<td>642.96</td>
<td>0.20</td>
<td>85</td>
</tr>
<tr>
<td>8</td>
<td>757.81</td>
<td>783.26</td>
<td>-3.36</td>
<td>95</td>
</tr>
<tr>
<td>9</td>
<td>783.77</td>
<td>794.68</td>
<td>-1.39</td>
<td>97</td>
</tr>
<tr>
<td>10</td>
<td>791.23</td>
<td>815.51</td>
<td>-3.07</td>
<td>92</td>
</tr>
<tr>
<td>11</td>
<td>982.23</td>
<td>995.68</td>
<td>-1.37</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 2. Measured natural frequencies and finite element predictions (Hz) for the hat of specimen #1.

The mode shapes of the test for the first specimen are presented in Fig. 3 and these were used later for MAC checks during the updating loop of the whole model. The natural frequencies for all the samples are shown in Table 4 where one can observe a standard deviation of about 1% for all the modes around the average for all the samples.
<table>
<thead>
<tr>
<th>SPEC No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>523.39</td>
<td>589.06</td>
<td>600.24</td>
<td>682.14</td>
<td>713.45</td>
</tr>
<tr>
<td>2</td>
<td>521.88</td>
<td>588.45</td>
<td>588.45</td>
<td>689.14</td>
<td>708.87</td>
</tr>
<tr>
<td>3</td>
<td>526.8</td>
<td>594.17</td>
<td>602.73</td>
<td>688.81</td>
<td>715.16</td>
</tr>
<tr>
<td>4</td>
<td>525.99</td>
<td>592.72</td>
<td>595.92</td>
<td>679.4</td>
<td>706.3</td>
</tr>
<tr>
<td>5</td>
<td>512.19</td>
<td>580.24</td>
<td>580.24</td>
<td>680.71</td>
<td>702.19</td>
</tr>
<tr>
<td>6</td>
<td>523.74</td>
<td>590.74</td>
<td>599.64</td>
<td>678.83</td>
<td>706.68</td>
</tr>
<tr>
<td>7</td>
<td>517.09</td>
<td>586.05</td>
<td>586.05</td>
<td>675.64</td>
<td>702.84</td>
</tr>
<tr>
<td>8</td>
<td>522.06</td>
<td>589.71</td>
<td>590.56</td>
<td>677.63</td>
<td>706.06</td>
</tr>
<tr>
<td>9</td>
<td>526.97</td>
<td>592.12</td>
<td>606.56</td>
<td>689.82</td>
<td>720.77</td>
</tr>
<tr>
<td>10</td>
<td>528.90</td>
<td>596.16</td>
<td>604.48</td>
<td>688.52</td>
<td>718.52</td>
</tr>
<tr>
<td>Mean</td>
<td>522.90</td>
<td>589.94</td>
<td>595.48</td>
<td>683.06</td>
<td>710.08</td>
</tr>
<tr>
<td>Std</td>
<td>5.04(0.96%)</td>
<td>4.49(0.76%)</td>
<td>8.76(1.28%)</td>
<td>5.45(0.80%)</td>
<td>6.50(0.91%)</td>
</tr>
</tbody>
</table>

Table 4. Natural frequencies for all the specimens (Hz).

Fig. 3. Welded structure modes for the first specimen.
4.4 Monte-Carlo simulation

A Monte Carlo simulation was carried up in order to validate the spot-weld model and determine its quantitative prediction capabilities. The backward problem requires reducing the output uncertainty by updating the statistical model through a comparison of the analytical and experimental frequencies.

The validation metric is based on the frequency error and depends in this case on the number of experimental replications of the first 5 modes reflecting the mean and variance of the data.

The Young’s modulus for the curved areas of the hat and the material constants (eq. 27) defining the properties of the interface elements (related to the sliding in tangential directions and to the displacement in the normal direction of the contact) were chosen as parameters which could explain the observed variability in Table. 4. The combination of these material properties can explain the uncertainty related to geometry of the specimens and the spot weld diameter, the location and the different material properties of the area around the nugget after welding respectively.

In order to quantify the input which explains a high percentage of the structure output variability, a cloud of models is created by varying the 5 material constants following a normal distribution. The assumption of normality for the input variables is dictated by the lack of experimental data for the geometry variation and for the spot-weld process (the interface element represents an equivalent model for the spot-weld and the material constants are not related directly to the actual characteristics of the spot-weld process). The

---

### Table 5. Frequency errors (%).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.2776</td>
<td>2.348</td>
</tr>
<tr>
<td>2</td>
<td>-7.7736</td>
<td>-2.020</td>
</tr>
<tr>
<td>3</td>
<td>-19.263</td>
<td>-0.499</td>
</tr>
<tr>
<td>4</td>
<td>-15.967</td>
<td>-0.666</td>
</tr>
<tr>
<td>5</td>
<td>-12.599</td>
<td>2.380</td>
</tr>
<tr>
<td>Mean</td>
<td>-11.776</td>
<td>0.308</td>
</tr>
</tbody>
</table>

---

**Fig. 5.** Analytical cloud and the target cloud in the output space of the first three frequencies (gradient-regression with euclidian distance)

a) initial b)after 10 iterations
simulation was run for 10 iterations and at each step the standard deviation of each material constant was considered to be 30% of the mean value.

The number of observations for the experimental and analytical models influences the size of the confidence ellipse and a stable shape is obtained by studying a $T^2$ statistic with the degrees of freedom $p, n_1 + n_2 - 2$. The stabilisation of the statistic $T^2$ is almost obtained for $n_2 = 200$ samples ($n_1 = 10$ experimental models) which becomes the dimension of the meta-model, to be displaced to the location in the output space given by the experimental data through the variation of the material constants.

The reduction of the distance between the models in the output space was achieved by reducing the Euclidian distance and the final results are shown in Table 5. The scatter plots of the initial and final cloud are presented in Fig 5. and the variation of the Euclidian distance and of the frequency errors in Fig.6. The material constants are initially at the value of the Young’s modulus of the steel components and the updated values are presented in Table 6. The strength of the relationship between input and output variables in the model has been measured by the Spearman correlation matrix and the analysis of the material coefficients shows the importance of the material constant related to the normal displacement between the welded shells. The material constant related to the sliding along the structure $k_{s2}$ is not modified because the target modes (Fig. 3) do not show deformations along the structure.

Fig. 6. Variation of the Euclidian distance and natural frequencies (gradient-regression with euclidian distance).

The analysis of the distance between the means can be performed by using the hypothesis testing and the concept of the confidence ellipsoid [8,23,24]. The two data sets are compared in order to check if the samples are from the parent population and a null hypothesis $H_0$ (equal means) is tested based on a $T^2$ statistic using the Mahalanobis distance. The norm of the observed difference between the means of the final analytical population and the experimental data is $d = |\bar{y}_\text{test} - \bar{y}_\text{ana}| = 24.65$, and the confidence region of $100(1 - \alpha)\%$ is bounded by a hyper ellipsoid with semi-axis equal with $c_i = \sqrt{\lambda_i T^2_{\alpha,p,n_1+n_2-2}}$

where $\lambda_i = \{298.64, 139.97, 1.773, 0.518, 0.042\}$ are the eigenvalues of the covariance matrix $S_\Delta = \left(\frac{1}{n_1} + \frac{1}{n_2}\right)S_{pl}$. With $T_{0.05,5,208} = 3.39$ the major semi-axis of the 95% confidence ellipsoid becomes 5.82. The distance between the two mean value of the populations can then be bounded by $|d| - 5.82 = 18.83 \leq |\delta| \leq |d| + 5.82 = 30.47$ showing that the two means are not equal.
The final model presents frequency errors which are comparable with the standard average of the test mean and consistent mode shapes (compared with the mode shapes of the specimen #1 in Fig. 3). An explanation for the fact that the distance between the two populations cannot be reduced further resides in the fact that the variation of the geometry for the specimens could not be measured and therefore was not taken directly into account in the optimization loop. The variation of the material constant for the spot-weld finite element model incorporated most of the uncertainty presented by all the specimens.

More results and an extended discussion of the statistical aspects related to this benchmark are presented in [25].

5 Conclusions

A stochastic model updating methodology is discussed and applied on a benchmark for spot-weld modelling. The spot-weld is modelled by using an interface zero-thickness element with the properties of a spring membrane at the contact between shell elements.

A hierarchical approach is used in the uncertainty study by analyzing different sources of uncertainty and their effect on model updating and error propagation through a finite element model in the case of multiple experimental data. A gradient-regression method based on the euclidian distance is used for displacing a meta-model in the output space. The results for a solution with final errors of the standard deviation magnitude for the experimental modes around the average, are presented in detail.

Acknowledgements

The research reported in this paper is supported by EPSRC grants GR/R26818 and GR/R34936.

References


