THE VIBRATION SIGNATURE OF CHORDAL CRACKS IN ASYMMETRIC ROTORS

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ABSTRACT

The study of asymmetric rotors is important because virtually all electrical machines have rotors which fall into this category and their have been a number of cracks in such rotors which have been reported in the literature. In a symmetric rotor, the diagnosis of a transverse crack from an examination of the free-free modes is relatively straightforward, but the presence of inherent asymmetry leads to a considerable complication of the situation. In this paper, a simple model is presented to represent this situation. The changes in natural frequencies, modes shapes and cross coupling of the original principal planes is examined.

1. INTRODUCTION

One method to establishing the integrity of a machine rotor is to study the free-free modes. This can be achieved in practice by suspending the rotor from a crane via a suitable cradle. The rotor is excited by means of a hammer or shaker applied horizontally and in this case the constraint due to the stiffness of the supports may be considered as minimal. For a symmetric shaft with no defects, clearly the natural frequencies measured will not depend on the orientation of the rotor.

If this symmetrical shaft now has a transverse crack, the stiffness of the body will be a function of the orientation. When the orientation of the rotor is such that the crack is vertically above the neutral axis of the rotor, then self weight bending will tend to open the crack and hence the flexural rigidity will be reduced leading to lower values for the natural frequencies. Each mode will be influenced to a different degree depending on the location of the crack with respect to the corresponding mode shapes of the (un-cracked) rotor.

This provides a convenient way of determining the presence of a crack in a rotor. Excitation applied with the rotor in two positions differing by 180° will show clearly the effect of a significant crack being open or closed and this technique has been applied in a number of practical investigations, some of which have appeared in the literature. For an axi-symmetric rotor there are no preferred principal axes of vibration: any pair of orthogonal directions may be chosen to describe the motion. When a transverse crack is present, and is open, the direction of the crack front and the vector orthogonal to it define the principal co-ordinates of the system.

However, in electrical machines, the rotors do not have axial symmetry, typically deviating by a few per cent. Such an asymmetric structure will have two defined principal axes which are defined with respect to the rotor itself and any further asymmetry introduced by a transverse crack with give rise to coupling of the motion even in the (un-cracked) principal directions. This effect has led to some confusion in the analysis of vibration data and this is illustrated by reference [1]. The present paper seeks to describe the phenomena in some detail.

In section 2 the basic model of the crack is described. The analysis closely follows the approach of Mayes and Davies [1-3] which is in turn a development of a fracture mechanistic description first given by Paris and Sih [4]. In section three, numerical examples are given describing the natural frequencies of an asymmetric rotor and illustrating the change in these natural frequencies as the orientation of the rotor is varied. The analysis of these examples is somewhat complicated by the presence of the original asymmetry and this makes the technique difficult to apply in an inverse sense. However, this is of great practical importance since modal testing of the rotor is a common step in the diagnosis of a crack. In section four, a method is
presented which has been used to solve this inverse problem.

2. THE MODEL

The behaviour of a rotating cracked shaft received extensive discussion from 1975 onwards with notable early contributions from Mayes and Davies [1-3], Gasch [5] and Henry [6]. It was noted that, in the case of a horizontal rotor, the self weight bending plays a very important role in the dynamics. At any stage in the cycle, the crack may be open or closed - indeed if operating very close to a critical speed, the crack may remain open throughout a full shaft revolution if the unbalance is very high. However, for modest levels of unbalance and away from critical speeds, the equations governing the shaft motion may be linearised by observing that the bending moment due to self-weight bending will dominate over inertial terms. Therefore, those portions of the crack below the neutral axis at any instant may be considered to be open, whilst those above the axis will remain closed. It is assumed that only the open portions of the crack change the rotor's stiffness and hence this stiffness will be a function of the orientation of the rotor.

This is also true for the case where the rotor has been taken out of the machine for examination and hung (horizontally) from a cradle. There are two steps to be established in the methods: first the reduction in stiffness due to a given crack is established and second, the size of effective crack for each orientation is discussed.

In a very elegant analysis, Mayes and Davies [1] related the change in natural frequency to the crack depth and location. It was shown that

\[ \Delta \left( \omega^2_N \right) = -R y_N^* (s_c) \]

where

\[ y_N^* (s_c) = \left[ \frac{d^2 y_N}{dx^2} \right]_{x=s_c} \]

is the second derivative of the mode shape at the crack location \( s_c \), and \( R \) is a function of crack and rotor geometry. \( y_N \) is the \( N \)th mode shape.

In [2] the same authors extend their study using dimensional analysis to describe the stress concentration factor at the crack front. Their study leads to the simplified expression

\[ \Delta \left( \omega^2_N \right) = -4 \left( \frac{EI^2}{\pi r^3} \right) (1-v^2) F(\mu) y_N^* (s_c) \]

where \( \mu \) is the non-dimensional crack depth (relative to the local shaft radius), and \( F \) is a function independent of all other parameters and is a universal function for a given shape of crack. The other parameters in this equation are

- \( E \): Young's modulus
- \( I \): Second moment of area
- \( r \): Shaft radius
- \( v \): Poisson's ratio

In principle, the function \( F(\mu) \) can be derived from the appropriate stress concentration factor, if this is known. A different approach was taken in [2] where the authors inferred the values of \( F \) from a series of experiments with chordal cracks. It was shown that to a very good approximation this function is just equal to the fractional change in the second moment of area as measured at the crack face.

In performing this calculation, however, it is important to note that the second moment of area of the remaining portion of the face is referenced to the new geometric centre rather than the original one. Hence \( F \) is a very non-linear function of \( \mu \). Having established the change in stiffness and natural frequency arising from a crack, it is straightforward to represent these factors in a finite element model. In [2] it is shown that a crack may be represented by reducing the second moment of area of a single element by \( \Delta I \). Using a Rayleigh type of approach it can be shown that, if the second moment of area of the element concerned is \( I_0 \), then

\[ \frac{\Delta I/I}{1 - \Delta I/I} = \frac{r}{l_{el}} (1-v^2) F(\mu) \]

where \( l_{el} \) is the length of the section with reduced properties. Note that this parameter is at the discretion of the modeller within a reasonable range but the choice determines the value of second moment of area. For any given chordal crack, there are two values of \( F \) and two orthogonal directions in the plane of the crack. Hence the parameters of a representative model are now fully specified.

3. A NUMERICAL EXAMPLE

When the crack is fully open, it will have a chordal form and the corresponding properties are readily calculated from the above analysis. Figure 1 shows the relevant compliances. The appendix gives the detailed integrals which are needed.
For each orientation of the rotor, a suitable crack model must be derived and there are several steps to achieving this. For the rotor under non-rotating conditions it is reasonable to assume that the portion of a chordal crack lying below the neutral axis will be opened under the influence of self weight bending. The actual portion of the crack open under these circumstances will in general be a rather complicated shape having its own stress intensity factor. In general these factors are not known, but the details are not of great importance.

At each orientation of the rotor, for a given size of chordal crack, the area of open crack must be calculated. The direction of the weaker axis bisects the angle of the open crack portion. The compliance functions in the two orthogonal directions are then evaluated by assuming equivalence with the chordal crack of equal effective area. It is argued that this relatively simple procedure is sufficient to reflect the main physics of the situation. Whilst it could be argued that the equivalence should be based on second moments rather than area, it must be remembered that we do not have the appropriate stress concentration factors to use and this being the case, it is desirable to keep the calculation as simple as possible. The two extremes of crack open and closed are modelled correctly and to a large extent, precise evaluation of the intermediate values is not necessary. Figure 2 shows graphically the open and closed portions for a particular crack depth and orientation.

The example chosen to demonstrate the effect of the crack was based on a uniform circular rotor, 1 m in length with a diameter of 10 mm. Two disks, of diameter 75 mm and thickness 15 mm were mounted 400 mm from each end. The rotor is shown in Figure 3. The rotor was made asymmetric by milling flats onto both sides of the rotor, where the sector height removed was 2 mm. This was done between 100 mm and 300 mm from both ends. The rotor was then modelled using 21 standard Euler beam finite elements as shown in Figure 3.

A crack of dimensionless depth of 1.0 was added to the centre of the shaft. Note that this corresponds to a situation in which half the area of the cross-section has been lost to the crack. The orientation of the crack front is 45° to the principal axes. Figure 4 shows the variation of the decrease in natural frequency as the shaft is tested in different orientations for the symmetric rotor (that is before the flats are milled onto the shaft). It is clear that the crack is fully open at an orientation of 45° and fully closed at 225°. Figure 5 shows the variation in the frequency reduction of the asymmetrical shaft, and now the minimum natural frequency does not occur in the same place (45°) for all the modes.
Figure 4. The Change in Natural Frequencies due to a Crack in a Symmetrical Rotor for μ = 1.0

Figure 5. The Change in Natural Frequencies due to a Crack in an Asymmetrical Rotor for μ = 1.0

For the uncracked shaft the modes are either vertical or horizontal (aligned with the principal axes), and these modes do not couple. The crack has introduced coupling between the vertical or horizontal directions. Figures 6 to 11 show the first six modes of the cracked asymmetric shaft. The left plots show the side views of the modes in the vertical and horizontal planes, and the right plot shows an end view. It is clear that the orientation of the modes has changed from the purely vertical or horizontal directions for the uncracked shaft. However it seems as though the mode vibrates in a tilted plane. This is approximately true, and some out of plane motion is just visible in modes 5 and 6. Modes even higher in frequency display this out of plane motion even more clearly, since local stiffness asymmetry becomes more important.

The third and fourth modes remain horizontal and vertical, since these modes have a node at the crack location and are therefore unchanged.
Suppose the non-dimensional crack depth is now reduced to 0.5. Figures 12 and 13 show the variation in natural frequency for this case, and are equivalent to Figures 4 and 5. Apart from the fact that the change in natural frequencies is now much less, the crack is also completely open and closed for ranges of shaft orientations. With the non-dimensional crack depth equal to 1 (Figures 4 and 5) there is only one orientation where the crack is completely open, and one orientation where the crack is completely closed.

Figure 11. Sixth Mode of the Cracked Asymmetrical Rotor

4. CONCLUSIONS

The problem of interpreting vibration data for an asymmetric rotor has been discussed and it is shown that with care, simple data of resonant frequencies in a single direction can be used to infer the state of the rotor under investigation. The way in which a transverse crack influences the dynamics the rotor is rather complicated and it has been shown that this leads to cross coupling of the principal planes. Indeed it is this factor which has, on a number of occasions lead to difficulties of interpretation.

Although not investigated in the present work, given accelerometers placed on both principal planes, a measure of the observed cross coupling could be a valuable source of insight into the state of the rotor.

Figure 12. The Change in Natural Frequencies due to a Crack in a Symmetrical Rotor for μ = 0.5

Figure 13. The Change in Natural Frequencies due to a Crack in an Asymmetrical Rotor for μ = 0.5

5. ACKNOWLEDGEMENTS

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6. REFERENCES


## Appendix

### Basic Geometric Parameters for the Calculation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Integral</th>
<th>Result</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Crack half angle</td>
<td>$\frac{\alpha}{2} \int r^2 \sin^2 \theta d\theta$</td>
<td>$\cos^{-1} (1 - \mu)$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Sector area</td>
<td>$\int_0^{\alpha} 2r^2 \sin^2 \theta d\theta$</td>
<td>$\alpha - (1 - \mu) \sqrt{1 - (1 - \mu)^2}$</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>Centroid</td>
<td>$\frac{I_{x1}}{A_s}$</td>
<td></td>
</tr>
<tr>
<td>$I_{x1}$</td>
<td>First moment of area</td>
<td>$\int_0^{\alpha} 2r^3 \sin^2 \theta \cos \theta d\theta$</td>
<td>$\frac{2}{3} \left( 2\mu - \mu^2 \right)^{3/2}$</td>
</tr>
<tr>
<td>$I_{x2}$</td>
<td>Second moment of area</td>
<td>$\int_0^{\alpha} 2r^4 \sin^2 \theta \cos^2 \theta d\theta$</td>
<td>$\frac{r^4}{4} \left( \alpha - \frac{\sin 4\alpha}{4} \right)$</td>
</tr>
<tr>
<td>$I_{y1}$</td>
<td>First moment of area</td>
<td>$\int_0^{\alpha} 2r^4 \sin^4 \theta d\theta$</td>
<td>$0$</td>
</tr>
<tr>
<td>$I_{y2}$</td>
<td>Second moment of area</td>
<td>$\int_0^{\alpha} \frac{2}{3} r^4 \sin^4 \theta d\theta$</td>
<td>$\frac{1}{4} \left( \frac{3}{2} \alpha + \sin 2\alpha + \frac{\sin 4\alpha}{8} \right)$</td>
</tr>
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From these basic geometric parameters, the moments about the centroids of the crack region and the remaining portion of the rotor are readily calculated.