The stiffening effect of laminated rotor cores on flexible-rotor electrical machines

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ABSTRACT

Almost all large electrical machines have laminated rotors. Any one of these rotors is constructed as a stack of steel laminations pressed onto either a solid shaft or a shaft with arms extending from the shaft outer diameter to the inner diameter of the core. When the margin between running speed and a major critical speed is small, it is critical that the stiffening effect of the stack of laminations on the rotor should be understood. It is obvious that the laminated core will offer some additional stiffening but it will not be as stiff as a solid steel body of the same dimensions. This paper describes how this stiffening effect can be included in a rotordynamic model and how the relevant parameters can be determined from experimental work. One particular outcome is the recognition that the simple rotordynamic models using unbranched shaft elements with fixed section properties are not suitable for representing these rotors. The alternative branched model structure is discussed.

§1 INTRODUCTION

Define a flexible rotor machine as any rotating machine for which the mode-shape associated with one or more of the low critical speeds (in the order of running speed or below) involves significant storage of strain energy in the rotor. For flexible-rotor machines, any uncertainty in the stiffness of the rotor will be manifest as corresponding uncertainty in at least one of the low critical speeds. In electrical machines (particularly in large machines), it is commonly acknowledged (e.g. [1]) that the rotor core presents a problem to the rotordynamicist in that the stiffness added by the core is not easily defined with any certainty.

The rotors of large electrical machines normally carry a substantial laminated rotor core. Figures 1a and 1b illustrate typical cross-sections of induction machine rotors and laminated salient-pole rotors. The electrical conductors are shown in both cases. In conventional topologies for large machines, there are several motivations for the machine-design engineer to tend towards longer machines in preference to larger diameter ones. Not least among these is the desire to maximise the proportion of active copper. In the ever more common case where machine rotational speed is not prescribed, power is directly proportional to speed and speed is firmly limited by rotor diameter. The ultimate limitation on length in this case becomes rotor stability. The effects of the laminated core in adding mass and rotary inertia are
relatively simple to assess. The stiffening effects are not so easy. In the case of a short stack of laminations on a thin shaft, we would expect that the stack would undergo almost pure shear when the shaft beneath it had some curvature as Fig. 2a shows. In the case of a relatively long stack of laminations on a relatively thick shaft (or 'spider'), we would expect that the stack would undergo almost pure bending when the rotor centreline was bent as Fig. 2b shows.

In reality, the stacks are not convincingly at either extreme and a model is needed which is capable of recognising that both shear and axial compression/extension occurs. This paper provides such a model together with a methodology for fitting parameters to it. The objective is to provide a sound representation for the stiffening effect of laminated stacks on rotors for rotordynamic purposes.

§2 ELASTIC STRESS-STRAIN RELATIONS FOR LAMINATED STACKS

If a full finite-element model of a machine rotor were to be prepared, the laminated stack would be modelled as a continuum with known equivalent elastic properties. Several of the key parameters describing the elastic properties of a laminated stack are determined primarily by the average qualities of the interfaces between adjacent laminations. Others are determined primarily by the material from which the laminations are composed. Assume that this material is isotropic — although in reality it does have some slight directionality. Fig. 3 shows a cuboid within a laminated stack. The z axis is normal to the plane of lamination and the other two axes are orthogonal to that and to each other. These three axes are the principal axes of this orthotropic material. Accept the following definitions:

\[
\begin{align*}
E_{\text{steel}} & : \text{Young's modulus of the lamination steel} \quad \text{(N/m}^2) \\
\nu_{\text{steel}} & : \text{Poisson's ratio for the lamination steel} \quad () \\
x & : \text{Stacking factor for the laminations} \quad () \\
S & : \text{Mean shear flexibility per interface} \quad \text{(m/(N/m}^2)) = \text{(m}^3/N) \\
C & : \text{Mean compressive flexibility per interface} \quad \text{(m/(N/m}^2)) = \text{(m}^3/N) \\
N & : \text{Number of interfaces per unit length} \quad \text{(m}^3) \\
\end{align*}
\]

Strain in the z direction is the sum of two contributions: normal strain due to the interfaces and normal strain of individual laminations. From this:

\[
\varepsilon_{zz} = C N \sigma_{zz} + \frac{(\sigma_{zx} x - \nu_{\text{steel}} \sigma_{xx} - \nu_{\text{steel}} \sigma_{yy})}{E_{\text{steel}}} \quad (1)
\]

Evidently \( \varepsilon_{xx} \) — the normal strain in the x direction — is governed by

\[
\varepsilon_{xx} = \left( \frac{(\sigma_{xx} / x) - (\nu_{\text{steel}} \sigma_{yy} / x) - (\nu_{\text{steel}} \sigma_{yy})}{E_{\text{steel}}} \right) \quad (2)
\]

A similar expression is obtainable for \( \varepsilon_{yy} \) by juxtaposing x and y. The x-z cross-coupling term between normal strains is the same in (1) and (2) — as required by reciprocity. Equations (1) and (2) provide a complete set of normal-stress normal-strain relations for the material given the quantities defined above (1). The shear-stress shear-strain relationships are also simple.
\[
\gamma_{xy} = \frac{2(1 + \nu_{\text{steel}})}{x E_{\text{steel}}} \tau_{xy}, \quad \gamma_{yx} = \left(\frac{2x(1 + \nu_{\text{steel}})}{E_{\text{steel}}} + SN\right) \tau_{yx}, \quad \gamma_{xx} = \left(\frac{2x(1 + \nu_{\text{steel}})}{E_{\text{steel}}} + SN\right) \tau_{xx}
\]

With these formulae and some knowledge of parameters \(\{x, C, S, N\}\), the engineer can define an equivalent orthotropic material for use in finite-element models. Quantities \(x\) and \(N\) are straightforward to obtain. Typically, \(x\) is around \(\frac{t_{\text{lam}}}{(t_{\text{lam}}+20\mu m)}\) and \(N\) can be determined from the thickness of the laminations as \(N = \frac{x}{t_{\text{lam}}}\). The additional flexibilities per interface, \(C\) and \(S\), can be determined experimentally. Previous work done by Garvey [2],[3] on evaluating the equivalent elastic properties of laminated stacks has shown that the effect of lamination (where the individual thicknesses were around 0.65mm) has been to reduce Young's modulus (for axial compression extension) to values in the order of \(0.8 \cdot 10^9\) N/m\(^2\) and to reduce the shear modulus to values in the order of \(0.3 \cdot 10^9\) N/m\(^2\). This is corroborated by other related investigations. For example, Long et al [4], found that the axially-skew symmetric flexural modes of a laminated stator core were almost imperceptibly higher in frequency than the axially-uniform stator flexural modes indicating that the shear stiffness of the stack in the r-z and o-z planes is very small. Delves [5] notes one measured equivalent Young's modulus for axial compression of the stack of \(1.8 \cdot 10^9\) N/m\(^2\). We compute \(C = 834 \cdot 10^{15}\) m\(^3\)/N and \(S = 2.09 \cdot 10^{12}\) m\(^3\)/N. A wide range of different effective moduli is obtainable depending on different clamping pressures and surface treatments of the individual laminations [3].

Fig. 4 shows side-elevation and an oblique view of a one-twelfth portion of a finite-element model of a rotor comprising a core on a solid shaft. Dimensions are given in Fig. 4. The free-free natural frequencies of this rotor are computed for the above nominal values of \(C\) and \(S\) and for sub-multiples of these. The results appear in table 1 below.

<table>
<thead>
<tr>
<th>(\frac{C}{(834E-15\text{ m}^3/\text{N})})</th>
<th>(\frac{S}{(2.085E-12\text{ m}^3/\text{N})})</th>
<th>Res. Freq. #1 (Hz)</th>
<th>Res. Freq. #2 (Hz)</th>
<th>Res. Freq. #3 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>307.19</td>
<td>310.27</td>
<td>494.87</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>309.22</td>
<td>310.29</td>
<td>526.47</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>308.14</td>
<td>369.98</td>
<td>544.83</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>311.77</td>
<td>370.06</td>
<td>620.96</td>
</tr>
</tbody>
</table>

**Table 1** Variation of free-free rotor resonances with flexibility parameters, \(C\) and \(S\)

### §3 MODAL TESTS ON LAMINATED ROTORS

One approach to modelling rotors having laminated cores is obviously to employ full three-dimensional finite-element analysis using equivalent elastic parameters for the laminated stack as described above. This presupposes that these parameters are available for the particular lamination materials and conditions prevailing on the rotor in question. A much more practical alternative for a machine builder is to carry out tests on an existing laminated-core rotor and to determine the two unknowns, \(C\) and \(S\) from these tests by using a model in an inverse sense. Then a reliable mechanism for predicting the effect of changing length will be created. At the very minimum, the first two free-free (non-rigid-body) resonances of the rotor should be measured — since there are two quantities to identify. Preferably, more than two resonances would be recorded and measurements would be conducted on several...
different rotors in order to estimate scatter. Fig. 5 shows two mode-shapes obtained from a real free-free rotor on which such a test has been performed.

§4 INTERPRETING MEASURED RESULTS USING AN UNBRANCHED MODEL.

The operative values of $C$ and $S$ can be determined from the modal tests described above by using a finite-element model in an inverse sense. Table 2 below shows the sensitivity of the model described in section 2 above to changes of 1% increases in $C$ and $S$. The units of the sensitivity are simply Hz.

<table>
<thead>
<tr>
<th></th>
<th>1% Increase in $C.$</th>
<th>1% Increase in $S.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta f_{N1}(\text{Hz})$</td>
<td>-41.6E-3</td>
<td>-752E-3</td>
</tr>
<tr>
<td>$\Delta f_{N2}(\text{Hz})$</td>
<td>-0.19E-3</td>
<td>-281E-3</td>
</tr>
<tr>
<td>$\Delta f_{N3}(\text{Hz})$</td>
<td>-1.570</td>
<td>-563E-3</td>
</tr>
</tbody>
</table>

Table 2 Sensitivity of rotor resonances to stack flexibility parameters, $C$ and $S$

Table 2 demonstrates that the fitting problem is well conditioned – at least in the present case. A lesson clearly evident from Table 2 is that information from these resonances should be used. The first two resonance frequencies in this case are very insensitive to $C$.

Traditionally, a coarsely-discretised (shaft-line) model is used to represent a rotor wherein the instantaneous deflection of the rotor is described fully through specifying the translations and rotations at each one of a finite number of positions on the rotor centreline. Although the rotor model may contain a relatively small number of degrees of freedom (compared with a 3-D finite-element model for example) the properties of each finite length of rotor may have been updated or computed carefully in the first instance in order that the rotor model is actually a very effective reduced model almost equivalent to the finite element model. Fig. 6 shows schematically the set of degrees in such a model. Fig. 6 is an unbranched model in the sense that each plane has an elastic connection to the planes immediately before and immediately afterwards but not to any other planes. Many rotordynamic models are attempted using an unbranched structure. A major purpose of this paper is to show that such a model structure is inappropriate for laminated rotors.

Consider the reference model again. Suppose that we desire to represent it using an unbranched model and that each section of "shaft" in the unbranched model is described by four section properties. We denote these properties using combinations of letters which suggest their nature:

- $EI$ : Flexural Stiffness of the Shaft Length (Nm$^2$)
- $GA$ : Shear Stiffness of the Shaft Length (N/rad)
- $\rho A$ : Linear Density of the Shaft Length (kg/m)
- $\rho I$ : Diametral Inertia per unit length of the Shaft Length (kgm)

For the solid-shaft sections, the values of these section properties are well-defined. Note that for the tapered sections, some equivalent values can be chosen. For the core section of the rotor, we must ascribe suitable values. Obviously, the total mass and total diametral inertia of the rotor must be correct and arguably, the quantities $\rho A$ and $\rho I$ are not subject to any arbitrary adjustment. We would intuitively expect that $EI$ and $GA$ should both be larger than
the values corresponding to the solid shaft underneath the core but there is a surprise in store here. With control of only two parameters, we could expect to get only two resonances exactly right. Table 3 shows the section parameters used (all shaft elements in this model were exactly 0.05m long).

<table>
<thead>
<tr>
<th>Shaft Section No.</th>
<th>EI</th>
<th>GA</th>
<th>ρA</th>
<th>ρl</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-8</td>
<td>9.802E+005</td>
<td>5.339E+008</td>
<td>5.881E+001</td>
<td>3.823E-002</td>
</tr>
<tr>
<td>9</td>
<td>1.716E+006</td>
<td>7.133E+008</td>
<td>7.857E+001</td>
<td>6.690E-002</td>
</tr>
<tr>
<td>10</td>
<td>4.338E+006</td>
<td>1.147E+009</td>
<td>1.264E+002</td>
<td>1.692E-001</td>
</tr>
<tr>
<td>11-18</td>
<td>6.432E+006</td>
<td>1.402E+009</td>
<td>1.488E+003</td>
<td>2.322E+001</td>
</tr>
<tr>
<td>19</td>
<td>4.338E+006</td>
<td>1.147E+009</td>
<td>1.264E+002</td>
<td>1.692E-001</td>
</tr>
<tr>
<td>20</td>
<td>1.716E+006</td>
<td>7.133E+008</td>
<td>7.857E+001</td>
<td>6.690E-002</td>
</tr>
<tr>
<td>21-28</td>
<td>9.802E+005</td>
<td>5.339E+008</td>
<td>5.881E+001</td>
<td>3.823E-002</td>
</tr>
</tbody>
</table>

Table 3  Section Properties for the Simple Model of the Laminated Rotor.

Table 4 shows the progress of such a process. Two important findings emerge from this table.

- We will never be able to fit three resonances accurately – the structure of the model is simply not correct.
- Evidently the effective GA value for the core-carrying section of the rotor is smaller than the GA value for the shaft alone. This appears to imply that adding the laminated core to the rotor shaft reduces the stiffness of the central section in some particular way. This cannot be true. We resolve this apparent anomaly shortly.

<table>
<thead>
<tr>
<th>EI of core section</th>
<th>GA of core section</th>
<th>EI of shaft alone</th>
<th>GA of shaft alone</th>
<th>Res. 1. (Hz)</th>
<th>Res. 2. (Hz)</th>
<th>Res. 3. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>336.204</td>
<td>433.734</td>
<td>976.004</td>
<td></td>
</tr>
<tr>
<td>300%</td>
<td>40%</td>
<td>40%</td>
<td>321.721</td>
<td>428.300</td>
<td>853.866</td>
<td></td>
</tr>
<tr>
<td>200%</td>
<td>20%</td>
<td>20%</td>
<td>303.441</td>
<td>418.126</td>
<td>740.250</td>
<td></td>
</tr>
<tr>
<td>142%</td>
<td>14%</td>
<td>14%</td>
<td>302.631</td>
<td>413.391</td>
<td>644.701</td>
<td></td>
</tr>
<tr>
<td>145%</td>
<td>10%</td>
<td>10%</td>
<td>301.647</td>
<td>404.572</td>
<td>556.748</td>
<td></td>
</tr>
<tr>
<td>155%</td>
<td>6%</td>
<td>6%</td>
<td>299.675</td>
<td>366.318</td>
<td>474.685</td>
<td></td>
</tr>
<tr>
<td>175%</td>
<td>4%</td>
<td>4%</td>
<td>297.281</td>
<td>314.723</td>
<td>452.254</td>
<td></td>
</tr>
<tr>
<td>200%</td>
<td>4%</td>
<td>4%</td>
<td>302.284</td>
<td>315.096</td>
<td>453.631</td>
<td></td>
</tr>
<tr>
<td>230%</td>
<td>3%</td>
<td>3%</td>
<td>276.530</td>
<td>297.011</td>
<td>446.416</td>
<td></td>
</tr>
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</table>

Table 4  Results from a Small Number of Passes Attempting to Fit Simple Model.

(Resonances computed from 3D F.E.A were: [307.19, 310.27, 494.87] Hz)

Of course, there is no real requirement to fit the first three resonances of the free-free rotor well. The principal requirement is that some model should be available which can reasonably predict the first critical speed of the rotor. Since we are dealing primarily with "flexible rotor" machines as defined earlier, we could be satisfied with a model able to estimate the first resonance of the pinned-pinned rotor where the pin-joints are located at the bearing centres. However the following major shortcoming applies to the use of the simple model.
If a simple unbranched model is made to fit a given laminated rotor well by judiciously selecting the section parameters for the lengths carrying the core, a different set of section parameters will be found if the length of core is different.

Since a major motivation for fitting the model is to enable consideration of designs with longer rotor core-lengths, the above shortcoming is a serious one.

§5 THE STIFFNESS MATRIX FOR A FINITE LENGTH OF SHAFT.

The (4x4) stiffness matrix, \( K \), for a symmetric finite length, \( L \), of free shaft can be written in terms of two angular stiffness, \( K_S \) and \( K_A \). Fig. 7 shows the convention used here for the motions at the ends (in a single plane). The convention for the forces and moments acting at the two ends is implicit from this. By symmetric in this case, we mean a shaft which has the same section properties at positions \( 2 \) and \( (L-2) \). We do not provide a derivation here but Fig. 7 allows some insight into the fact that the stiffness matrix obviously has two zero singular values (corresponding to rigid-body deflections) and it can thus be composed from two rank-1 components. Define

\[
\nu_s = \begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix}^T \\
\nu_A = \begin{bmatrix} \frac{2}{L} \\
1 - \frac{2}{L} \end{bmatrix}^T
\] (4)

These deflection vectors correspond to deflection shapes 3 and 4 in Fig. 7. With obvious definitions for the angular stiffness \( K_S \) and \( K_A \), we have

\[
K = \frac{1}{2} \left[ K_S (\nu_s^T \nu_s) + K_A (\nu_A^T \nu_A) \right]
\] (5)

In the case of a uniform length of shaft whose elastic behaviour is characterised by the section constants \( EI \) and \( GA \), \( K_S \) and \( K_A \) are given by

\[
K_S = \frac{2EI}{L}, \quad K_A = \left( \frac{GAL}{2} \right)^{-1} + \left( \frac{6EI}{L} \right)^{-1}
\] (6)

Equations (4), (5) and (6) are consistent with the standard definitions of stiffness matrices for uniform shaft-lengths (with no shear-deformation) as given in [5], for example. We shall use (4), (5) again in the following section. The following is evident from (5) and (6).

It is possible that for a given length, \( L \), of shaft, section property \( GA \) can be reduced without the static stiffness of the shaft length being decreased in any way provided that a sufficient increase is made in the other section property, \( EI \). Usually, \( GA \) contributes only a very small amount to the flexibility of a length of shaft and the definition of stiffness \( K_A \) in (6) can be seen as the inverse of the sum of two flexibilities – one due to \( GA \) and one due to \( EI \). Table 4 suggested that it was necessary to multiply the shear flexibility by a factor in the order of 30 to fit the resonances of a simple model to the resonances computed from a detailed finite-element model. This action does not reduce the static stiffness of the shaft in any way when it is coupled with the increase in \( EI \) also suggested in Table 4. This apparent anomaly is resolved.
Figure 8 shows a conventional unbranched rotor model (having 4 degrees of freedom per plane) carrying what we shall call *inertia-rings*. There are eight of these inertia-rings in the model. Each one appears above and below the rotor axis in the section. Each inertia-ring, with the exception of the two at the ends, is elastically connected to both a shaft-station beneath it and to its two neighbouring inertia-rings. This model is *branched* in the sense that forces and moments can be transmitted along the central portion partly through the rotor shaft and partly through the subsystem of inertia-rings and associated elastic connections.

The inertia rings represent the bulk of the mass of the laminated core. Since the core is invariably prismatic, each inertia ring will have identical mass and inertial properties. Similarly, each one of the elastic connections between adjacent inertia-rings will have identical properties and each of the elastic connections between the shaft centreline and the inertia ring will be identical to each other one. Note that all of these elastic connections are capable of reacting angular motion as well as transverse motion. The elastic connections between adjacent inertia-rings act like shaft elements in the sense that radial motion of one end of the connection relative to the other causes a moment at each end and angular motion of one end relative to the other requires lateral forces to act at the two ends. It is sensible to characterise each one of these (ring-to-ring) connections as a shaft-element whose stiffness is determined by specifying the scalar angular stiffnesses $K_S$ and $K_A$ though (4) and (5).

In attempting to fit a model of this structure to a set of measurements, an initial decision is to choose how many discrete inertia rings will be used to represent the rotor core. It is straightforward to ensure that a sufficient number has been chosen to represent the rotor behaviour accurately up to the highest frequency of interest. The values of mass, polar-inertia and diametral inertia for the inertia-rings are easily computed. It then remains to assign values to four separate stiffness quantities:

- $K_S$ and $K_A$ to define a stiffness matrix for the ring-to-ring connections using (4) and (5)
- Independent angular stiffness and translational stiffness for the ring-to-shaft connections.

We may reasonably assume that the translational stiffness of the ring-to-shaft connections is sufficiently high that each ring undergoes almost exactly the same transverse translation as the shaft beneath it. This can either be built into the model as a constraint or a suitably high value can be used for this stiffness – taking care not to cause numerical problems in the model by using an excessively high number. Thus the number of unknown elastic parameters is effectively three. We denote the angular stiffness of the connection between any inertia-ring and the shaft beneath it as $K_X$.

We can make initial estimations of these parameters based on the flexibility parameters, $C$ and $S$ characterising the laminated stack as a continuum. For the simple rotor examined already in this paper, we can now examine how successfully this model structure can fit the results from the finite-element model. Table 5 contains results from a small number of trial analyses using the branched model.
Table 5 shows that $K_d$ is by far the most significant parameter. For this rotor, the other quantities play only a very weak role. The value of $K_d$ which reproduces the most representative sets of resonance frequencies is nearly twice as large as our original estimate. The original estimate is dominated by the shear term of the expression in (6) and it was computed with no recognition that the laminations must deform substantially from plane in order to take up an angle which is not normal to the shaft centreline.

<table>
<thead>
<tr>
<th>$K_x$</th>
<th>$K_y$</th>
<th>Res 1 (Hz)</th>
<th>Res 2 (Hz)</th>
<th>Res 3 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>97.15E6</td>
<td>1.403E6</td>
<td>0.0</td>
<td>255.713</td>
<td>297.758</td>
</tr>
<tr>
<td>97.15E6</td>
<td>1.403E6</td>
<td>0.5E6</td>
<td>257.730</td>
<td>297.780</td>
</tr>
<tr>
<td>120.0E6</td>
<td>1.403E6</td>
<td>0.5E6</td>
<td>257.776</td>
<td>298.061</td>
</tr>
<tr>
<td>120.0E6</td>
<td>1.55E6</td>
<td>0.5E6</td>
<td>264.563</td>
<td>298.245</td>
</tr>
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<td>0.5E6</td>
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<td>2.5E6</td>
<td>1.0E6</td>
<td>301.538</td>
<td>317.767</td>
</tr>
</tbody>
</table>

Table 5 Results from a Small Number of Passes Attempting to Fit Branched Model.
(Recall, resonances computed from 3D F.E.A: [307.19, 310.27, 494.87] Hz)

The matches to the resonance frequencies produced by the detailed finite-element model are not significantly better in the present case than those produced using the unbranched model. The difference is that the branched model should extrapolate well for all lengths of laminated core whereas the simple unbranched model would have to be fitted again for each different core length.

The unbranched model could be tuned considerably more. The results quoted in Table 5 above were found by attributing all of the mass and inertia of the laminated core to the inertia-rings. Intuitively, we would expect that some proportion of the mass of the laminated core should be constrained to move with the rotor shaft – perhaps the innermost one-third of the radial thickness of the laminated core.

§7 DISCUSSION AND CONCLUSIONS

We have observed in this paper that the dynamic behaviour of machine rotors is strongly influenced by the elasticity parameters describing the stack of core laminations. We have described such parameters and given typical values. Representative values have been utilised in a detailed finite-element model of a simple rotor to produce predicted free-free resonances and to demonstrate the sensitivity of the overall rotor stiffness to these parameters.

An equivalent unbranched model for a laminated rotor was subsequently considered and it was found that by selecting appropriate cross-section parameters, $EI$ and $GA$, for the lateral stiffness of the central portion of the rotor, the free-free resonances can be matched to some extent. An interesting outcome of this is that the resulting $GA$ parameter transpires to be much smaller than the original value. This anomaly is explained by viewing the stiffness of a finite length of shaft from a particular useful perspective.
An equivalent branched model for the general class of laminated rotors from electrical machines was then proposed. When a fit to the resonance data was attempted with this, the fit was still surprisingly poor. However, the branched model has the advantage of being scaleable according to the length of core whereas the unbranched model does not scale. Order-of-magnitude estimates for the connection stiffnesses in the branched model were shown to be obtainable using direct computation based on the elastic parameters for the laminated stack.

The same difficulty arose in fitting both the equivalent branched and the equivalent unbranched models to the resonance data given by the finite-element model – namely that the first two resonances from the F.E. model are significantly closer together than appears to be possible from either of the equivalent models. This may be connected with the representation of the tapered section of shaft on either side of the core-carrying section and as such, quite unconnected with the modelling of the core-carrying section.

The rotor model examined up to now has been a trivially simple example for illustrating the effects of laminated cores on shaft dynamics and the modelling thereof. In the introduction and in Fig. 1, we observed the usual structure of rotors of electrical machines having laminated cores. In the case of laminated-rotor synchronous (or reluctance) machines, the laminated core is not geometrically similar to a cylinder but it is nevertheless possible to model such rotors using the branched model structure having inertia-rings as described above.

Experience has shown that the connection stiffnesses operative in a branched model for an induction machine will be strongly influenced by the presence of rotor bars. These act to increase significantly the compression-extension stiffness of the stack in the axial direction and have little effect on the shear stiffnesses.

Rotor damping has not been touched on in the paper and yet this is potentially an area of considerable importance. It is known that damping in the rotor can be destabilising. Flexible laminated-rotor machines store a substantial proportion of their peak strain energy in the laminated core where damping is relatively high. Further attention is needed in this area.

REFERENCES:


Figure 1. Cross-sections of laminated-rotor induction machine and synchronous machine.

Figure 2. Two extremes of possible deflection of rotor core:
(a) pure flexure, (b) pure shear.

Figure 3. Elemental cuboid of laminated stack.

Figure 4. Side Elevation and Oblique View of Example Rotor.

Figure 5. Measured Mode Shapes (Mass Normalised) from a 1.8m long Rotor (Laminated Core from 0.5m – 1.15m).

Figure 6. The Degrees of Freedom in an Unbranched Rotor Model.

Figure 7. Four Independent Deflection Patterns for a Shaft Length.

Figure 8. Schematic of an Appropriate Branched Model.
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